Operating Window for Channel-Assisted Pinch Propagation

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Operating window

Limits for

- chamber pressure
- chamber length
- relative density reduction

Reasons:

- energy loss of the ion beam in the channel
- channel clearance by $\mathbf{j} \times \mathbf{B}$ force
- prevent breakdown to the walls $\rightarrow n_{ch}/n_0$

Energy loss - Basics

Stopping power $\rightarrow$ Bethe formula

$$\frac{dE}{dx} = -\frac{1}{4\pi\epsilon_0}\left(\frac{Z_{\text{eff}}\; e\; \omega_p}{\beta\; c}\right)^2 \ln\left(\frac{2m_e(\beta c)^2}{I}\right)$$

Plasma frequency

$$\omega_p = \sqrt{\frac{n_{\text{Xe}}\; Z_{\text{Xe}}\; e^2}{\epsilon_0\; m_e}} \rightarrow \frac{dE}{dx} \propto n_{\text{Xe}}$$

$\rightarrow$ upper limit for chamber pressure and length

Charge state of the ion beam

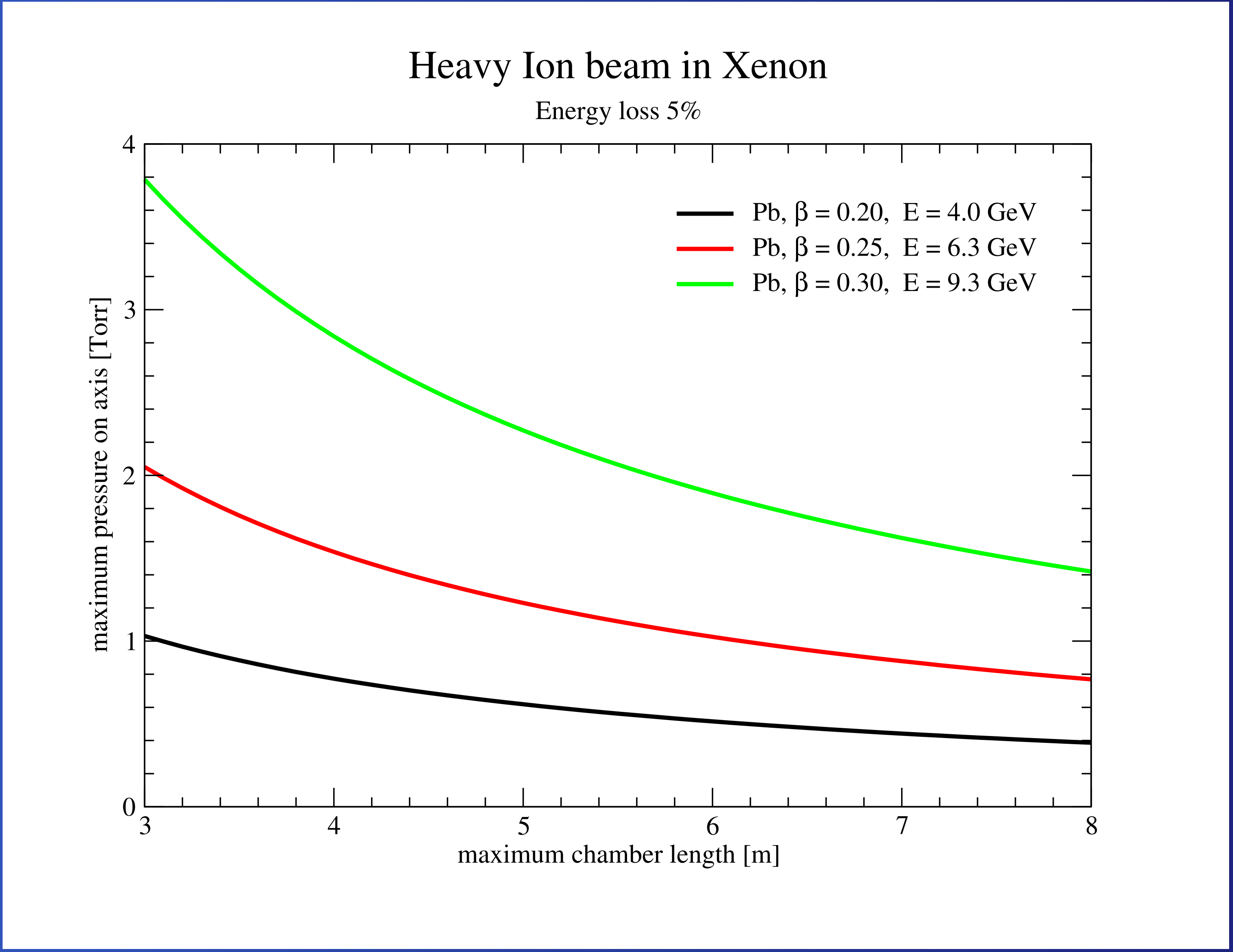
Estimated with empirical Betz formula

$$Z_{\text{eff}}(\beta) = Z_{\text{ion}} \left(1 - \exp\left[-0.555 \left(\frac{137\ \beta\ c}{Z_{\text{ion}}^{0.517}} \right)^{1.175} \right] \right)$$

