Recent Progress in Configuration Development for Compact Stellarator Reactors

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Highlights of Accomplishment

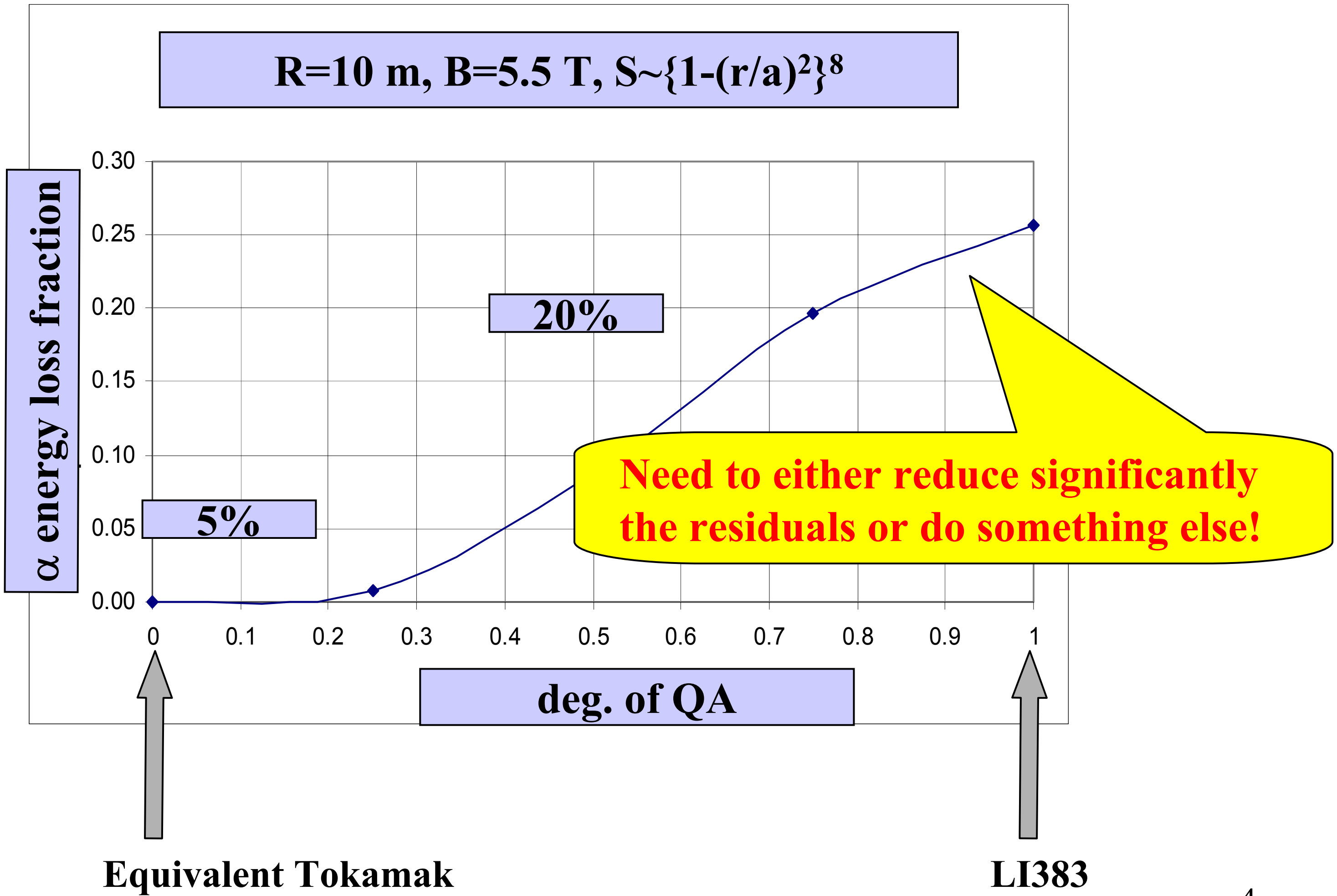
- We have built into the configuration optimizer five figures of merit useful for optimizing α confinement.
- We have studied the effectiveness of these measures in guiding the development of α-loss optimized plasma geometry.
- We have found configurations with improved α particle confinement characteristic (relative to NCSX).

Background

In the October 4, 2002 review of reactor aspects of NCSX, We reported that:

- Fast ion confinement in NCSX is not as good as we'd like (It was not part of the optimization strategy).
- For a device of volume 1000 m^3 and a field of 5.5 T, we estimated that α losses could amount to > 25%.
- To find an attractive CS reactor, it is necessary to couple an effective figure-of-merit into the configuration search to improve the energetic particle confinement characteristics.

α Energy Loss Versus QA Calculated for LI383 (The Baseline Configuration for NCSX)



- We also pointed out other important considerations in designing an optimal reactor configuration:
  - MHD stable at beta >5%,
  - Identify most attractive aspect ratio and iota,
  - Need minimizing COE with a proper figure-of-merit,
  - Need effective figure-of-merit for assuring flux surface quality,
  - Assess impact of Δ, the plasma-coil separation, on the configuration design (plasma shaping).

- In the initial phase of our efforts, we have chosen to concentrate on the α confinement issue, trying to find ways to understand how we can improve the losses. In addition, we have also started to survey the aspect ratio-rotational transform (A-ι) landscape.

Today, we shall present:

- definitions of various measures for QA and α confinement built into the optimizer;

- correlation plots showing how these measures stack up against full α loss calculations using configurations developed in the course of NCSX work;

- initial results of searching for better configurations along a constant iota plane but with varying aspect ratios and field periods, keeping the same plasma MHD characteristics of the NCSX;

- comparison of properties of an α loss improved 3-field period configuration to an unimproved one, illustrating features that distinguish a “good” versus a “poor” plasma geometry.

We posit that:

- Almost all measures considered correlate weakly to the full α loss.
  - Configurations with good QA have all good measures, but converse is not true.

- Good α confinement at high β can be achieved in QA devices.
  - The MHD stability constraints limit how good the confinement can be.
  - There may exist an optimal aspect ratio at a given ι for QA reactors.

- Good QA is a sufficient condition for good α confinement, but is not a necessary condition. There might be intriguing roles played by the mirror, B(0,1), B(0,2), and helical, B(1,1), B(1,-1), field components.

A. Figures-of-Merit for QA and α Confinement

- Minimization of residuals in magnetic spectrum: weighted and un-weighted.
- Pseudo-symmetry (PS): minimization of ripple well areas.
- Effective ripple: equivalent effects of helical ripples in $1/\nu$ transport.
- Second adiabatic invariant, $J_{||}$: minimization of contour losses to outside flux surfaces.
- Reduction of initial loss of collision-less fast ion orbits.

A.1. Minimization of Residuals in the Magnetic Spectrum

- In a straight field line coordinate, usually the Boozer coordinate, we write

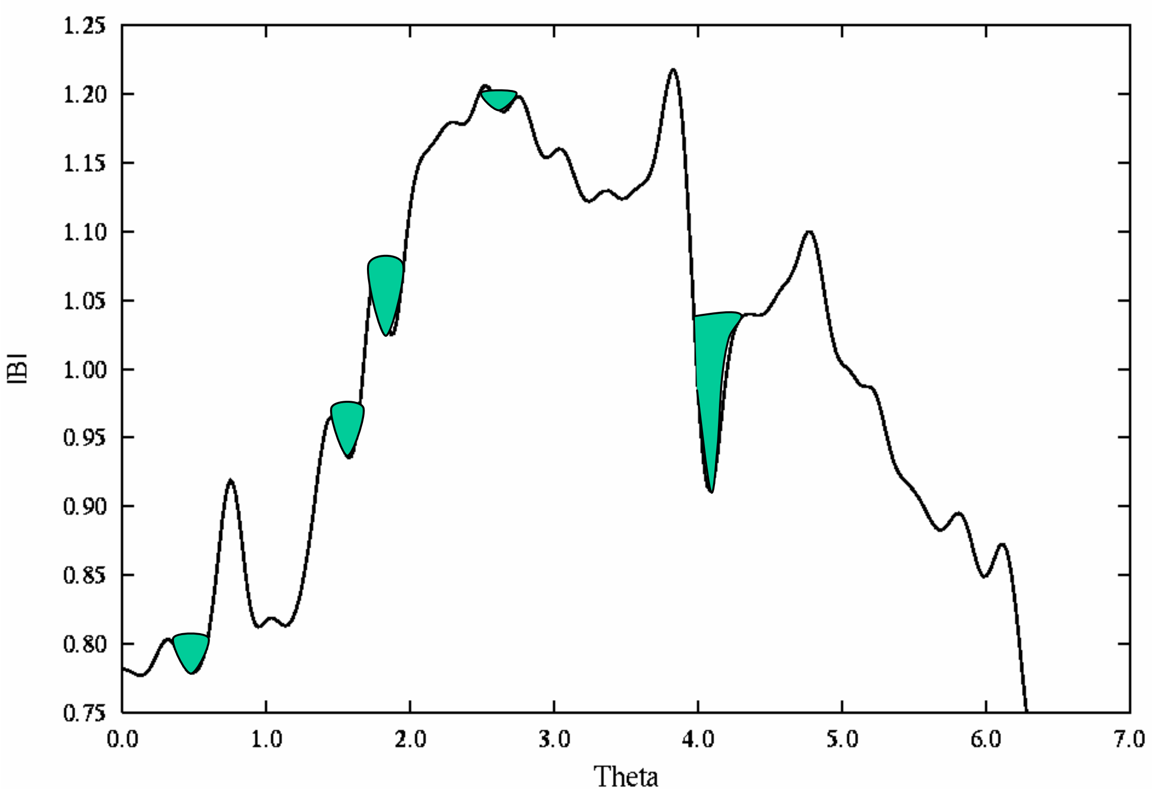
$$B(r) = \sum_{m,n} B_{m,n}(r) \cos(m\theta - n\varphi)$$

- For QA, want to minimize the residuals

$$\sum_{m,n\neq 0} \text{weight}(r; m, n) \cdot B_{m,n}(r) \cdot B_{m,n}(r)$$

- We observe that, for α losses in LI383 (R=10 m, B=5.5T, birth distribution~$\{1-(r/a)^2\}^8$):
  - Include all m, n the energy loss is 25.6%;
  - Include only n=0 terms plus $B_{2,1}$ (1.8% @r/a=0.9), $B_{3,2}$ (1.3%), the largest two harmonics, the loss is 9.2% or ~40% of the total;
  - Include n=0 terms plus $B_{2,1}$, $B_{3,2}$, $B_{1,1}$(0.3%), $B_{1,-1}$(0.4%) the loss is 7.3%;
  - Include additional higher n terms $B_{3,3}$(0.6%), $B_{4,3}$(0.5%), $B_{1,3}$(0.4%) the loss is increased to 16.8%, or ~65% of the total.
- Clearly, there is a dependency on n, and perhaps m, but we have not yet had enough data to effectively set the proper weights or intelligently discriminate one from the other.

A. 2. Pseudo-symmetry measure: minimization of ripple well area along a field line



But, which part of the field line to follow and for how long so as to be both physically meaningful and computationally effective?

A. 3. Semi-analytic formula for 1/ν transport: effective ripple

- The formulation provides an effective and efficient means of calculating local transport coefficients in 1/ν regime, being able to take into account all classes of trapped particles (Nemov V. V., Kasilov S. V. and Kernbichler W.; PoP, 6, 4622, December 1999).

- How effective is ε_{eff} when applying to α transport, particularly when the loss is dominated by collision-less mechanism?
  - For LI383, ~86% loss is collision-less.

