STELLARATOR REACTOR OPTIMIZATION AND ASSESSMENT

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TOPICS

- Stellarator Reactor Optimization
- 0-D Spreadsheet Examples
- 1-D POPCON Examples
- 1-D Systems Optimization with Self-Consistent Electric Fields and Fueling
- Suggested Approach

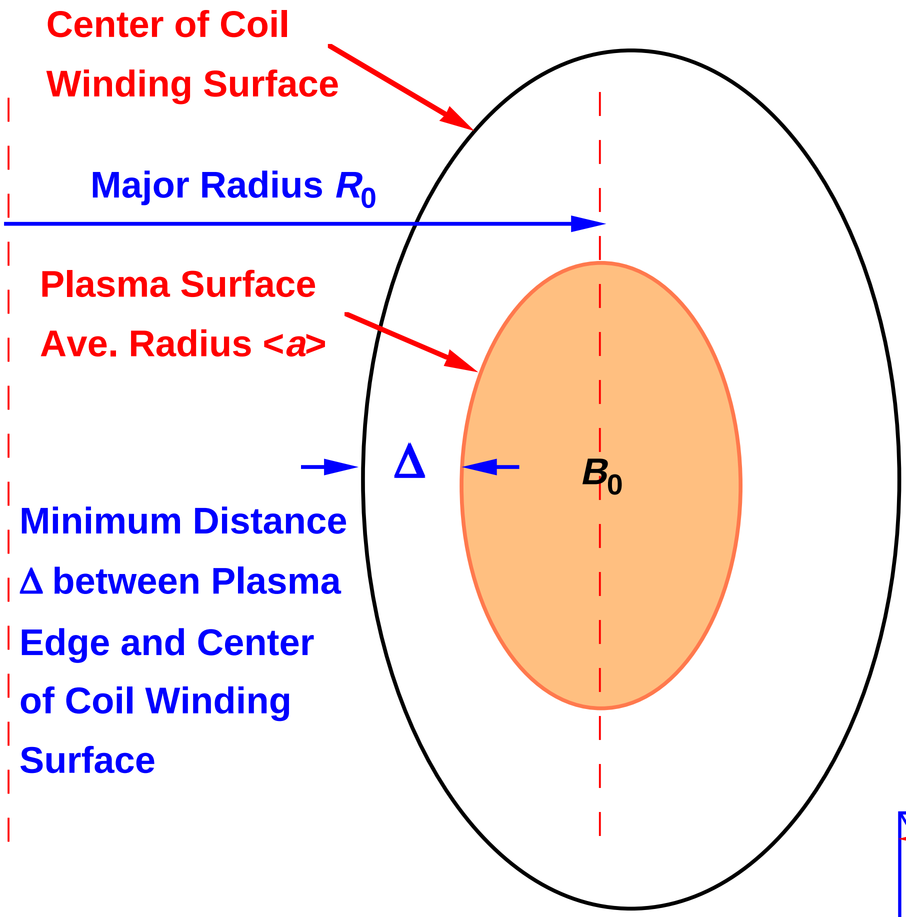
Stellarators Have Complex 3-D Magnetic Fields and Coils

- No simple scaling laws for β limits, confinement
  - depends on details of the magnetic configuration
- Divertor and maintenance requirements are more complex than for axisymmetric systems
- Systems codes must incorporate complex coil geometry and stellarator physics
  - complicated optimization and assessment
  - no geometry scalings possible

Reactor Core (Plasma and Coil) and Operating Point Optimizations are Separate

- **Reactor core optimization leads to a *fixed* plasma and coil geometry**
  - **integrated 3-D plasma/coil optimization code**
  - **⇒ plasma shape, aspect ratio, coil geometry**
  - **⇒ β limits, helical ripple, edge geometry**
  - **⇒ plasma-coil and coil-coil spacings, etc.**
- **Operating point optimization leads to plasma parameters, profiles, field and component sizes**
  - **1-D systems code incorporating complex plasma and coil geometry and stellarator physics modules**

Minimum Reactor Size Is Determined by Δ



Center of Coil Winding Surface

Major Radius R_0

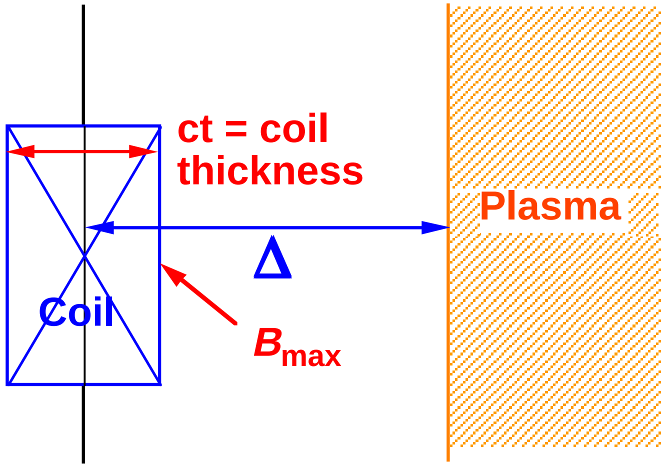
Plasma Surface Ave. Radius <*a*>

Δ

B_0

Minimum Distance Δ between Plasma Edge and Center of Coil Winding Surface

- A configuration is characterized by the ratios $A_\Delta = R_0/\Delta$, $A_p = R_0/\langle a \rangle$, and B_{max}/B_0
- The minimum reactor size is set by $R_0 = A_\Delta(D + ct/2)$ where D is the space needed for scrapeoff, first wall, blanket, shield, coil case, and assembly gaps
- Cost ∝ surface area ∝ A_Δ^2/A_p



ct = coil thickness

Plasma

Δ

Coil

B_{max}

Lowest <*R*>/<*a*> Does Not Necessarily Lead to the Most Compact Reactor

- **For most reactor studies (ARIES, HSR, SPPS) <*a*> = 1.7-1.8 m**
  ⇒ lowest A_p (= <*R*>/<*a*>)
  - **for stellarator configuration with A_p = 2.6, this would give <*R*> ≈ 4.6 m, which is impossible**

- **The argument is OK for configurations where distance Δ between LCFS and coil center is *not* a hard constraint**
  - **OK for *truly* axisymmetric systems, *not* for QA or QP systems**
  - **for CS's there is a maximum feasible Δ for a given <*R*> before**
    - **the coils become too kinky and not buildable**
    - **B_{max}/B_0 becomes large**

- **A certain distance D (≈1.6 m) ≤ Δ is needed between LCFS and coil center**
  ⇒ minimum <*R*> = A_ΔD where A_Δ = <*R*>/Δ

0-D Spreadsheet Calculations

- **Fixed plasma and coil geometry**
  - R/a, $\iota(2/3\ a)$, R/Δ, B_{max}/B_0
- **Input parameters**
  - max H-ISS95, max β, max $T(0)$
  - max B_{max}, target P_{fusion}, max neutron wall loading ($\Gamma_{n,\ max}$)
- **Minimize *R* for target P_{fusion} by varying *n* and H-ISS95 with constraints: parameters ≤ max. allowed values**
  - H-ISS95, β, $T(0)$, n/n_{Sudo}
  - plasma-coil distance, j_{coil}, Γ_n
- **Calculated quantities**
  - R, a, n, T, β, ν^*, H-ISS95
  - plasma-coil distance, coil j, coil thickness, P_{fusion}, Γ
- **Useful for size scaling for *fixed* plasma and coil geometry and comparing reactor configurations**