$\rightarrow$ charge state of Pb ($\beta = 0.2$, 4 GeV)

$$Z_{\text{eff}} = 69.3$$

Energy loss - results



Channel clearance

Ion beam induces return current $\rightarrow \mathbf{j} \times \mathbf{B}$ force

$$\rho \frac{d\mathbf{v}}{dt} = -\nabla p + \mathbf{j} \times \mathbf{B}$$

$\rightarrow$ lower limit for gas density on axis

Estimate: neglect pressure term

$$\Delta x = \left(\frac{\mu_0 \, I_{\text{beam}} \, I_{\text{net}}}{4\pi^2 R_0^3 \, \rho} \right) \, t^2$$

Lower bound for density

Example:

$I_{\text{net}} = 50\,\text{kA}$, $I_{\text{beam}} = 3\,\text{MA}$, $R_0 = 5\,\text{mm}$, $t = 10\,\text{ns}$, $\Delta x \leq 1\,\text{mm}$:

$$\rho \geq 3.8 \cdot 10^{-3}\,\text{kg/m}^3\ (0.5\,\text{Torr})$$

Better calculation:

- expansion slowed down by wall of increased gas density $\rightarrow$ smaller limit
- hydrodynamical simulation necessary

Breakdown

Breakdown condition for xenon:

$$(E/p)_{\text{Xe}} = 60\,\text{V}/(\text{cm Torr})$$

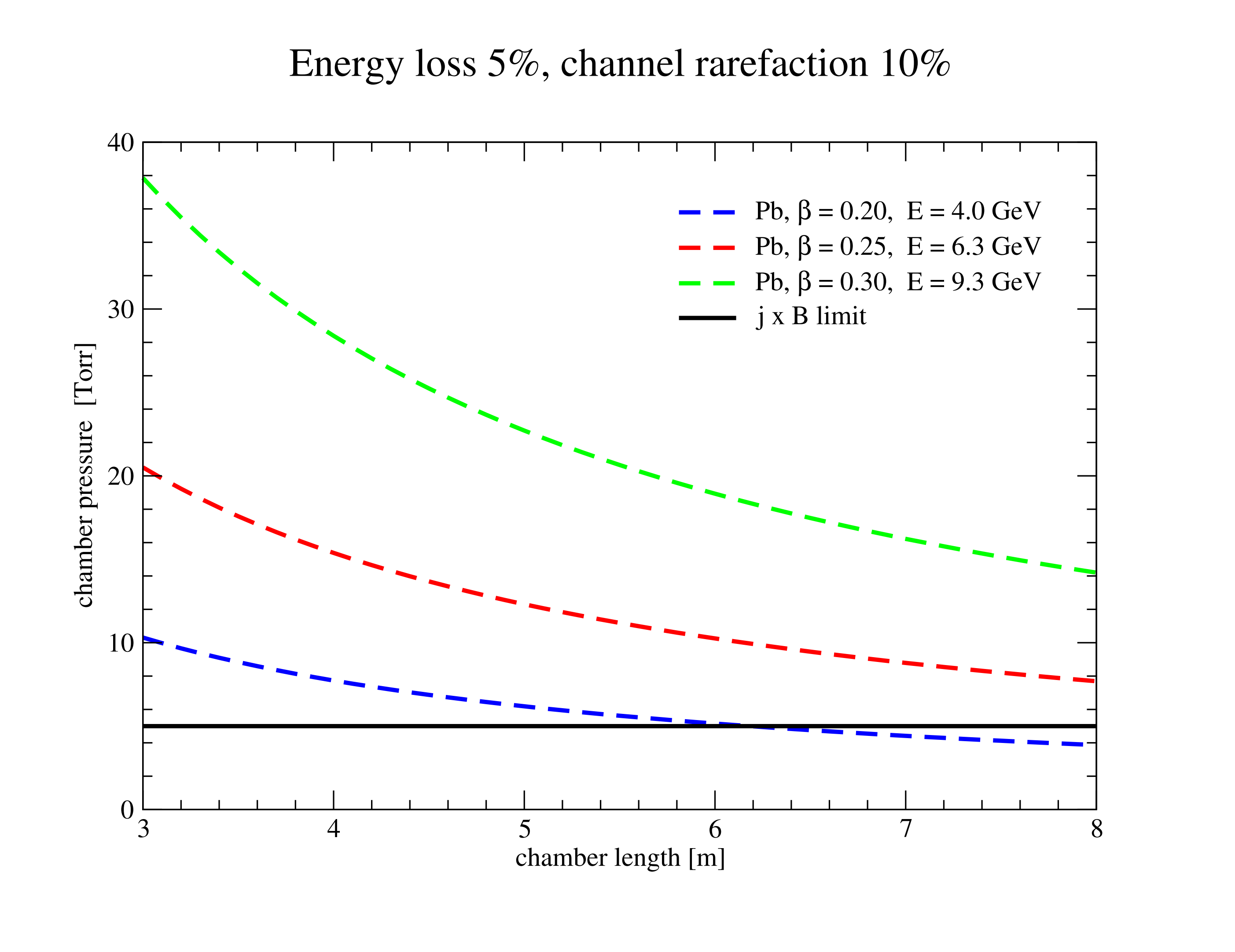
Prevent breakdown to chamber wall:

$(E/p)_{\text{channel}} > (E/p)_{\text{Xe}}$ and $(E/p)_{\text{chamber}} < (E/p)_{\text{Xe}}$

$\longrightarrow$ criterion for rarefaction:

$$\frac{n_{\text{channel}}}{n_{\text{chamber}}} < \frac{R_{\text{port}}}{R_{\text{chamber}}} \approx 0.1$$

Resulting operating window



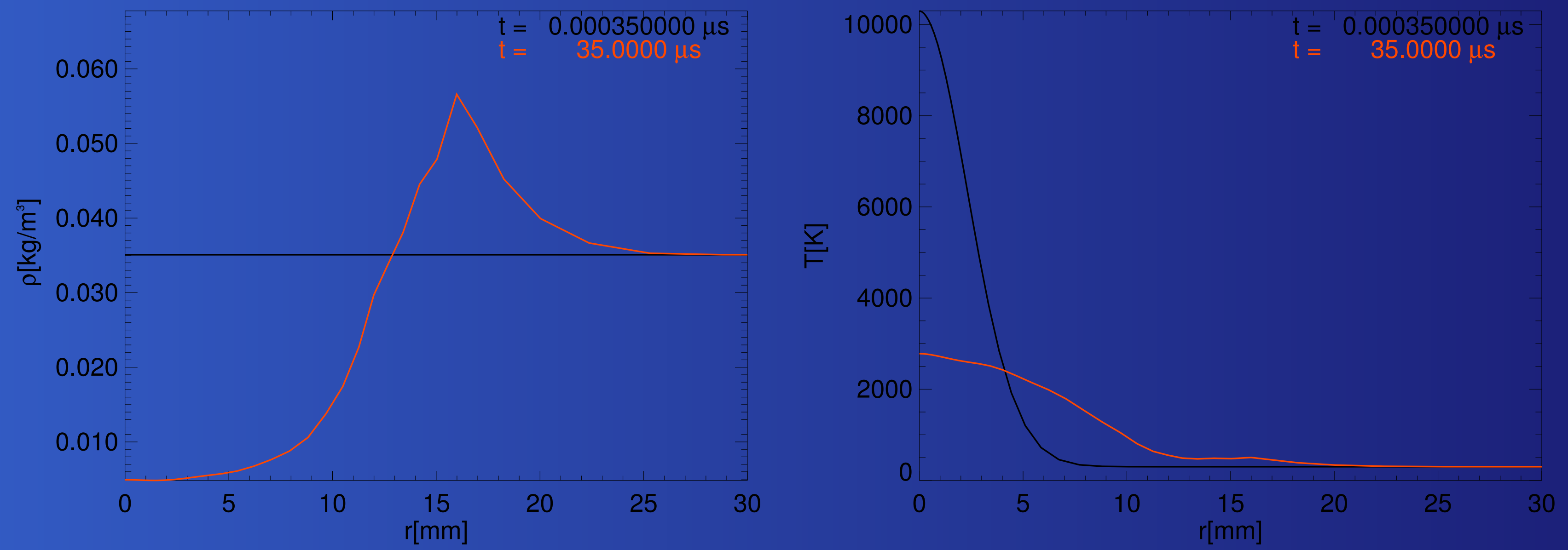
Simulation

Critical point: rarefaction on axis achievable ?