A. 4. Second adiabatic invariant: minimizing α loss by closing *J* contours

- $$J = \oint \mathrm{v}_{\parallel} \cdot dl$$

  $$\mathrm{v}_{\parallel} / \mathrm{v} = \sqrt{1 - \mathrm{B}\overline{\mu} / \mathrm{W}}$$

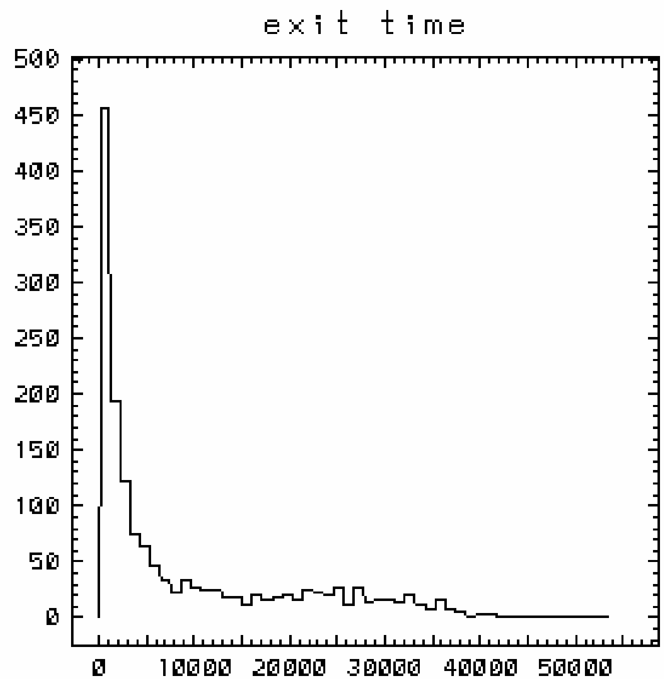
  – Where $\overline{\mu}$ is the magnetic moment, proportional to the perpendicular energy of source particles, hence the pitch angle.

- For how many pitch angles and how many segments of a field line do we need to sample to make it an effective and efficient measure?

A. 5. Minimizing initial orbit loss: maximizing the resident time of particles.

- $$f = \frac{\text{Particle Lost}}{\text{Particle Started}} \cdot \{1 - \frac{\text{Average Toroidal Transit Completed}}{\text{Toroidal Transit Expected}}\}$$

- For a configuration having poor α orbits, most losses occur rapidly.



- Simulating the guiding center motion of particle orbits is a Monte Carlo process in numerical computation subject to the statistical fluctuation.

- What sample size and cutoff would be needed to be effective and efficient, i. e. not fooled by the statistical noise?

  – Computation time directly proportional to the sample size, but the statistical error proportional to the square root of the sample size.

  – Increasing step size in derivative calculation to minimize the contamination of statistical error may give a false direction of steepest descent in the gradient search of the optimum.

B. Test effectiveness of the five penalty functions using configurations in our inventory.

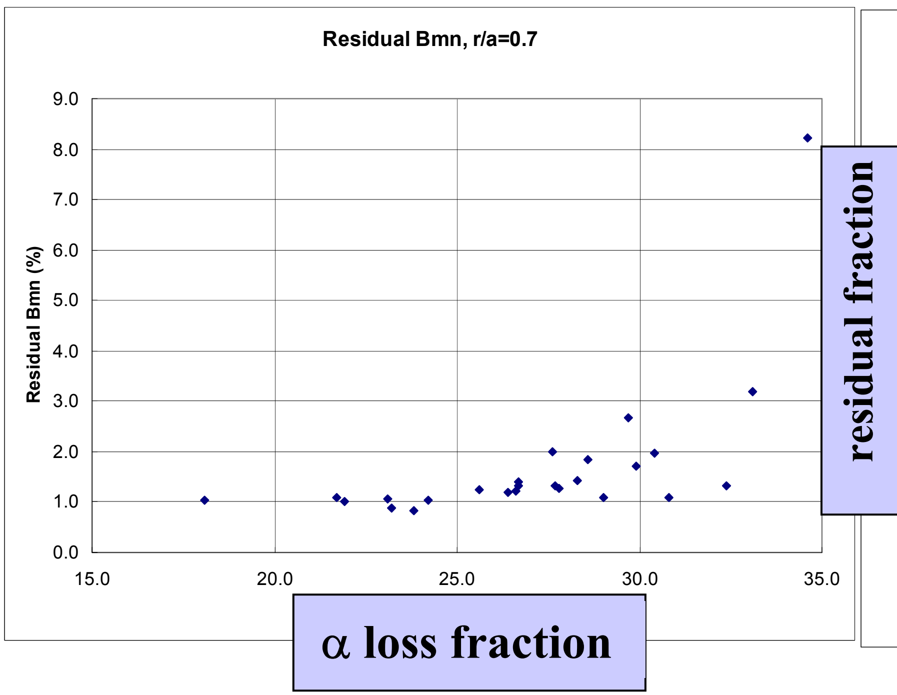
- Configurations generated during the course of NCSX development serve us a useful data base for testing the “goodness” of the five measures.

- These configurations range in aspect ratio from 2.2 to 5.4, average iota from 0.35 to 0.65, and field period from 2 to 3, with varying degree of “QA”ness, but all are at ~4% β and mostly stable to ballooning and kink modes.

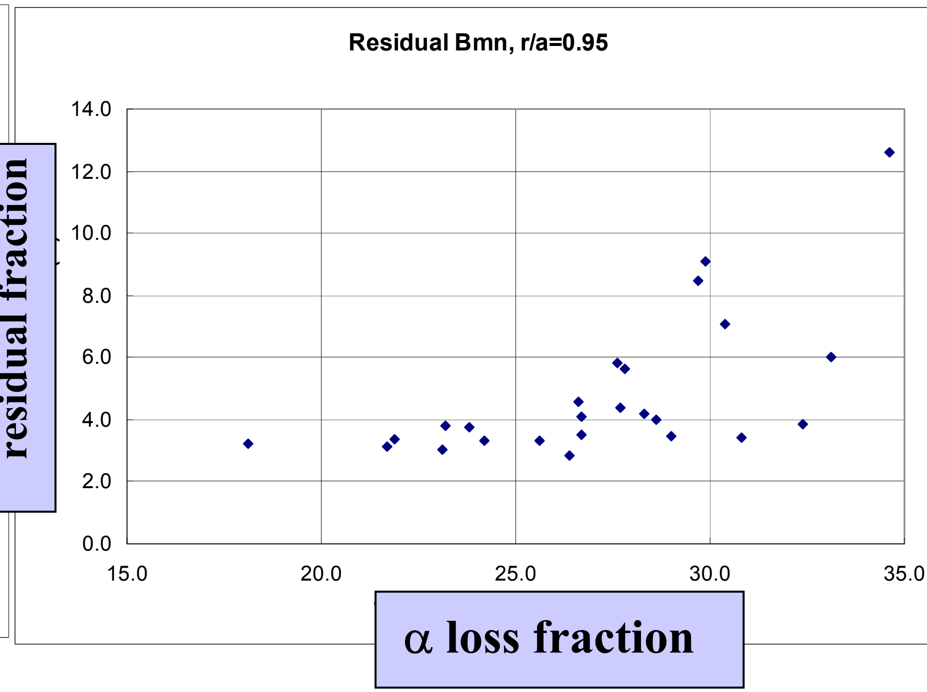
B. 1. Residual Bm,n

As expected, the magnitude of residuals in the magnetic spectrum does not correlate well with the α loss except for the very good or very bad cases.

r/a=0.7

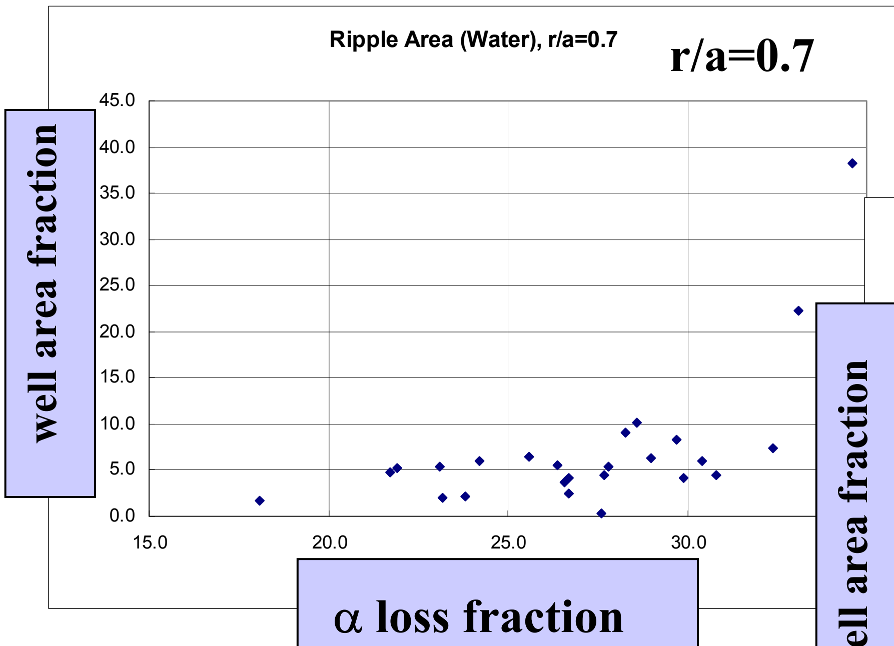


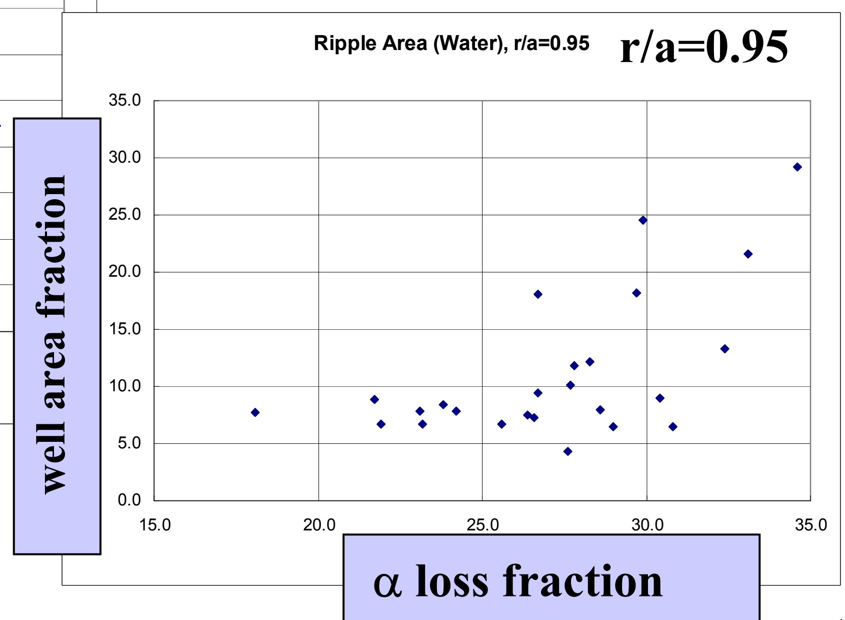
r/a=0.95



B.2. Ripple Well Area (PS)