Extrapolation of Compact Stellarators to a Reactor

- **Vary distance Δ for compact stellarator configurations**
  - **calculate sheet-current solution at distance Δ from plasma that recreates desired plasma boundary**
  - **calculate B_{max}/B_0 at distance ct/2 radially in from current sheet**
- **Choose maximum credible distance Δ ⇒ $R_0 = \mathrm{A}_\Delta(\mathrm{D} + \mathrm{ct}/2)$**
- **$R_0^3 \propto P_{fusion}/B_0^4$, so want high B_0 for smaller reactor; *however***
  - **B_0 *decreases* with increasing Δ (B_{max}/B_0 increases)**
  - **Coil complexity (kinks) *increases* with increasing Δ**
- **Choose minimum ct/2 that satisfies two constraints**
  - **Ampere's law: $B_0 = 2\mu_0 \mathrm{N} j \mathrm{ct}^2/(2\pi R_0)$; coil aspect ratio = 2 assumed**
  - **$B_0 = (16\ \mathrm{T})/(B_{max}/B_0)$; B_{max}/B_0 increases as ct decreases**
- **B_{max}/B_0 is larger for actual modular coils, so use $1.15B_{max}/B_0$**
- **Need to redo for real modular coils**

Scaled 1-GW Compact Stellarator Reactors
with B_{max} = 12 T, $\langle\beta\rangle \leq \beta_{limit}$, H-95 ≤ 5

	QA#1	QA#2	QP#1	QP#2
Plasma aspect ratio R/a_p	2.96	4.4	2.70	3.70
Volume average β limit $\langle\beta\rangle_{limit}$ (%)	4	4.1	10	15
Average major radius R (m)	**8.22**	**9.93**	**7.34**	**7.84**
Average plasma radius a_p (m)	2.78	2.26	2.72	2.12
Plasma volume V_{plasma} (m^3)	1250	1000	1040	690
On-axis field B_0 (T)	**5.41**	**5.65**	**5.23**	**5.03**
τ_E/τ_E^{ISS95} multiplier H-95	**2.65**	**2.62**	**3.61**	**4.42**
Volume average beta $\langle\beta\rangle$ (%)	**4**	**4.1**	**4.6**	**6.2**
Energy confinement time τ_E (s)	2.69	2.41	2.49	2.01
Vol.-ave. density $\langle n\rangle$ (10^{20} m^{-3})	1.31	1.50	1.40	1.70
Density-average $\langle T\rangle$ (keV)	11.1	10.8	11.3	11.5
Neutron wall load Γ_n (MW m^{-2})	**1.34**	**1.37**	**1.54**	**1.85**

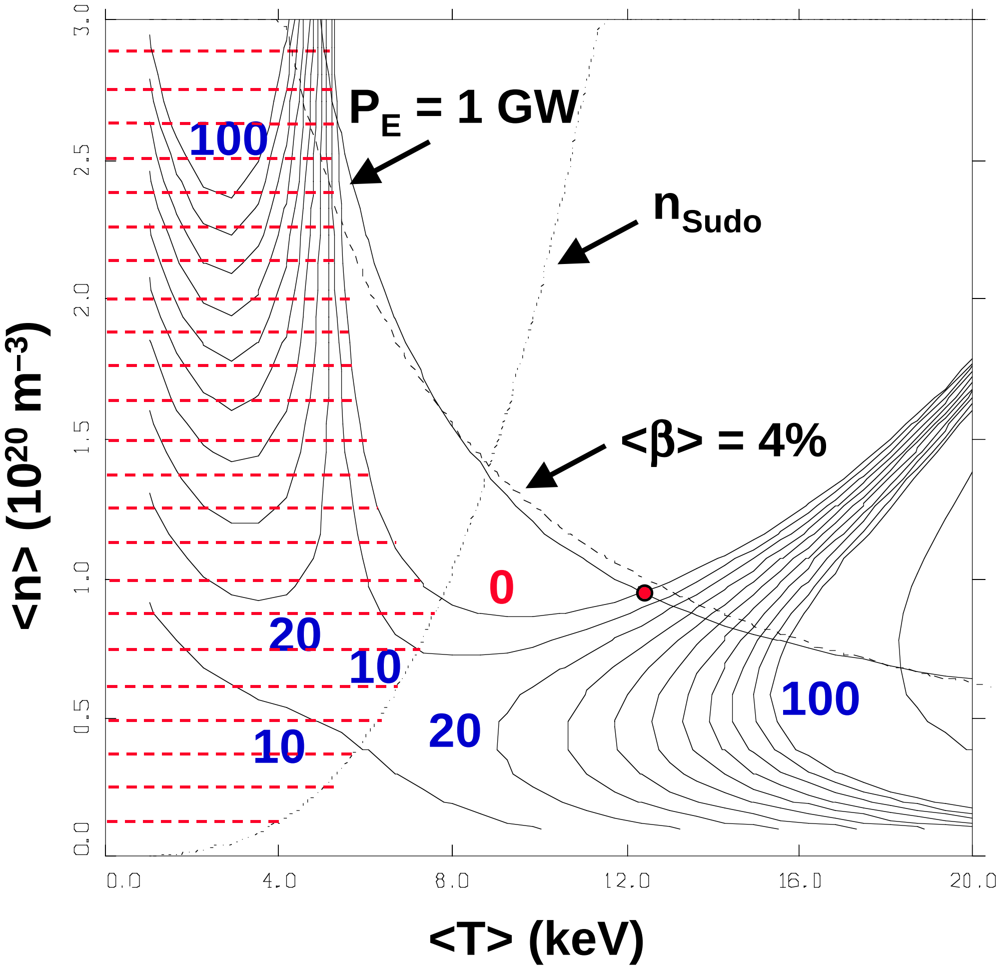
Comparison with Other Stellarator Configurations

- **The same assumptions were used with the plasma and coil configurations corresponding to the HSR, MHR-S, SPPS , QA and QP stellarator reactors**
- **The modified “HSR*” had R = 17.4 m (instead of 22 m because B_{max} was increased from 10.6 T to 12 T), H-95 = 3.06, $\langle\beta\rangle$ = 4.9%, and Γ_n = 1.24 MW m^{-2}**
- **The modified “MHR-S*” had R = 18.6 m (instead of 16.5 m because of the ARIES-AT blanket and shield assumptions), H-95 = 2.87, $\langle\beta\rangle$ = 5%, and Γ_n = 0.62 MW m^{-2}**
- **The modified “SPPS*” had R = 20.8 m (instead of 14.0 m because B_{max} was decreased from 16 T to 12 T), H-95 = 3.13, $\langle\beta\rangle$ = 5%, and Γ_n = 0.60 MW m^{-2}**
- **For the same modeling assumptions, the compact stellarator configurations lead to reactors with a factor of 2 to 3 smaller major radius and a factor of 1.4 to 3 higher wall power loading**