- hydrodynamical simulation code Cyclops
  - cylindrical symmetry $\rightarrow$ one spatial dimension
  - lagrangian formulation
  - thermal conductivity and other properties calculated with fits to measured values
- compare with measurements and analytic models

Simulation results

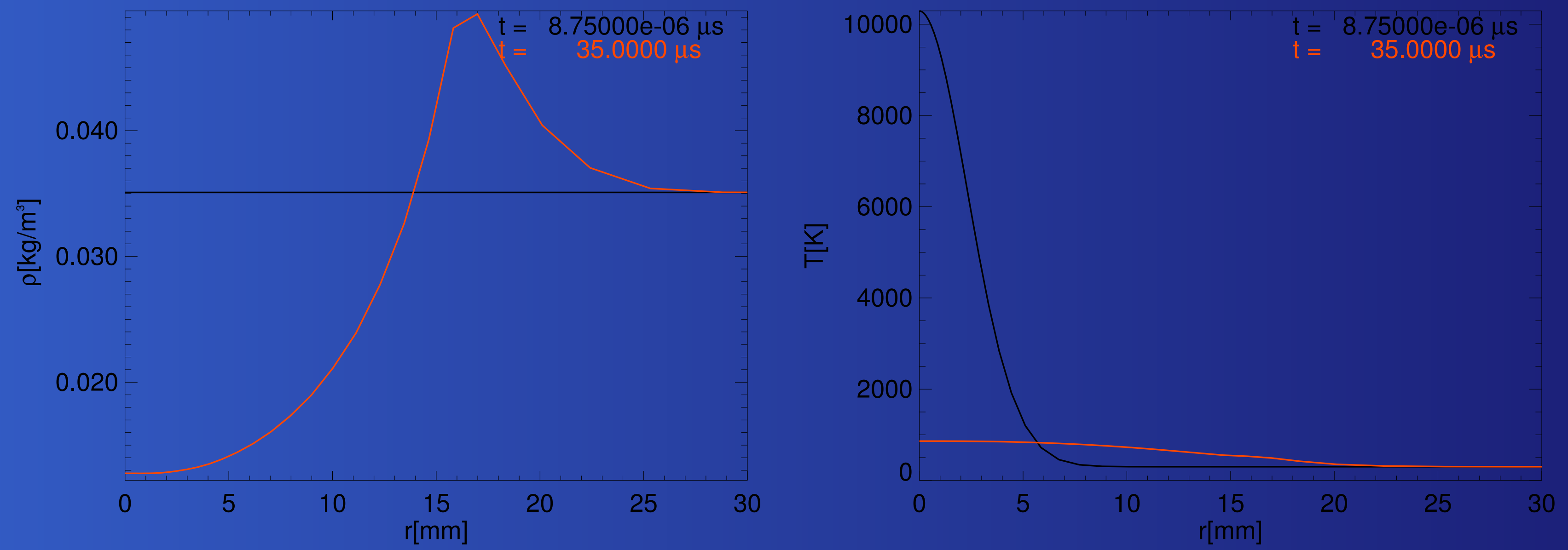
5 torr xenon, no thermal conduction



10,000 K lead to $n_{\mathrm{ch}}/n_0 = 14\%$

Simulation results II

5 torr xenon, with thermal conduction



10,000 K only lead to $n_{\text{ch}}/n_0 = 34\%$

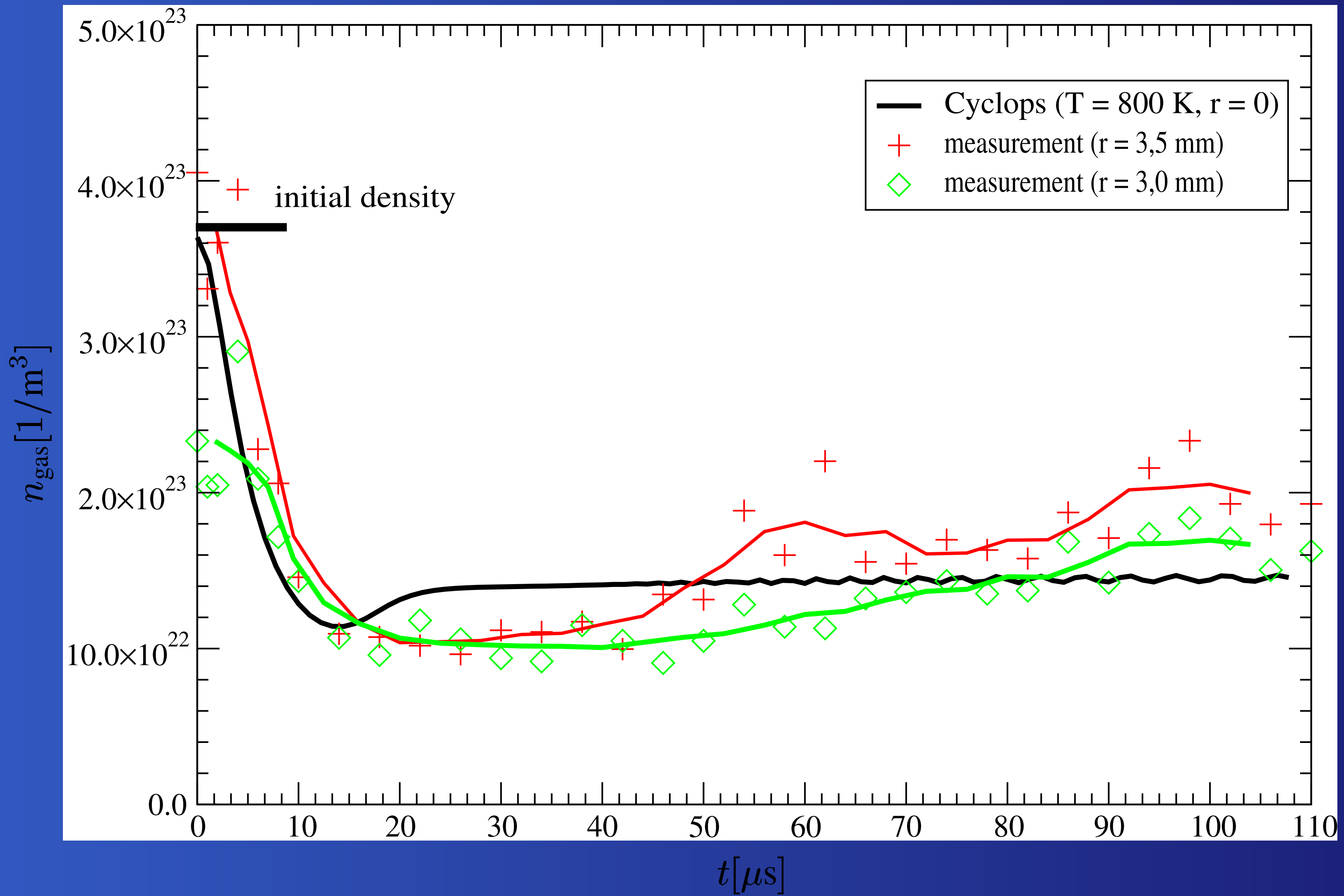
$\rightarrow$ effect of thermal conduction too large

Density measurements

Measurements were performed at GSI z-pinch experiment

- density was determined by the scattering of a heavy ion beam
- applicable to hydrodynamic expansion but not for discharge
- experiments with ammonia: good agreement between measurement and simulation
- could be applied for xenon discharge

Measurements in ammonia



Analytical channel model

Assumptions

- starting with continuity equation and force equation
- homogeneous current density inside channel
- surrounding gas is neglected