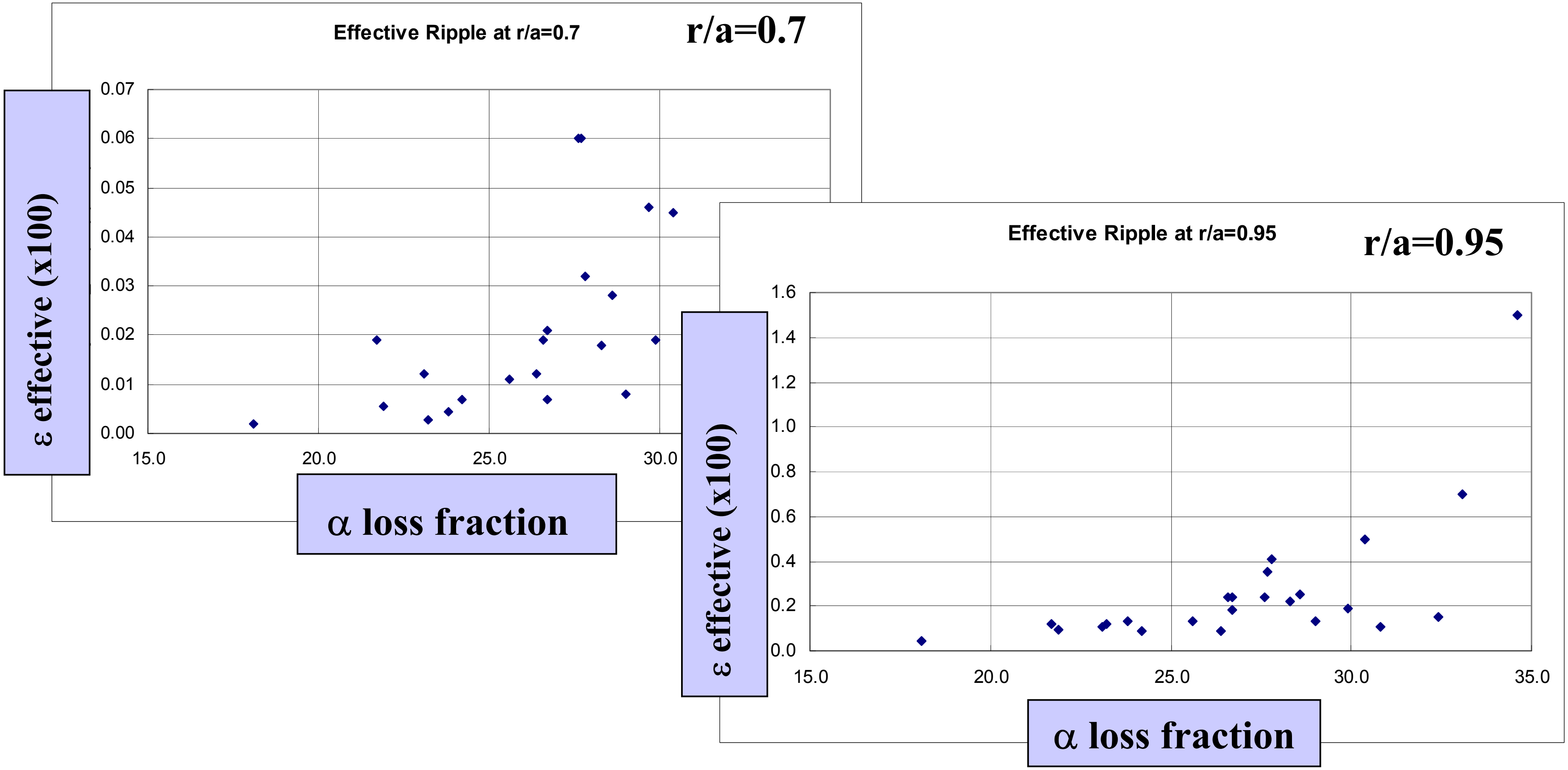
The PS measure as presently implemented shows a weak correlation near the boundary, but shows no correlation in the interior region of the plasma except for the very good and very bad cases, as before.





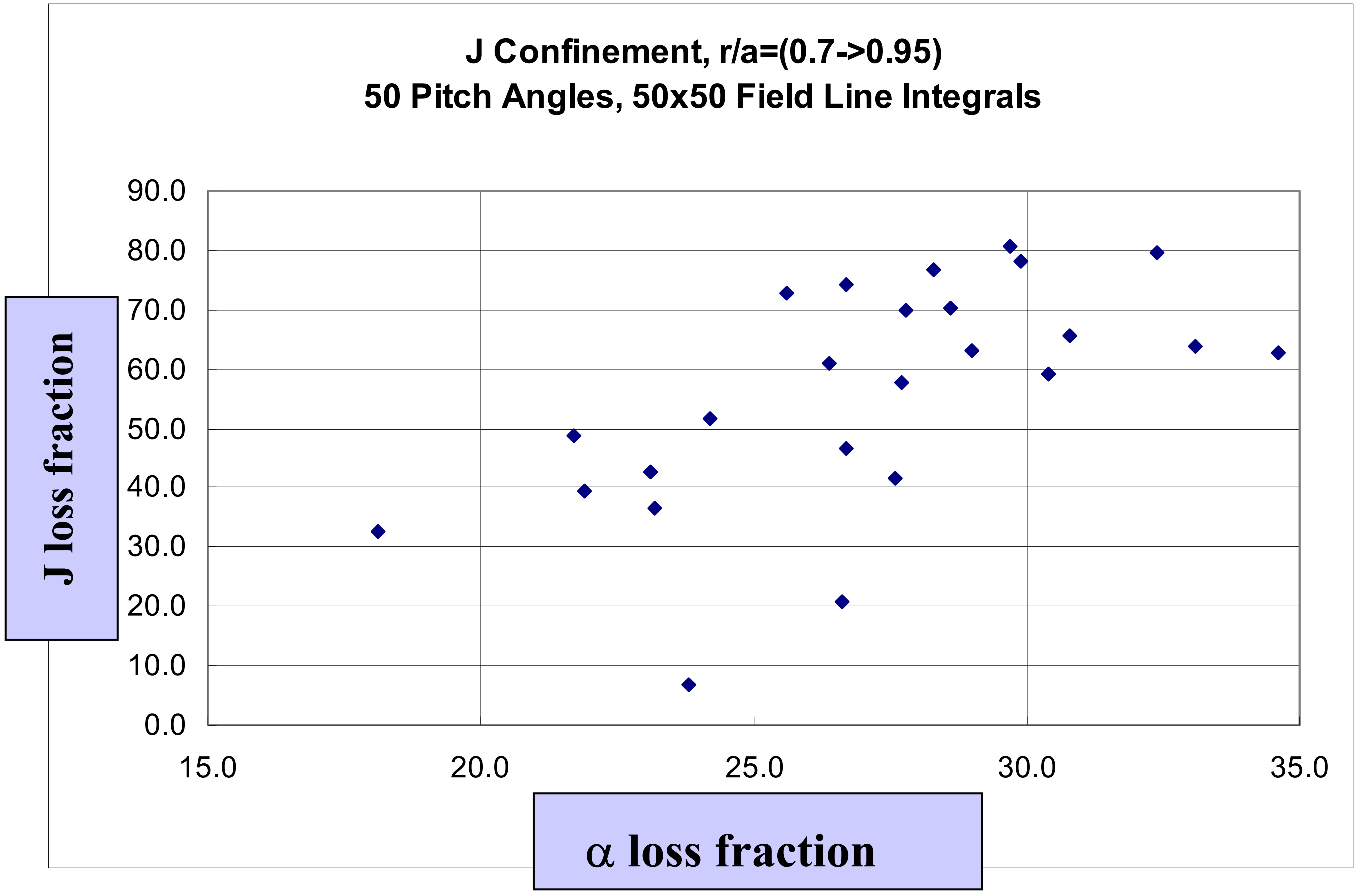
B.3. Effective Ripple (ε effective)

The ε_{eff} is not an effective indicator, as the α losses probably are all collision-less for these configurations.



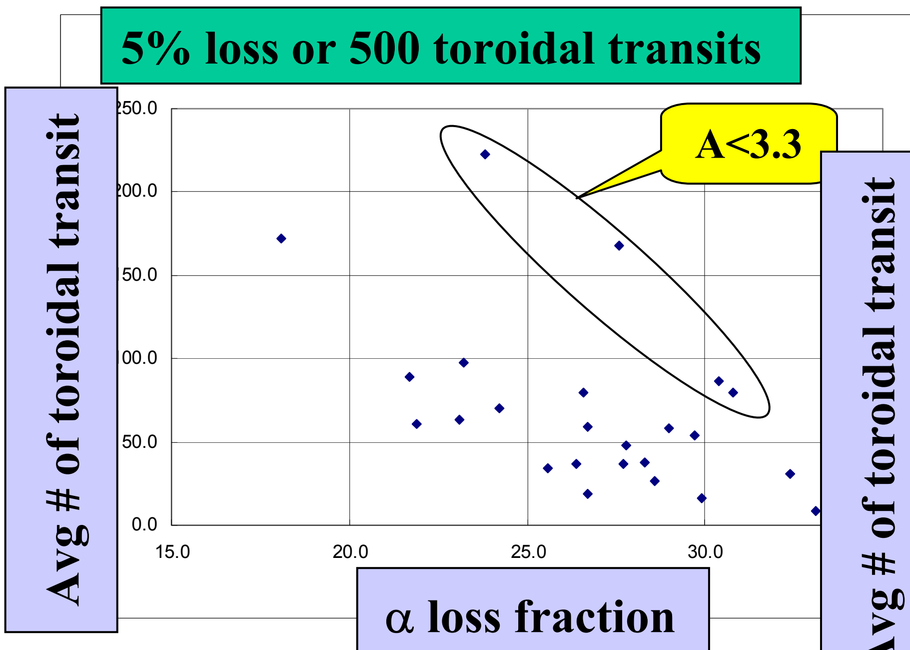
B.4. J|| Confinement

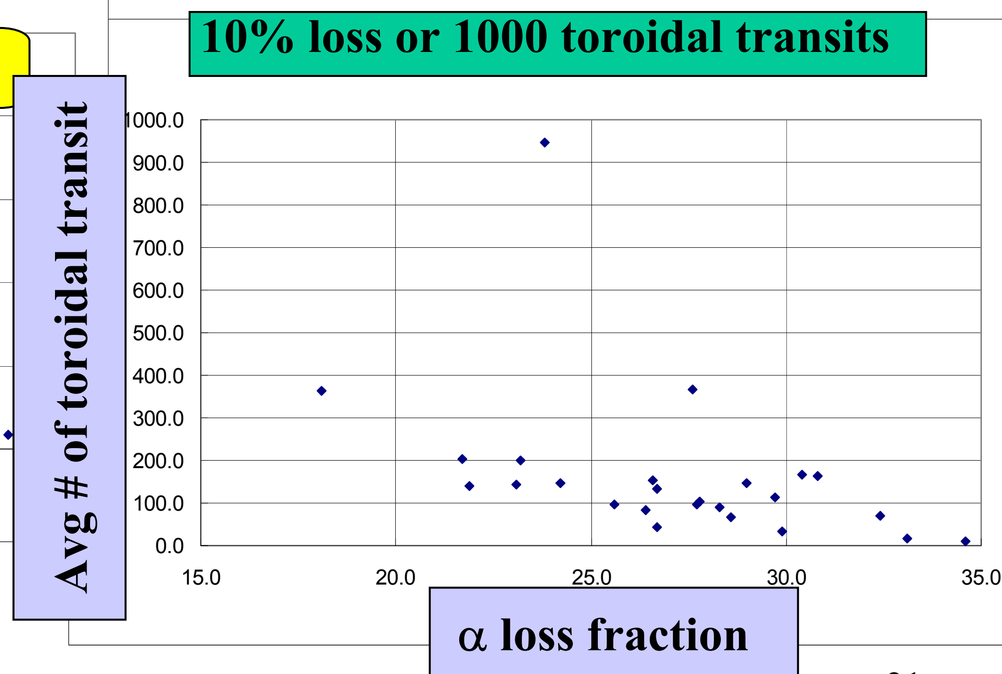
The "leakage" of J contours from a source surface @ r/a=0.7 to a target surface @ r/a=0.95 shows a weak, positive correlation to the α loss, but the scatter is too wide. Tests using this measure to improve α confinement have not shown promise.