1-D POPCON Calculations

- **Fixed reactor parameters: B_0, $\langle R \rangle$, $\langle a \rangle$**
- **Plasma models for τ_E or χ, radiation, etc.**
- **Fixed plasma assumptions: β_{limit}, n_{limit}, $\tau_{\mathrm{He}}/\tau_E$, impurity fraction, α losses, etc.**
- **Calculates ignition contour and contours of P_{heating} needed in $\langle n \rangle$ vs. $\langle T \rangle$ plane**
- **$\langle n \rangle$ is the volume-averaged electron density**
  $\langle T \rangle$ is the density-averaged temperature
- **Useful for understanding operating space, startup scenario, thermal stability**

Typical QA Reactor POPCON Case



Operating Point

$\langle n \rangle = 9.5\ 10^{19}\ m^{-3}$

$\langle T \rangle = 12.4$ keV

$\langle \beta \rangle = 3.6$ %

$P_{fus} = 1.73$ GW

Saddle Point

$\langle n \rangle = 4.9\ 10^{19}\ m^{-3}$

$\langle T \rangle = 6.1$ keV

$\langle \beta \rangle = 0.9$ %

$P_{aux} = 12$ MW

Ignition minimum

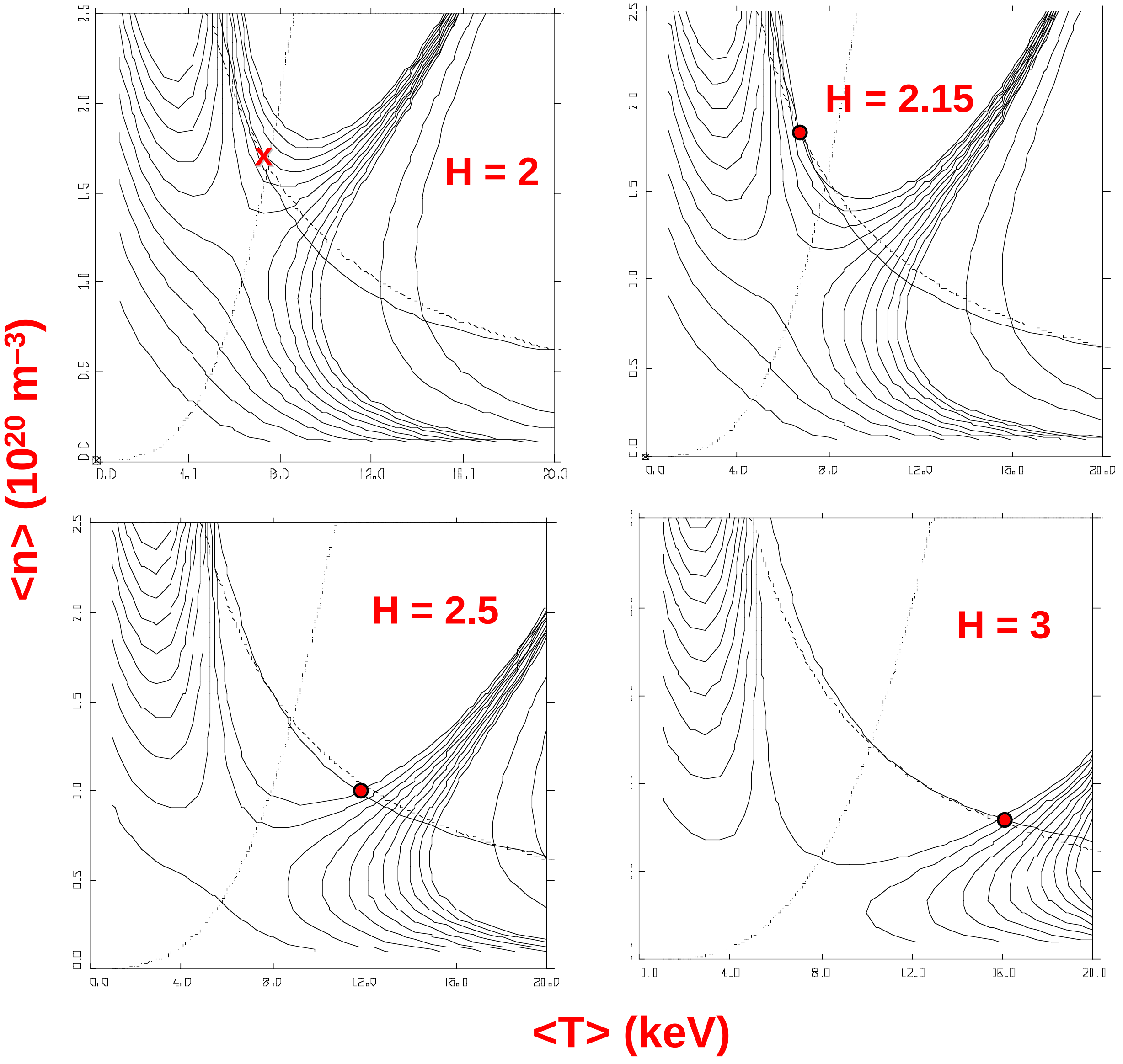
$\langle \beta \rangle = 2.2$ %

$P_{fus} = 0.6$ GW

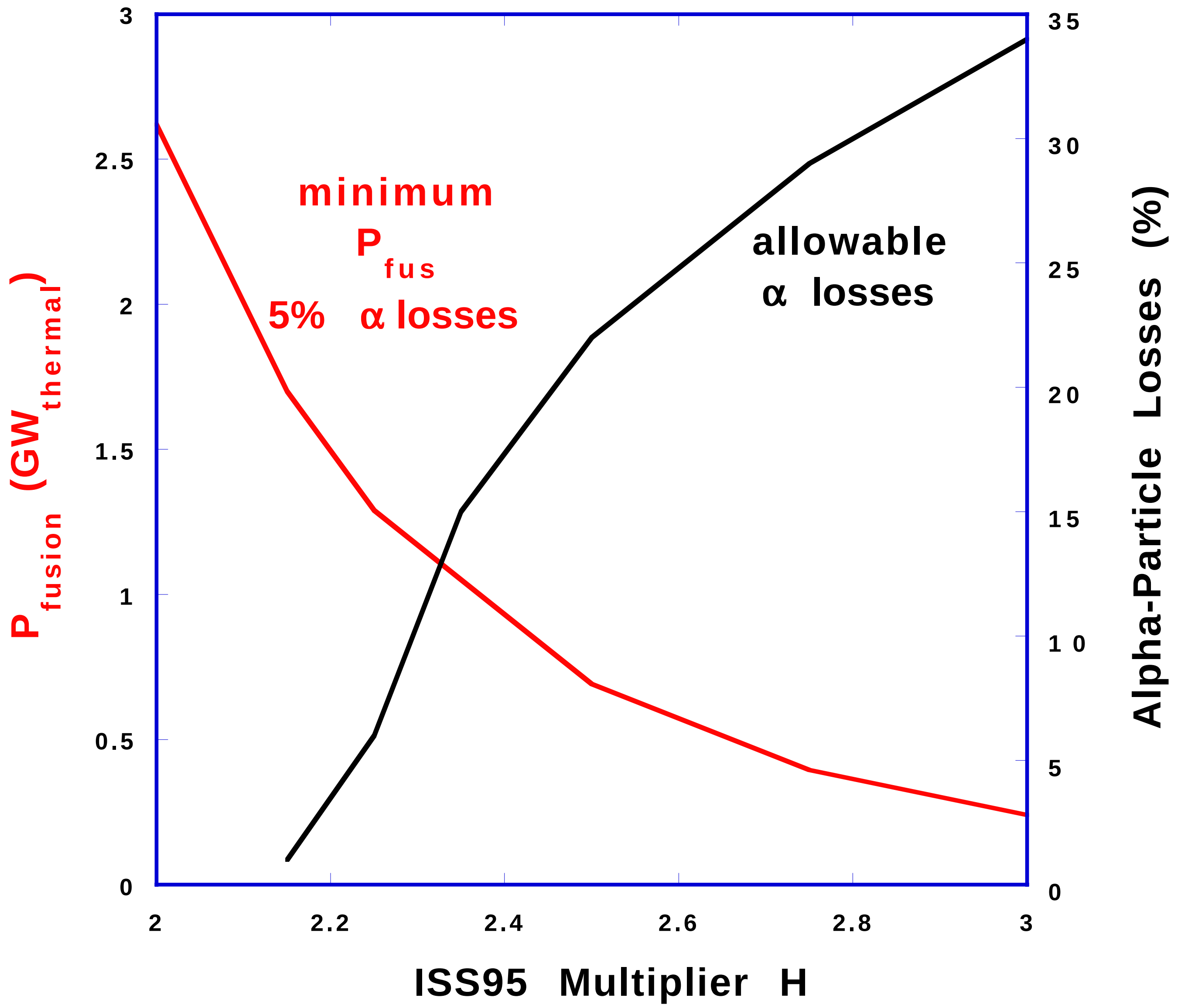
- $R = 9$ m, $B_0 = 5$ T ($B_{coil} = 12.7$ T), 2.5 x ISS-95, 5% α loss
  $\tau_{He}/\tau_E = 6$ ⇒ 5.3% He, $n_{DT}/n_e = 0.83$, $Z_{eff} = 1.5$