Averaged values

Calculate moments $\rightarrow$ integrate over radius

- mass line density

$$N = 2\pi \int r \, \rho \, dr = \pi R_0^2 \, \rho_0$$

- averaged radius

$$R^2 = \langle r^2 \rangle = \frac{2\pi}{N} \int r^3 \, \rho \, dr$$

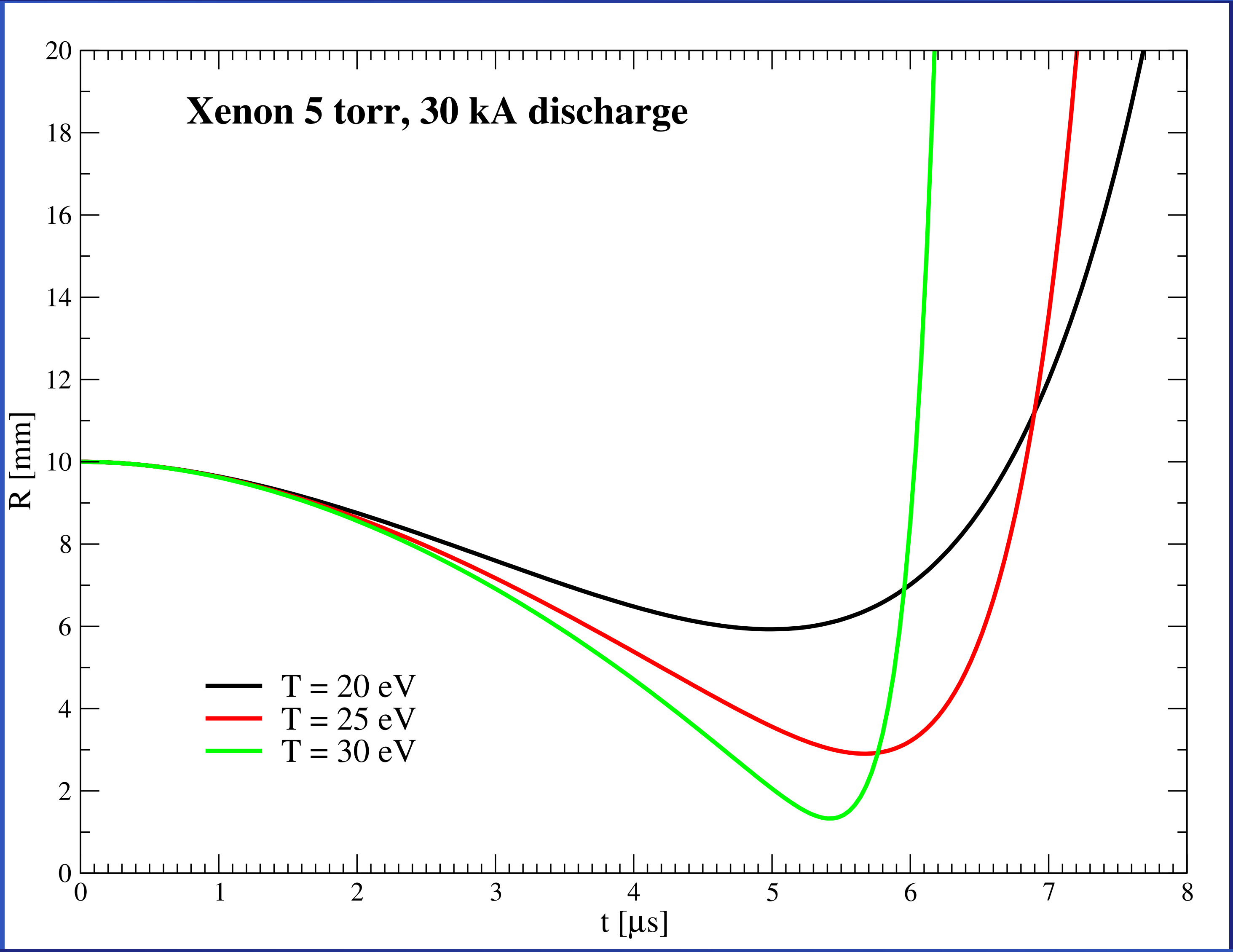
Envelope equation

Integrating the force equation yields

$$\frac{d^2R}{dt^2} = \frac{c_1}{NR^3} - \frac{c_2}{N}\frac{I^2}{R} + \frac{c_3 R}{N}\left(\int_0^t \frac{I^2(\tau)}{R^4(\tau)} d\tau\right)$$

- can be solved numerically (for constant current)
- compare with calculated moments from Cyclops runs (yet to be done)

Example – varied conductivity



Conclusions/Outlook

- calculations indicate good operating window
- MHD simulation of channel clearance necessary
- gas rarefaction is a critical point:
  - model for thermal conduction needs to be checked
  - analytical model useful for comparison
  - comparison with measurements