B.5. Initial α losses

The initial loss measure shows a much better correlation, clearly separating out the aspect ratio effect on the confinement. There is still a sizable scatter; however. Toroidal transit calculations are based on R=10 m, B=5.5 T, 1024 α's born at r/a=0.5.





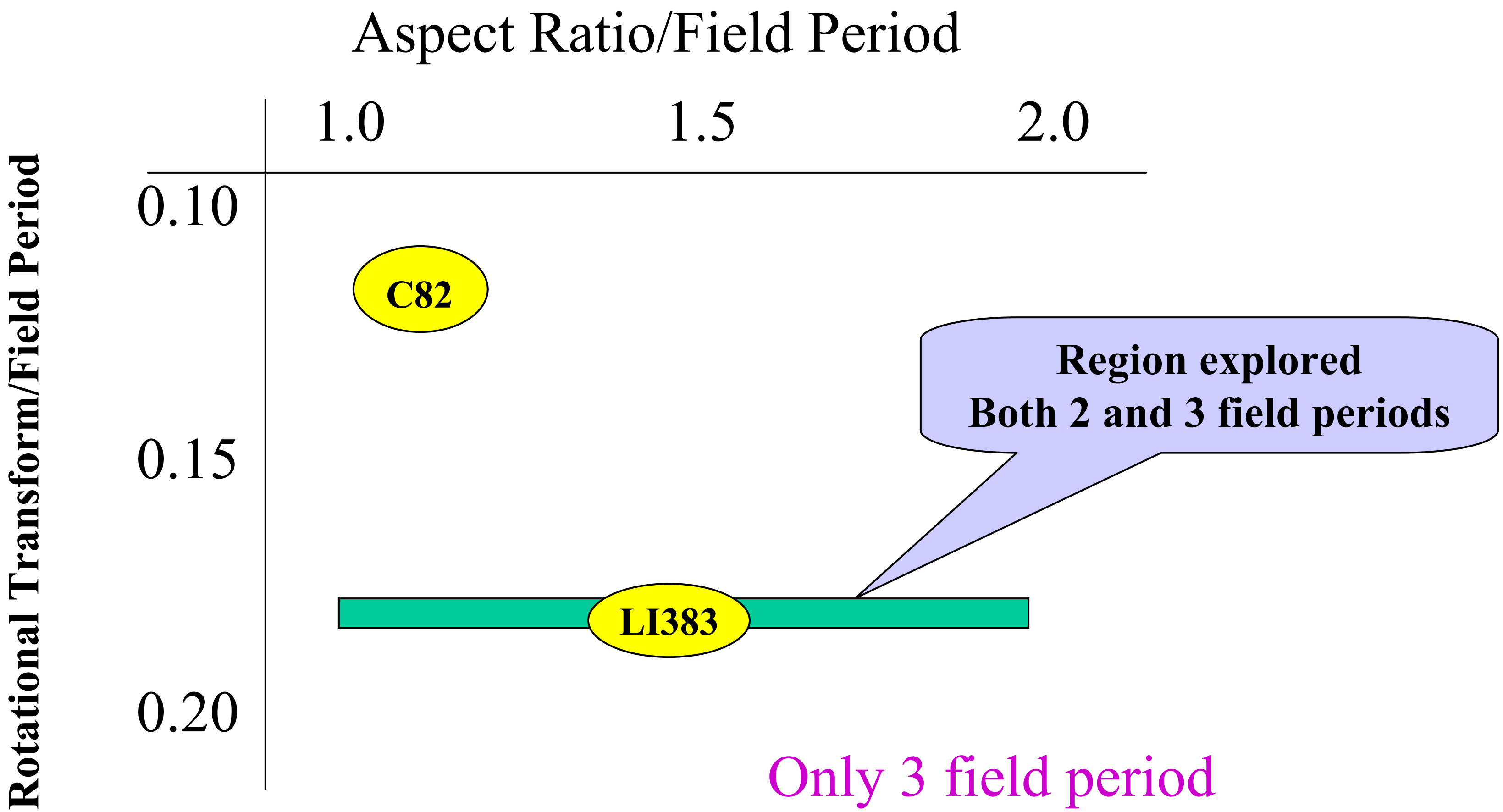
Conclusion:

- The study indicates that to obtain an α optimized QA configuration (if no other effective means are found) the most effective procedure would be
  - Minimize the residual Bm,n to reach the QA regime,
  - Minimize initial α loss using a cutoff of small loss fraction and limited number of toroidal transits.

Tests:

We explore one region in the aspect ratio-iota (A-ι) space with configurations having the same QA and MHD stability characteristics as NCSX.

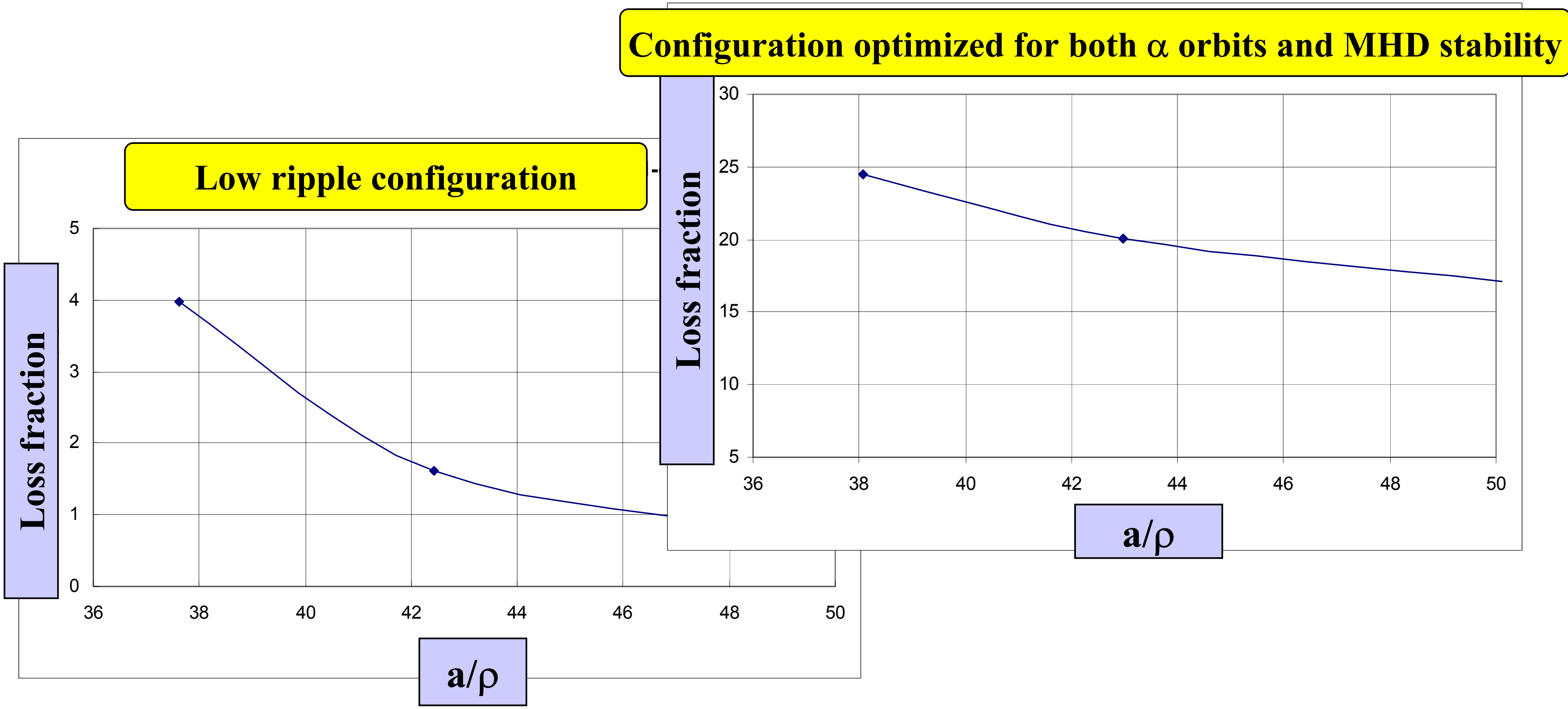
C. Initial configuration space explored for Aries CS



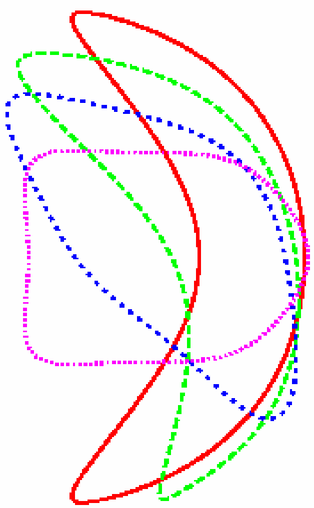
Only 3 field period configurations will be discussed today.

α losses depend on <a>/ρ, the ratio of average minor radius to the gyro-radius of the α particles. To provide an equal basis of comparison, we normalize all configurations to the same volume (1000 m^3).

Scaling of α loss as a function of device minor radius for A=5.8, B=5.5 T, S~$\{1-(r/a)^2\}^8$



C.1. Recently developed 3-field period configurations with MHD stability constraints and improved α orbits at beta=4%.



A=5.7

ι=(0.41, 0.64)

$\langle\kappa\rangle$=1.72,

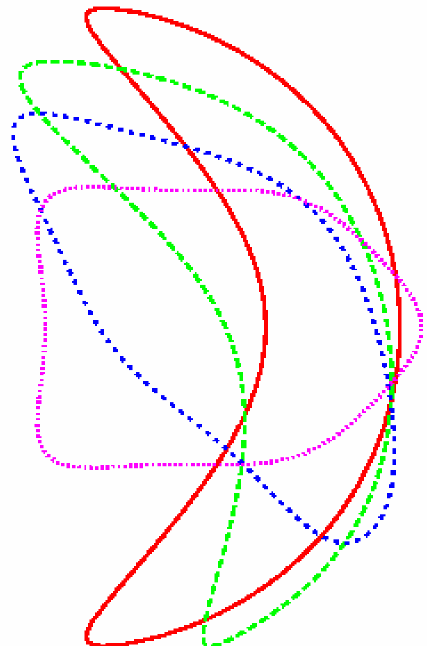
$\langle\delta\rangle$=0.63

Res(Bmn)=0.68%

ε_{eff}=0.094%

J loss=23.3%

α loss=18%



A=4.4

ι=(0.41, 0.67)

$\langle\kappa\rangle$=1.72,

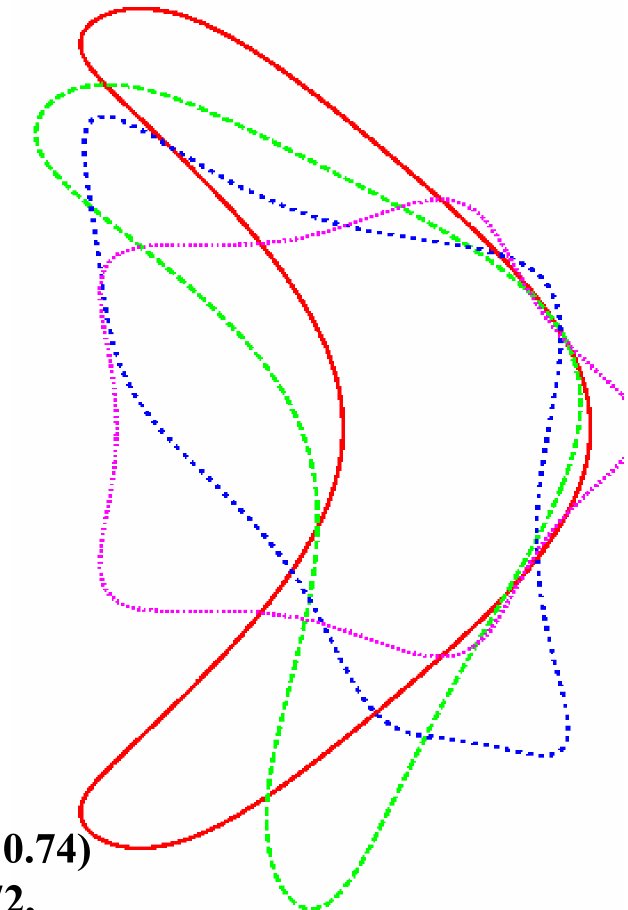
$\langle\delta\rangle$=0.67

Res(Bmn)=1.54%

ε_{eff}=0.21%

J loss=30.0%

α loss=16%



A=3.1

ι=(0.24, 0.74)