Operating Point Moves to Higher <*T*> as ISS95 Multiplier H Increases

- R = 9 m, B = 5 T, 5% α losses, τ_{He}/τ_E = 6

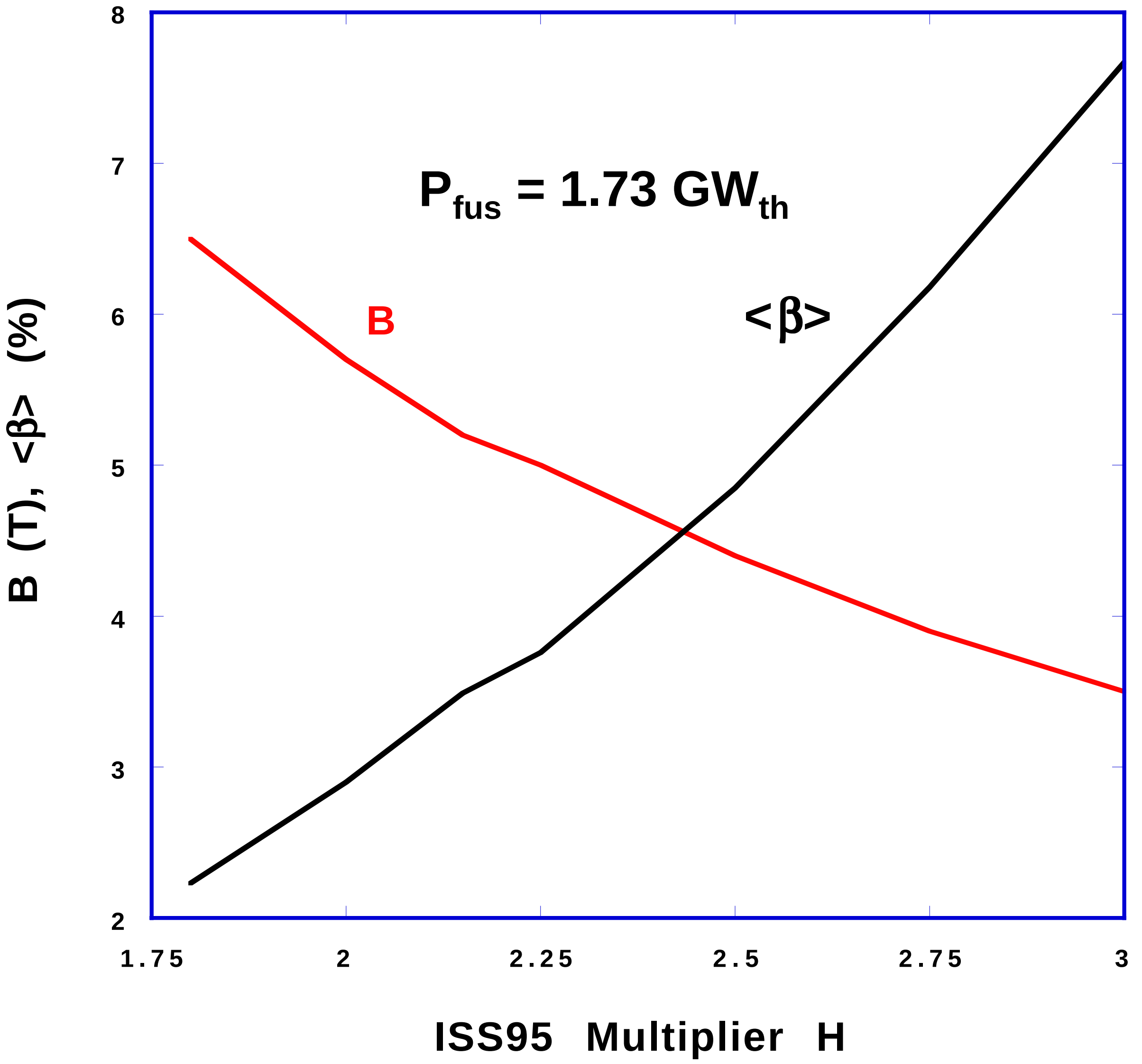


Operating Point Characteristics



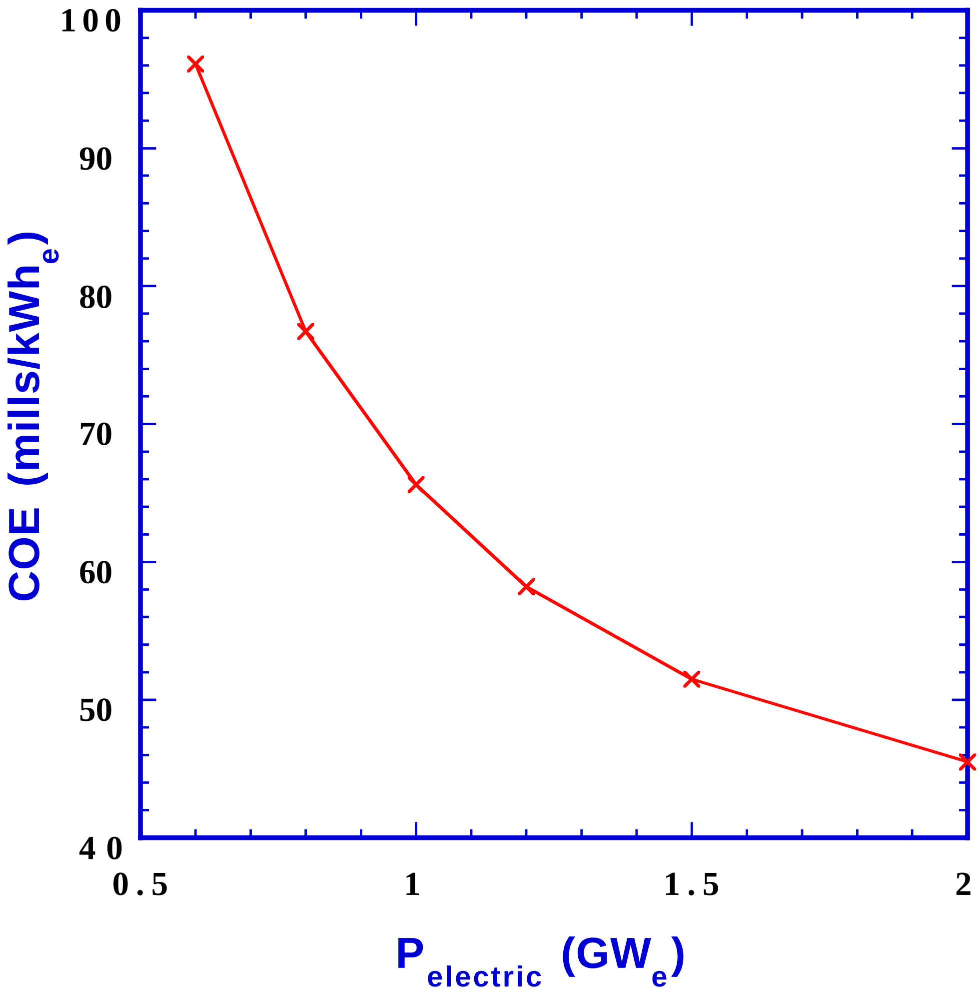
- $R = 9$ m, $B = 5$ T, $\tau_{He}/\tau_E = 6$

Higher B Required at Lower H



- R = 9 m, 5% α losses, $\tau_{He}/\tau_E = 6$

Cost of Electricity Could Decrease with Plant Size



- **Stellarator reactor example, similar for ARIES-RS**

Systems Code Integrates Physics, Materials, Cost Models

- Detailed physics models for
  - alpha-particle heating and losses (Fokker-Planck with losses)
  - radiation (coronal line radiation, bremsstrahlung, cyclotron)
- Stellarator transport options (ISS95 + Shaing-Houlberg)
  - (a) 1-D evaluations with fixed profiles
  - (b) solve for T_e(r) and T_i(r) with fixed n_e(r) and E_r(r)
  - (c) solve for T_e(r), T_i(r), n_e(r) and E_r(r) with fixed particle source
- ARIES magnet and reactor material assumptions
  - multi-region blanket and shield (except for divertor regions)
  - B_{max} vs. j in coil from ARIES studies
  - allowable stresses, reactor safety penalties, etc. from ARIES
- ARIES costing algorithms based on masses and cost per kg
  - ARIES-RS algorithms and accounts

Optimization Approach

- **Minimize cost (<*R*>) or COE with constraints for a particular plasma and coil geometry using a nonlinear constrained optimizer with a large number of variables**
- **Large number of constraints allowed, for example**
  - **ignition margin, β limit, H-ISS95, radial build, coil *j* and B_{max}, plasma-coil distance, blanket and shielding thicknesses, TBR, access for divertors and maintenance, etc.**
- **Large number of configuration, plasma parameters, transport model, costing, and engineering model parameters**

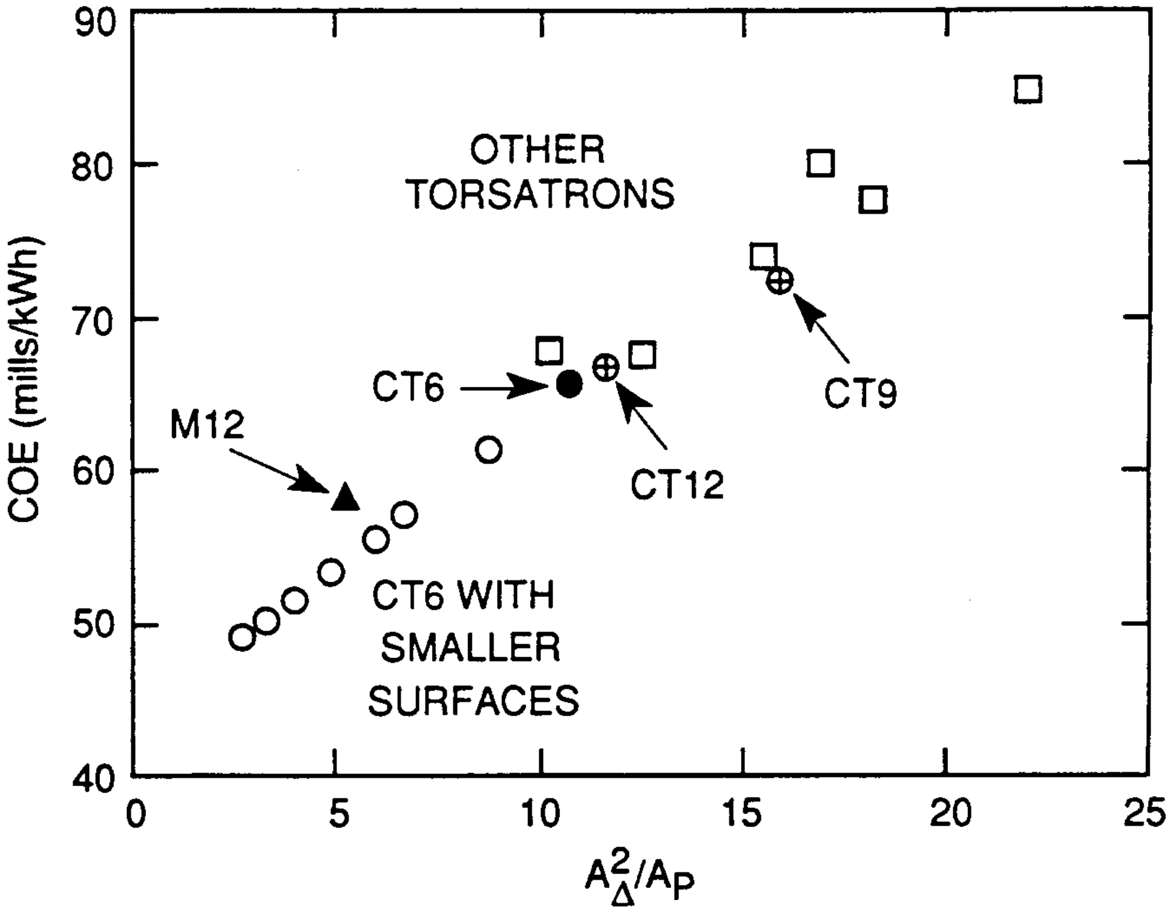
1-D Transport in Systems Code

- Steady-state 1–D integral-differential equations for the heat and particle fluxes for the ions (D,T) and electrons are solved for $n_e(r)$, $n_i(r)$, $T_e(r)$, $T_i(r)$, and $E_r(r)$:

$$\rho q_j(\rho) = a_p \int p_j(\rho^*)\rho^* d\rho^* \ , \quad \rho \Gamma_j(\rho) = a_p \int s_j(\rho^*)\rho^* d\rho^* \ ; \quad \mathrm{j} = \text{ions, electrons}$$