$\langle\kappa\rangle$=1.72,

$\langle\delta\rangle$=0.67

Res(Bmn)=2.45%

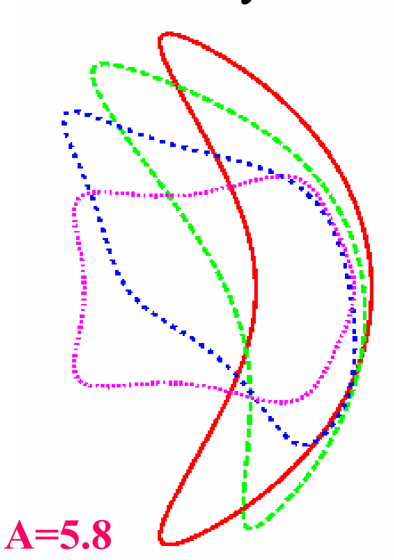
ε_{eff}=0.6%

J loss=77.9%

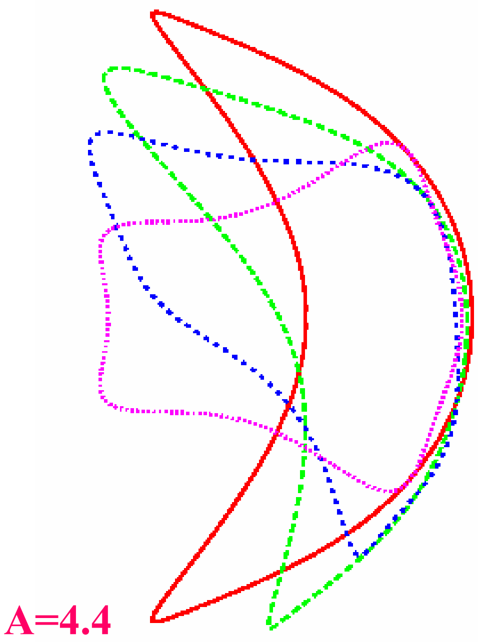
α loss=30.6%

C.2. QA and α confinement can be substantially improved if the requirement of MHD stability is relaxed.

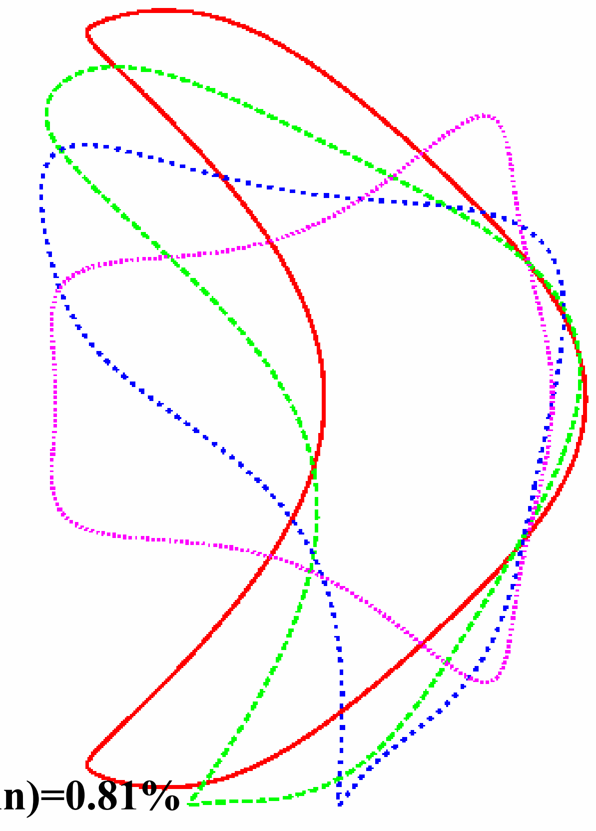
Three field period configurations as in slide 25, except QA is maximized while MHD stability is not considered.



A=5.8
Res(Bmn)=0.24%
ε_{eff}=0.02%
J loss=1.8%
α loss=1.6%

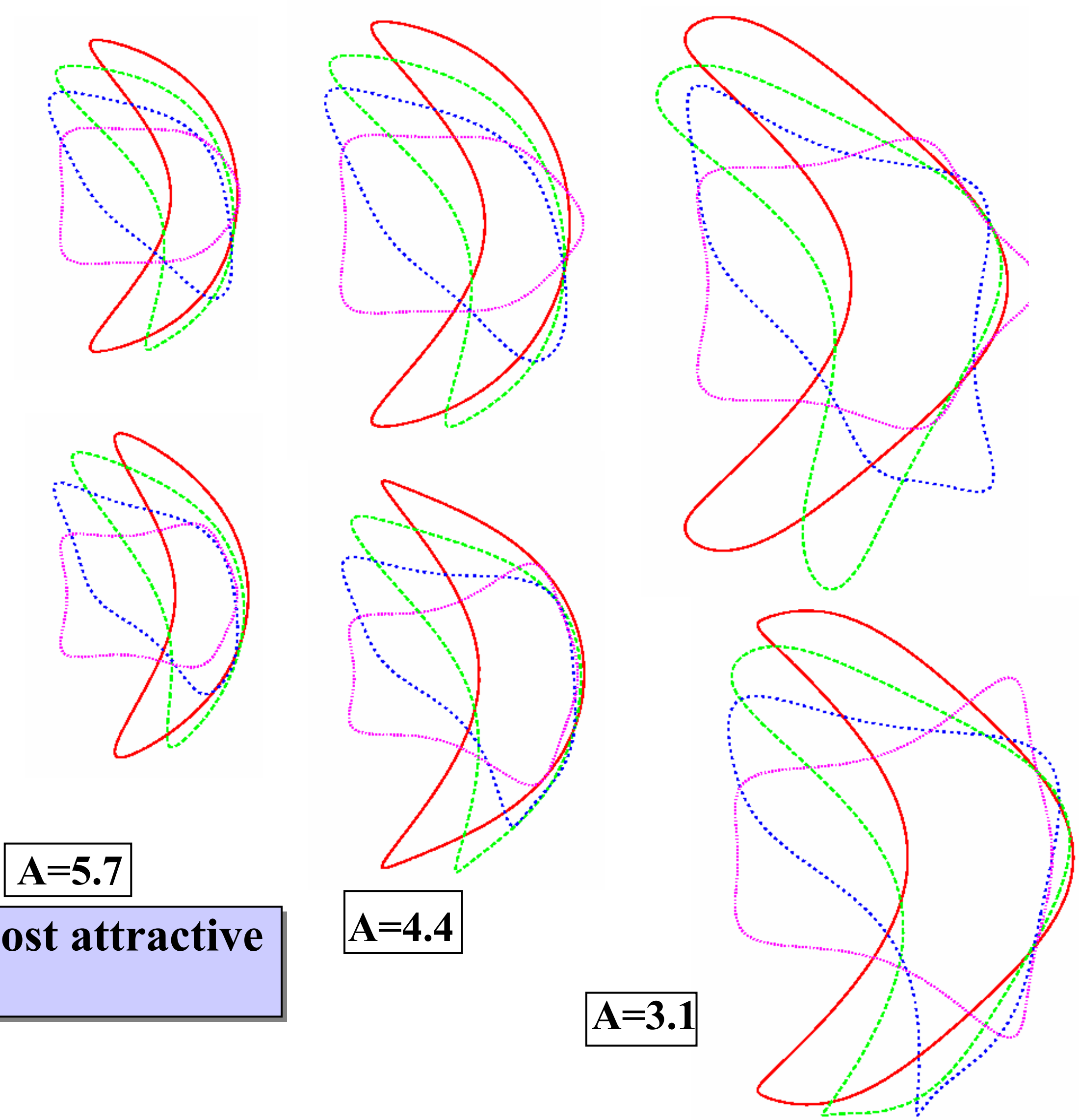


A=4.4
Res(Bmn)=0.24%
ε_{eff}=0.05%
J loss=0.1%
α loss=0.2%



A=3.2
Res(Bmn)=0.81%
ε_{eff}=0.08%
J loss=11.5%
α loss=2.1%

C.3. Comment: Optimizing QA without boundary curvature control may result in a plasma shape which is hard to produce with coils. The lower the aspect ratio is, the harder it is to achieve good QA and the more deformed the boundary is.



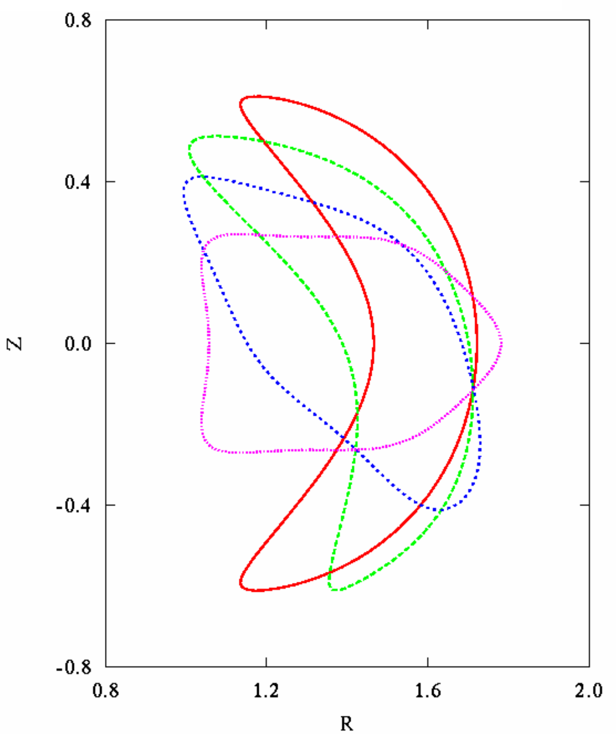
What aspect ratio is most attractive for a reactor?

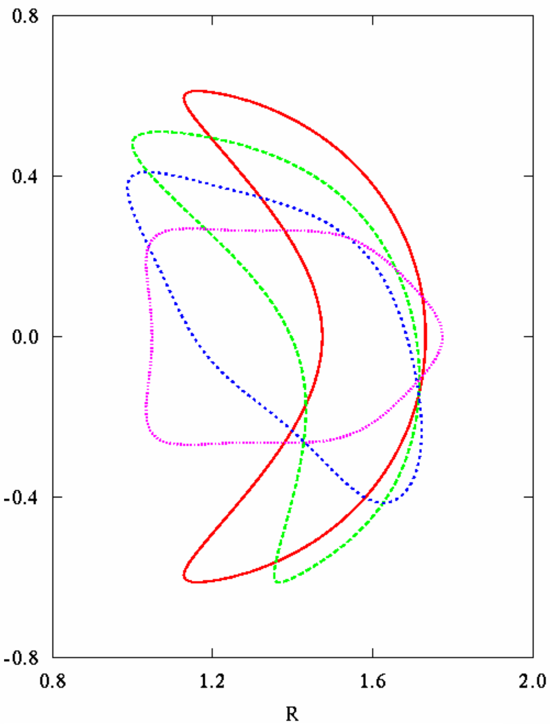
D. Analysis of properties of three 3-field period, A=4.4 configurations with different α confinement characteristics.