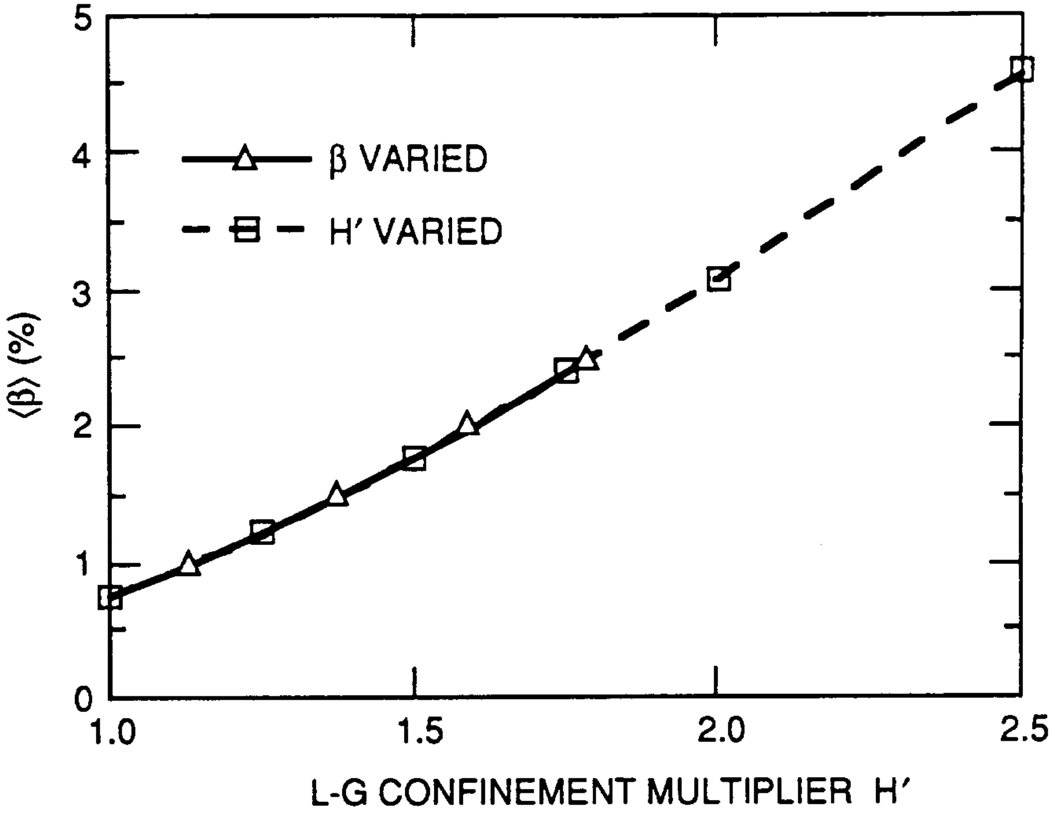
- Heat flux $q_j = -n_j \chi_j^T \nabla T_j - T_j \chi_j^n \nabla n_j - Z_j n_j \chi_j^\phi \nabla \phi$
- Particle flux $\Gamma_j = -n_j D_j^T \nabla T_j - T_j D_j^n \nabla n_j - Z_j n_j D_j^\phi \nabla \phi$
- The electric field is determined from the ambipolarity condition. The electric field E enters both through an E/B drift term in the denominators of χ and D, and directly through the sign-dependent $\nabla\phi$ term.
- The volumetric heat sources (and sinks) are the usual alpha-particle heating, electron-ion heat transfer, and radiation terms.
- The form for the particle source rate (s) is chosen to represent shallow or deep fueling of the plasma.

Cost of Electricity Depends on A_{Δ}^{2}/A_{p}



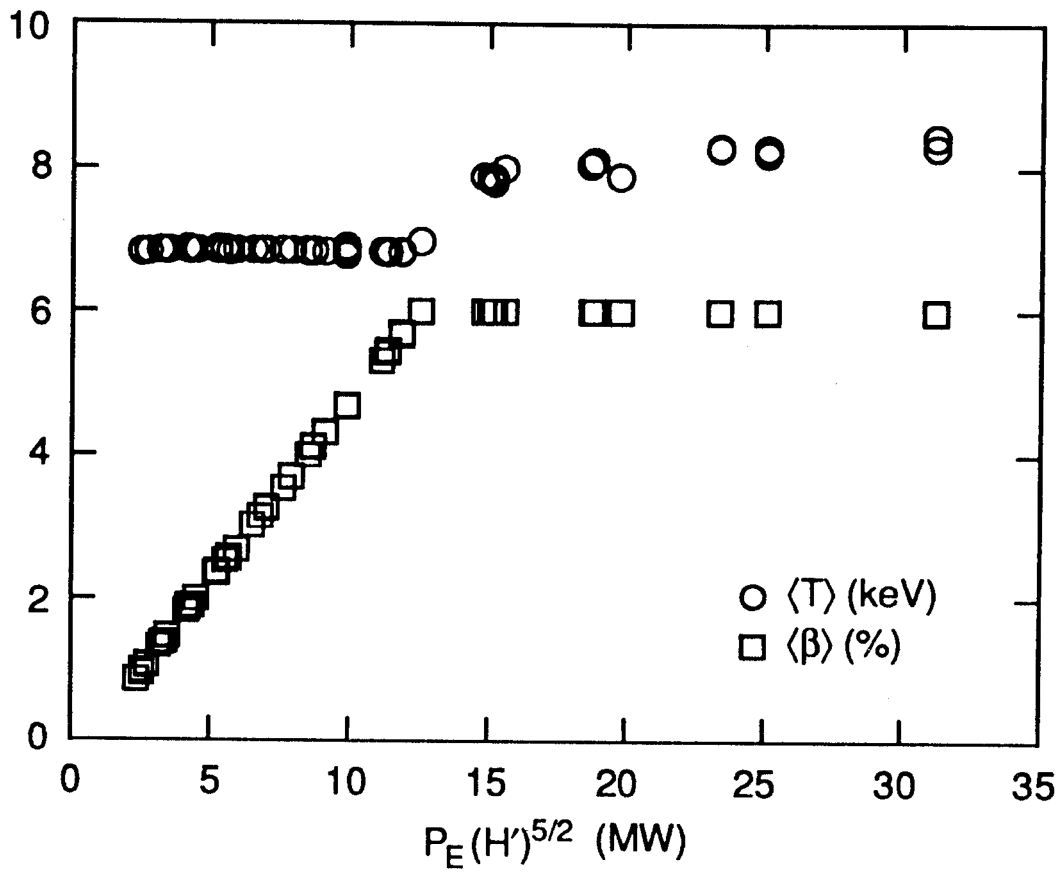
- **Minimized COE for fixed fusion power**

Beta and Confinement Multiplier are Coupled



- **Minimized COE for fixed fusion power**

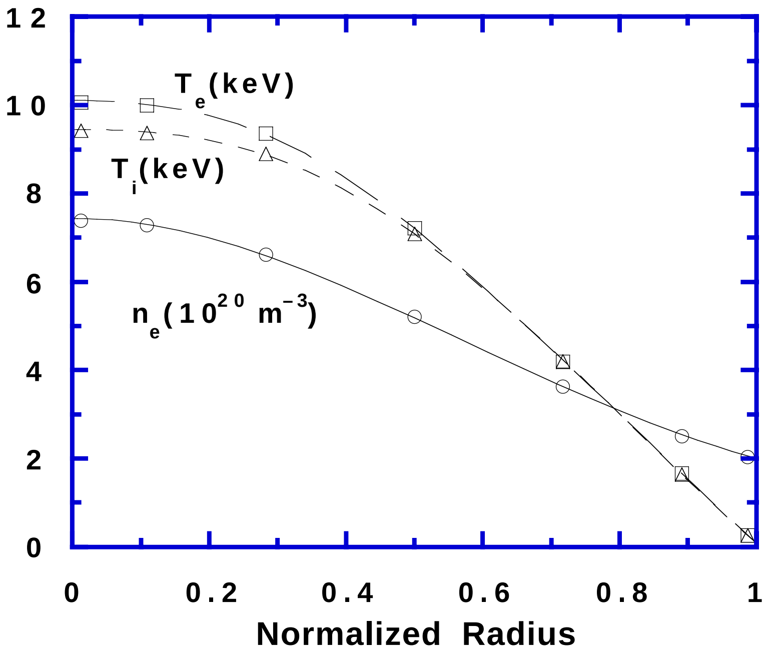
<T> Settles on Constant Value below β limit

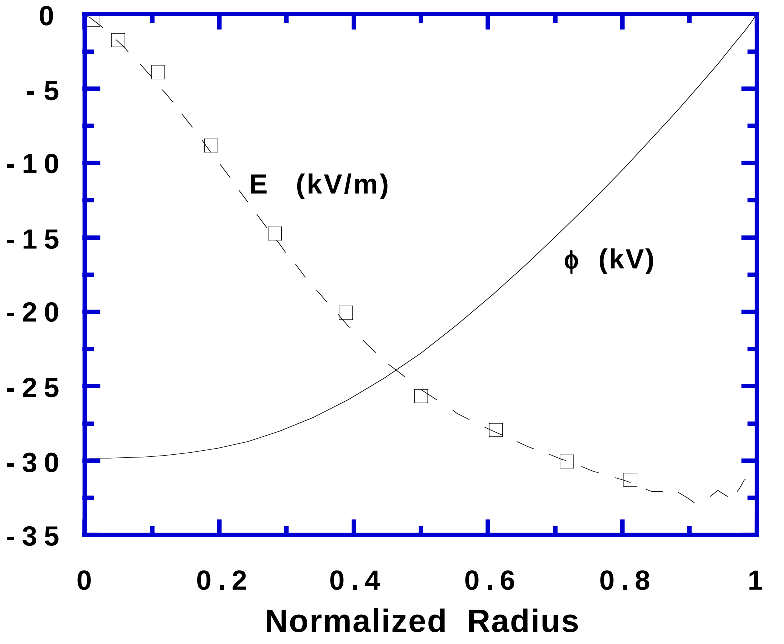


- **Minimized cost of electricity**

1-D Systems Optimization Calculations

- **Reference parameters: R_0 = 12 m, a_p = 1.5 m, B_0 = 7 T, P_{fus} = 3 GW (thermal), edge helical field ripple $\varepsilon_h(r = a_p)$ = 0.1.**





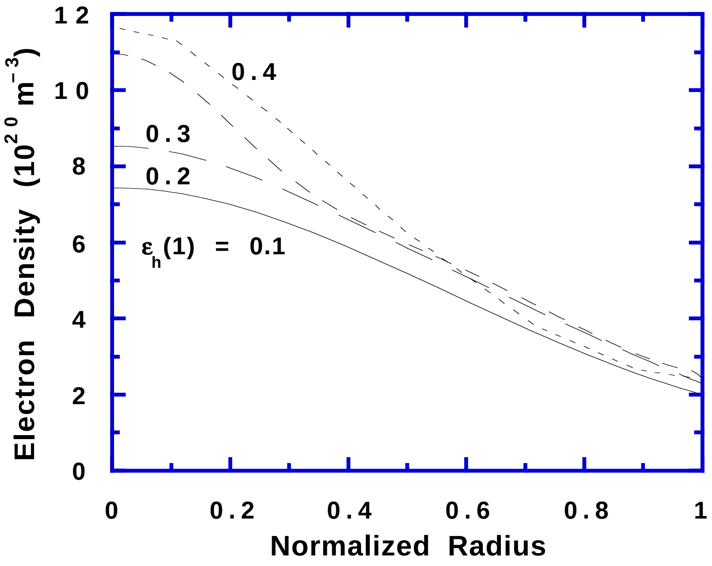
Sensitivity to Parameter Assumptions

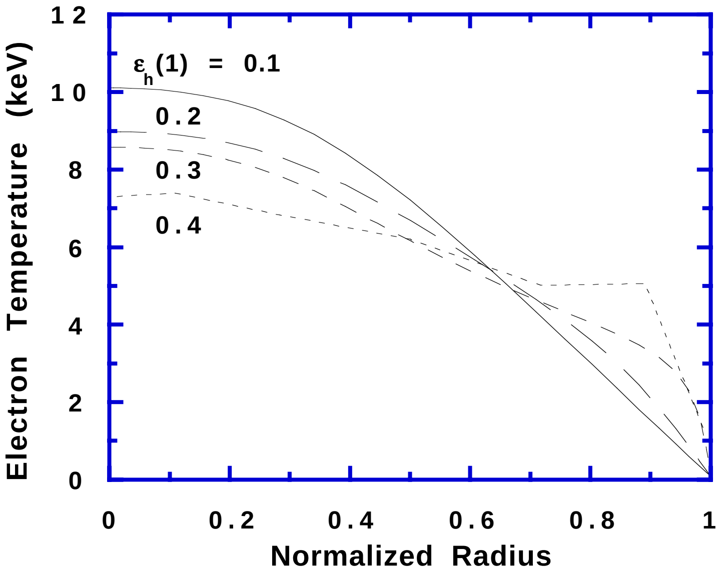
$\varepsilon_h(1)$	χ_{anom} (m^2/s)	$\langle n \rangle$ ($10^{20}m^{-3}$)	$n_D(0)$ ($10^{20}m^{-3}$)	$T_e(0)$ (keV)	$T_i(0)$ (keV)
0.1	2.60	1.85	3.53	10.07	9.43
0.2	1.49	2.11	3.92	8.97	8.51
0.3	0.81	2.11	4.99	8.08	7.72
0.4	0.66	2.21	6.08	7.31	7.09
0.6	0.43	2.01	7.93	7.00	6.86