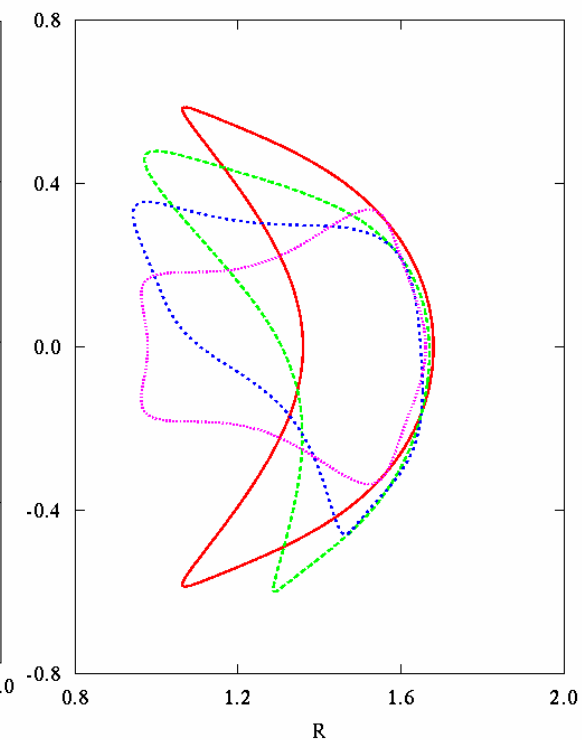
LI383
α loss=25.6%

N3AEC
α loss=16%

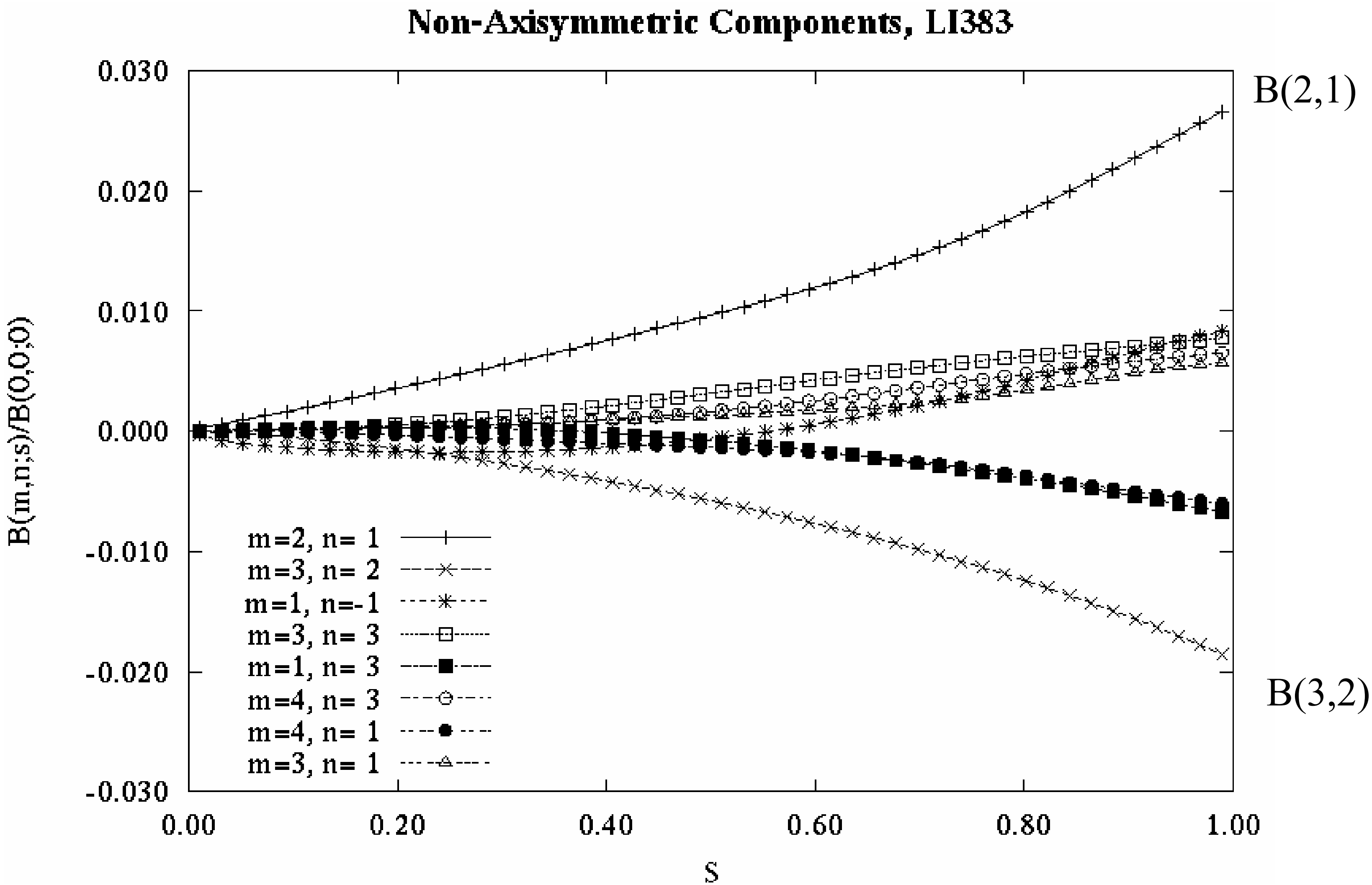
N3AQ2
α loss=0.2%



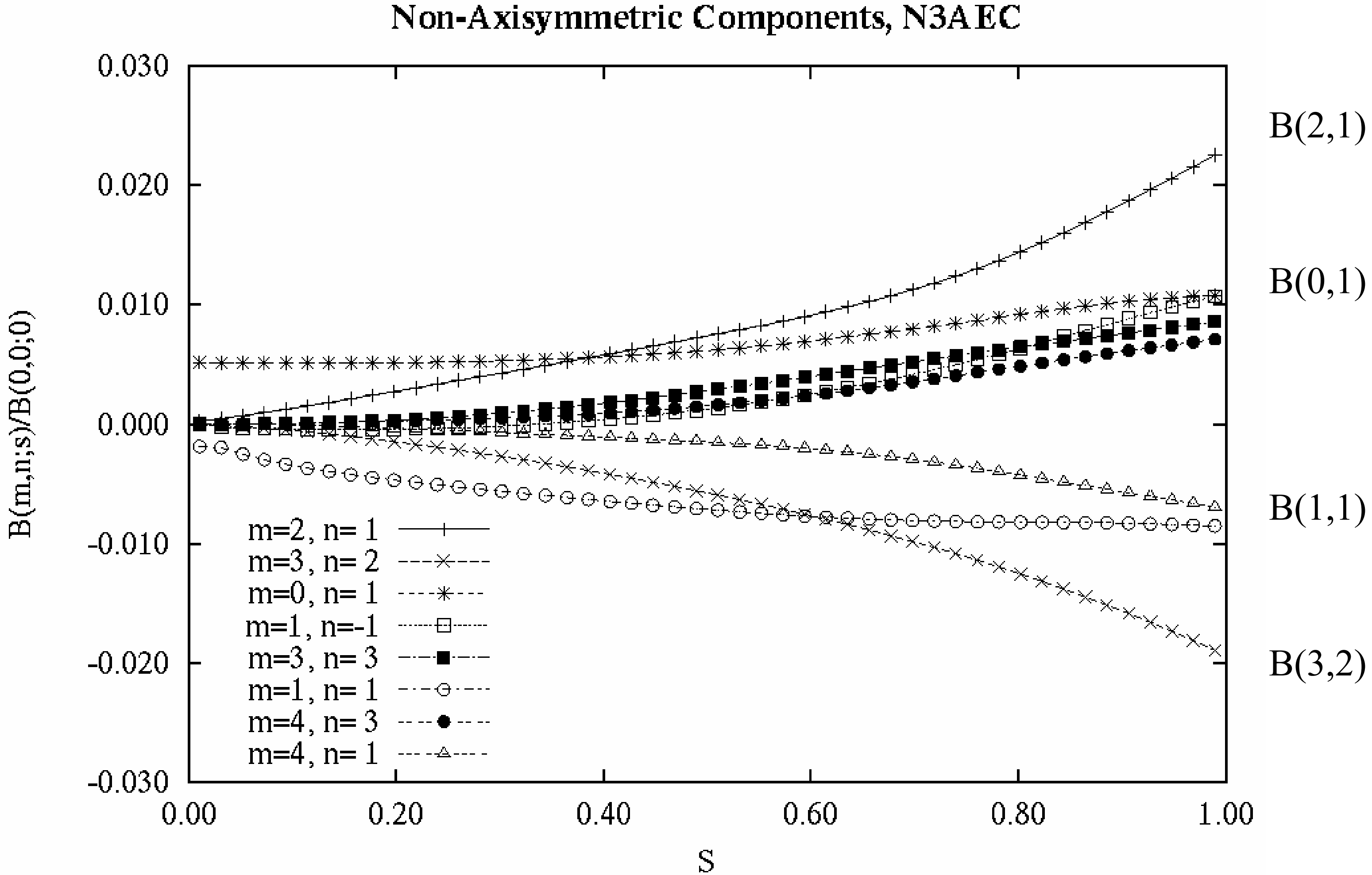




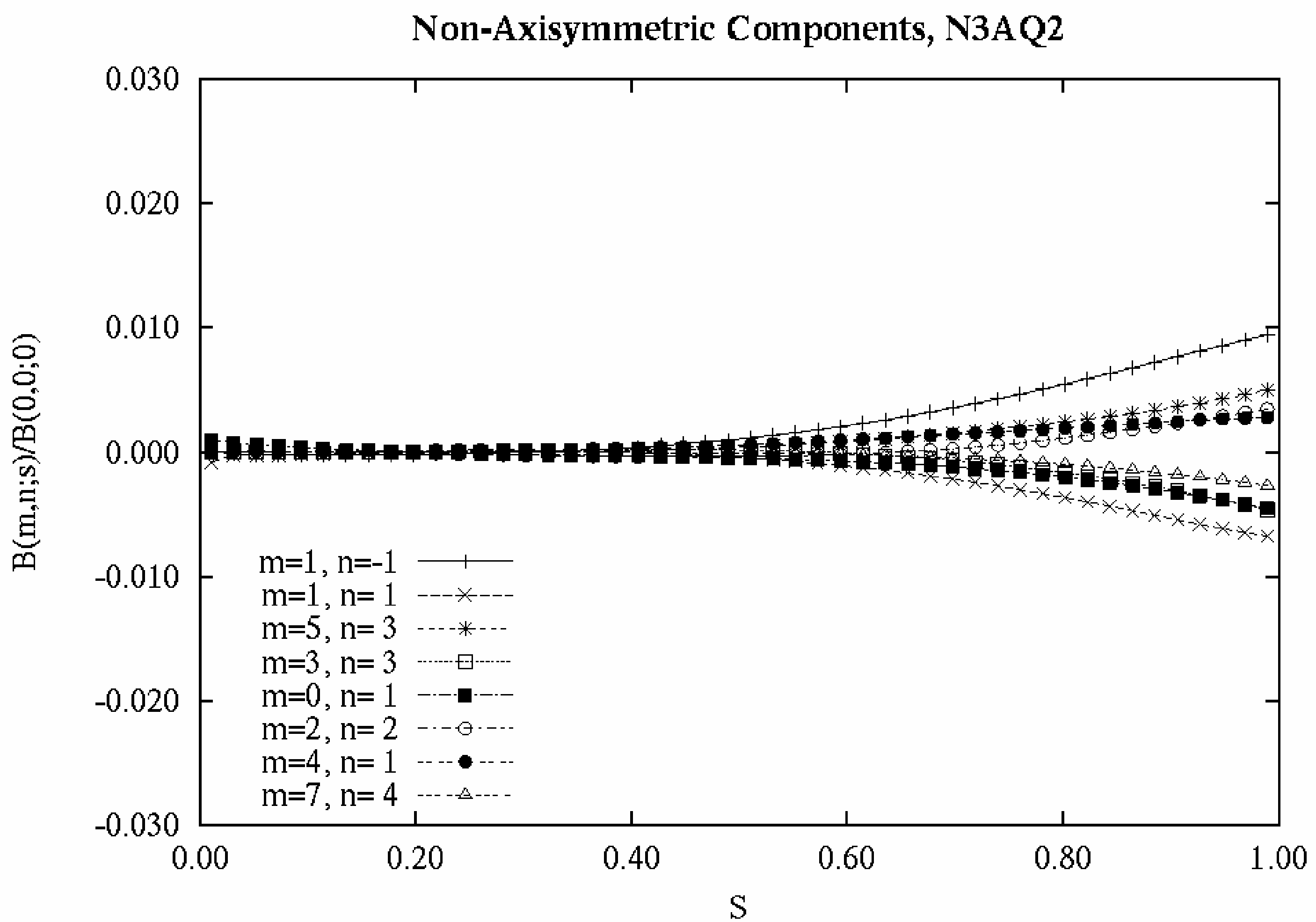
D.1. Eight Fourier harmonics with largest amplitude in the LI383 magnetic spectrum.



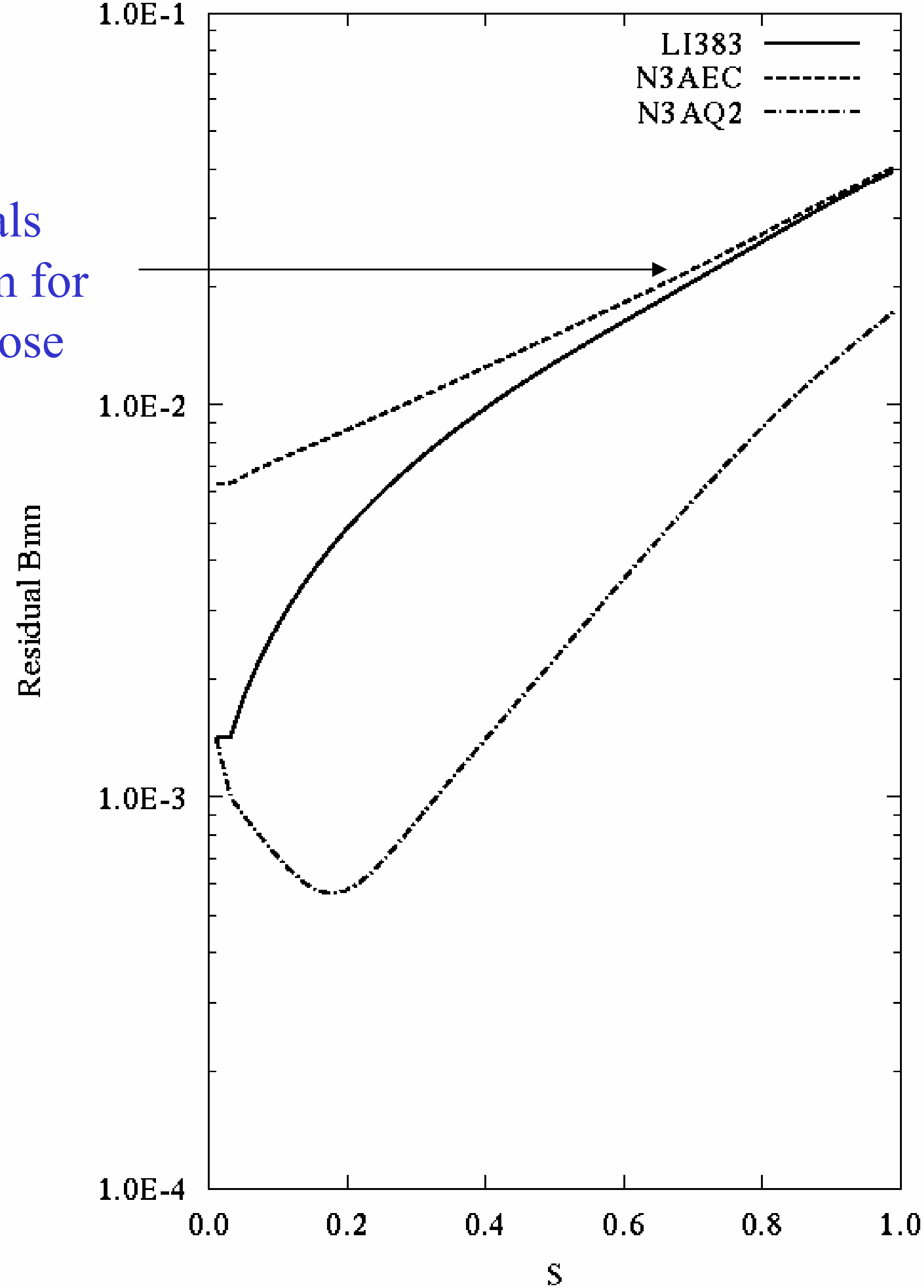
Eight Fourier harmonics with largest amplitude in the N3AEC magnetic spectrum. Note the prominent role of the primary mirror and helical terms. The largest components B(2,1) and B(3,2) have essentially the same magnitude as those in LI383.



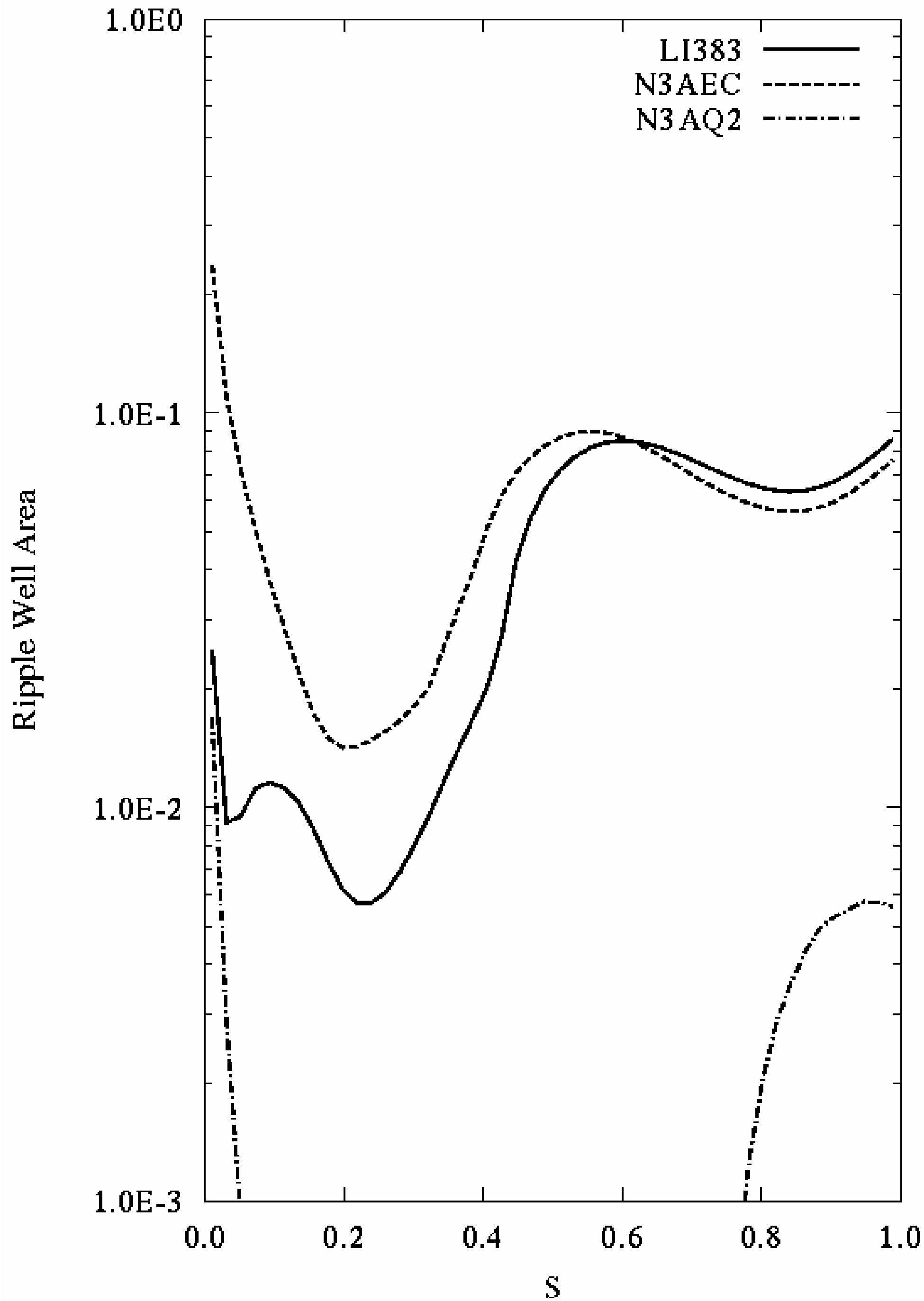
Eight Fourier harmonics with largest amplitude in the N3AQ2 magnetic spectrum. Note that many components are still ~0.5-1% in the outer region.



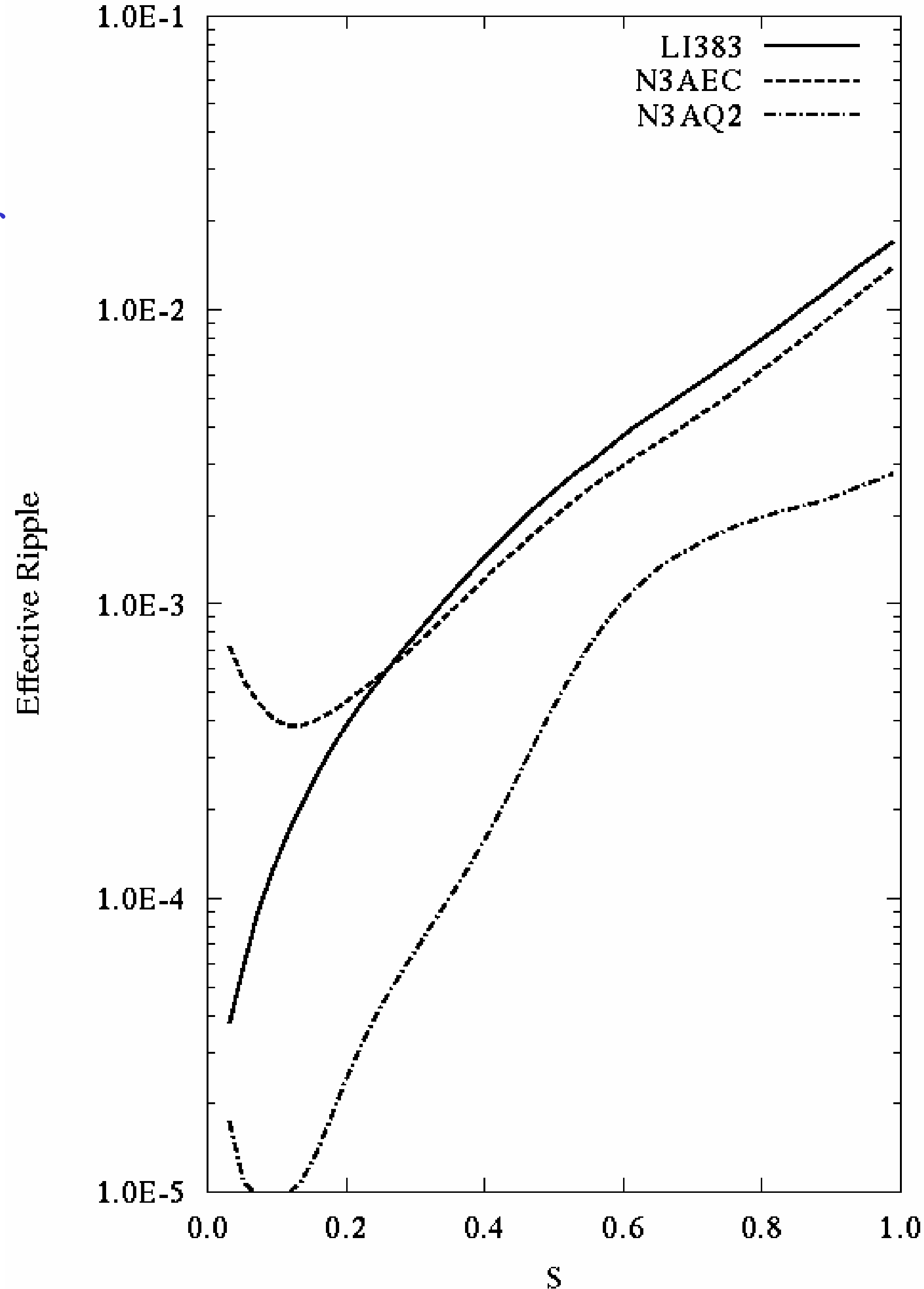
The un-weighted residuals of the magnetic spectrum for N3AEC is larger than those of LI383.