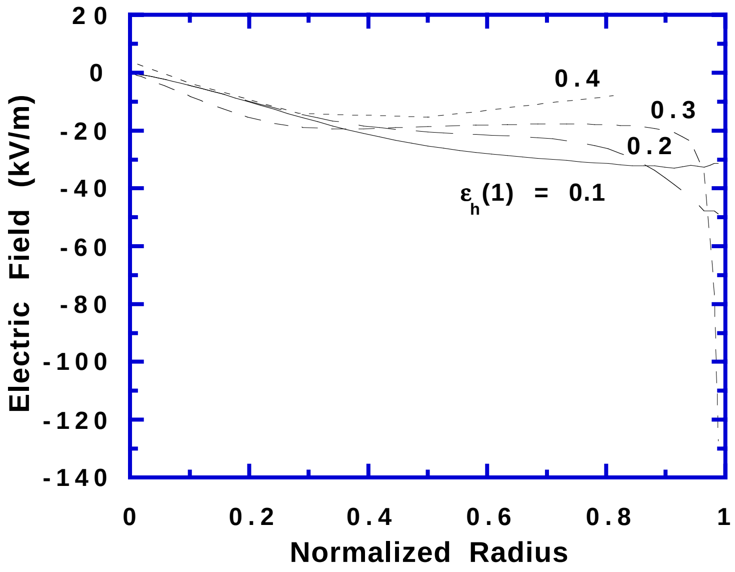
P_f (GW)	B_0 (T)	α –loss	χ_{anom} (m^2/s)	$\langle n \rangle$ ($10^{20}m^{-3}$)	$n_D(0)$ ($10^{20}m^{-3}$)	$T_e(0)$ (keV)	$T_i(0)$ (keV)
3	7	no	2.60	1.85	3.53	10.07	9.43
3	7	*yes*	0.79	1.68	3.11	10.34	9.72
3	*5*	no	1.47	1.85	3.47	9.47	8.85
4.5	7	no	4.32	2.27	4.32	10.22	9.57
$\chi_{anom} \propto 1 + 19\rho^3$			0.23	1.25	3.21	13.92	12.33

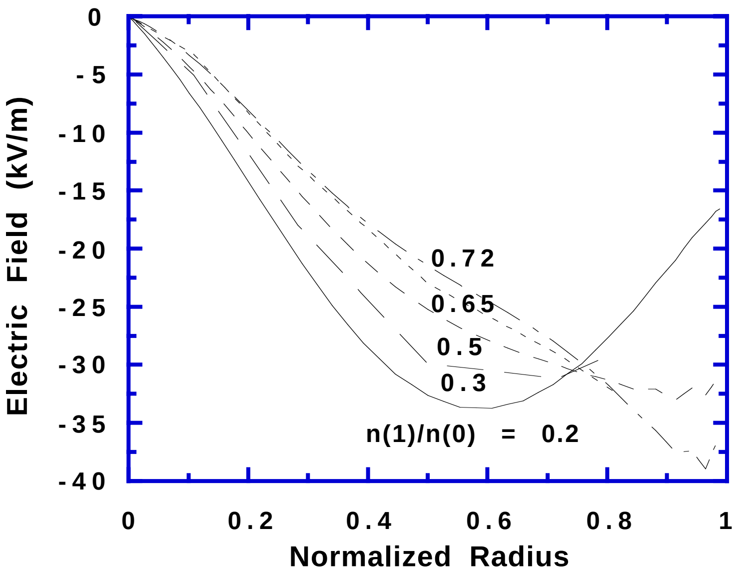
Here χ_{anom} ($\propto 1/n$) is the largest value for ignition

1-D Systems Optimization Calculations









Lessons Learned

- **3-D stellarator magnetic fields means more complex divertor and maintenance geometry, no simple scaling laws, no geometry scaling studies with a simple systems code**

- **Systems codes must incorporate complex coil geometry and stellarator physics**
  - **optimization and assessment more complicated**

- **Geometry scaling studies are *not* possible**
  - **plasma: shape, aspect ratio, plasma profiles**
  - **coils: plasma-coil and coil-coil spacings**

Most Important Measure of Reactor Attractiveness is COE

Reactor	Type	$R_0/<a>$	R_0 (m) a(m)	p_{wall} MW/m^2	COE mills/kWh	Q_{eng}	B_{max}/B_0	B_0(T)
W7-X based HSR	high-A stellarator	12.2	22 1.8	0.5	>110		2.11	4.8
W7-X like SPPS	modular stellarator	8.6	13.9 1.6	1.3	75	19.3	2.94	4.9
ARIES-IV	2nd stability tokamak	2.8	6.0 2.1	2.7	68	5.2	2.09	7.6
ARIES-RS	reverse shear tokamak	3.1	5.5 1.8	4.0	76	5.9	1.98	8.0
ARIES-ST	spherical tokamak	0.87	3.2 3.7	4.1	>76	3.1	3.55	2.1

- **Higher value of Q_{eng} can compensate for R_0, p_{wall}**

Reactor Comparisons

Configuration	R_0/Δ ⇒ $R_0/<a>$ ⇒	R_0 (m) a(m)	1.15 x B_{max}/B_0 ⇒	B_0(T)	P_{elect} GW ⇒	p_{wall} MW/m ^{2}
QA C82	**5.8** **3.4**	**8.9** 2.6	**2.54**	**6.3**	**2.0**	**4.7**
QA A4.1	**5.8** **4.1**	**9.0** 2.2	**2.20**	**7.3**	**2.6**	**7.0**
QA C93	**5.8** **3.4**	**9.0** 2.7	**2.14**	**7.5**	**4.2**	**9.5**
ARIES-RS **tokamak**	**3.4** **3.1**	**5.5** 1.8	**1.98**	**8.0**	**1.0**	**4.1**
SPPS **stellarator**	**7.0** **8.6**	**13.9** 1.6	**2.94**	**4.9**	**1.0**	**1.3**

- **Closer to ARIES-RS than SPPS**
- **B_{max} = 16 T and $<\beta>$ = 5% leads to large P_{elect}**

The Coils are the Key to a CS reactor

- **Plasma-coil spacing Δ (for blanket and shielding) and coil bend radii ρ are more important than the plasma configuration or aspect ratio**
  - $R \propto \Delta$ (for blanket and shielding) and cost $\propto \Delta^2$
  - $\rho \Rightarrow B_{\mathrm{max}}$ on the coils $\Rightarrow P_{\mathrm{fusion}} \propto B_{\mathrm{max}}{}^4$
- **Can't just enlarge an experiment to reactor size**
  - Optimization is based on different needs
  - W 7-X, LHD, NCSX, QPS *don't* extrapolate to good reactors

A Phased Approach

- **Development of optimization tools and modules highest priority**
- **Need combined optimization of plasma and coil configurations**
  - no point in optimizing plasma and then finding coils
  - need to include reactor physics (α losses, divertor, etc.)
  - minimum cost implies minimum major radius and simplest coils, *not* minimum plasma aspect ratio
  - the key parameters are minimum values of R/Δ and B_{max}/B_0; small R/a is a secondary factor
- **Concerns**
  - pace of reactor concept development restricted by very limited funding ($200k PPPL, $70k ORNL)
  - have to proceed at slow pace or drop some parts