D.2 The pseudo-symmetry measure of N3AEC does show improvement, albeit small, in the outer region.



D.3. The effective ripple of N3AEC is about 20% smaller for r/a>0.7, while that of N3AQ2 is x4 better.



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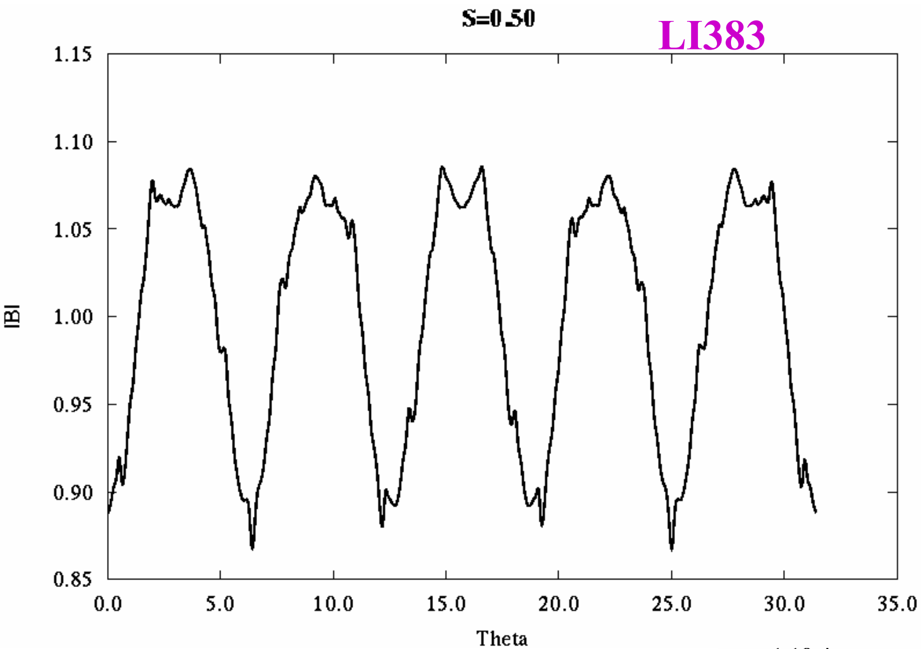
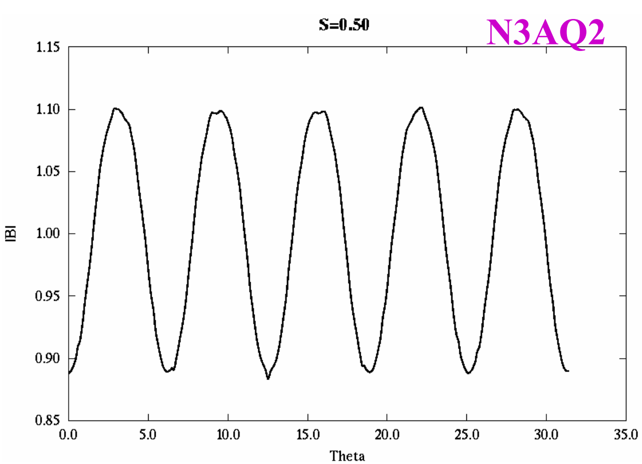
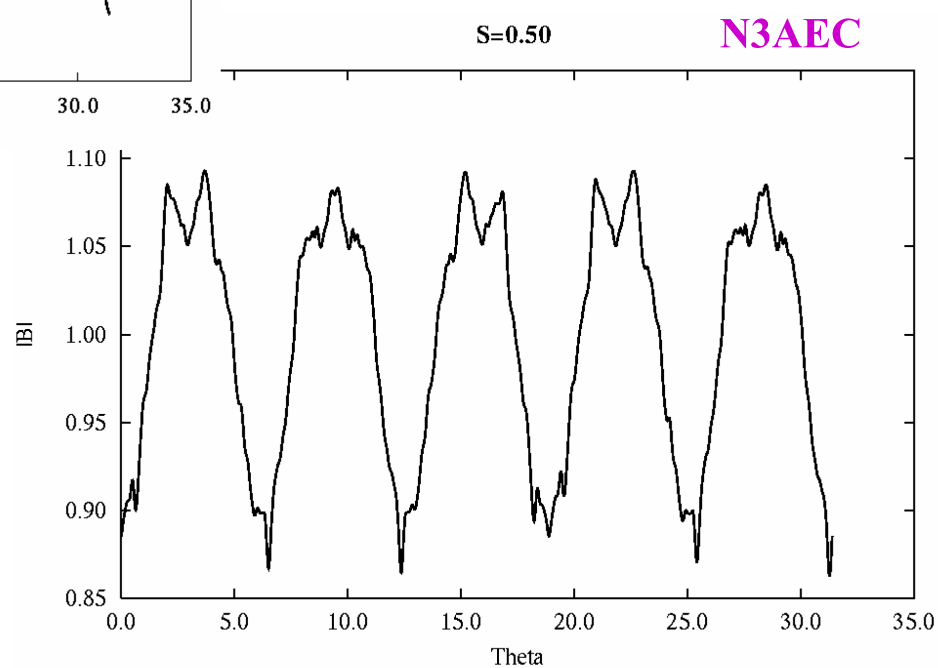
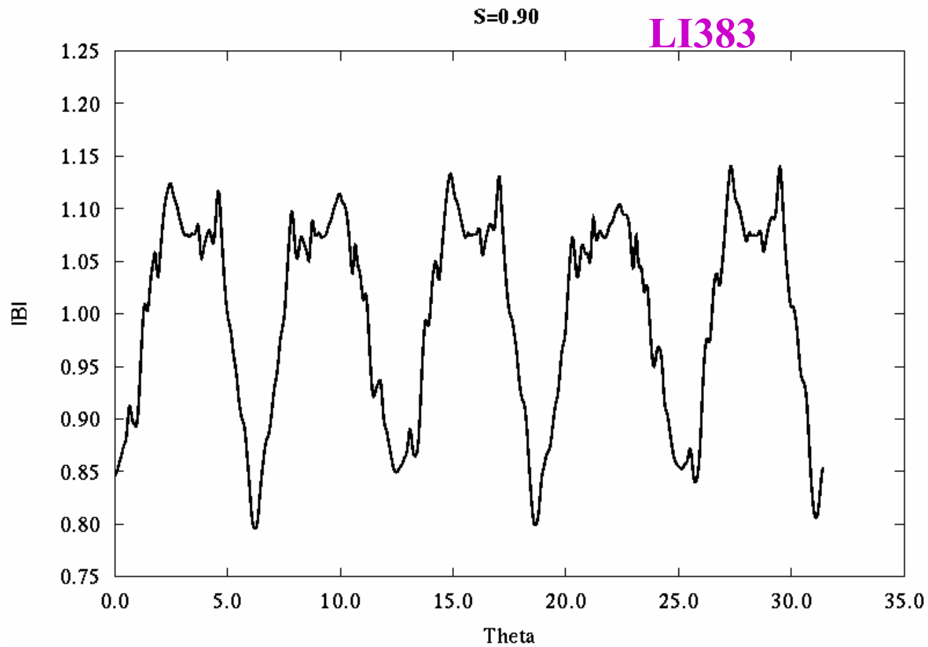
D.4. Mod B along field line on a surface at r/a=0.7



"Ripple" along a magnetic field line for N3AEC is not diminished.

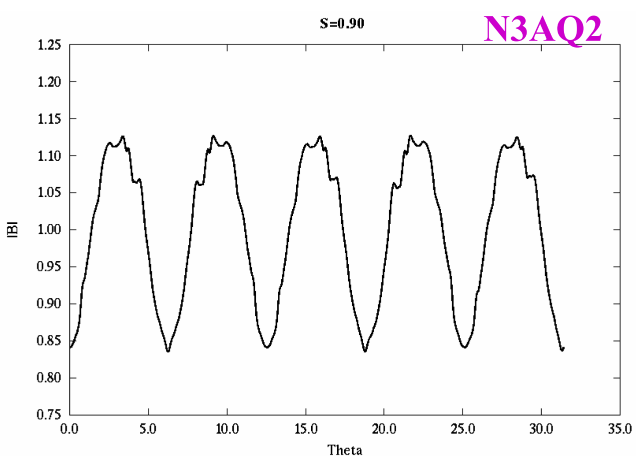


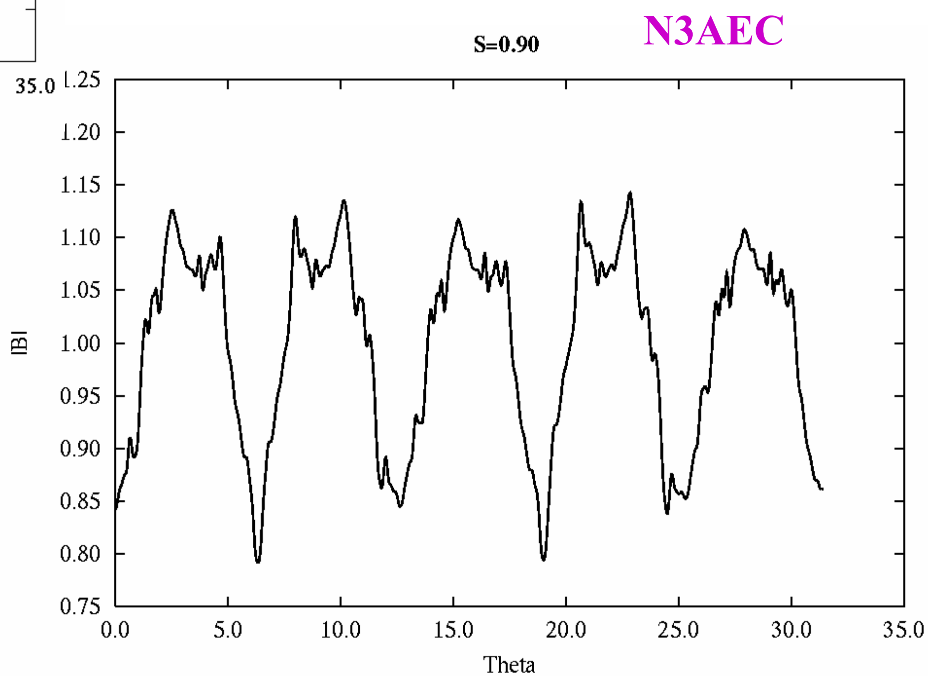




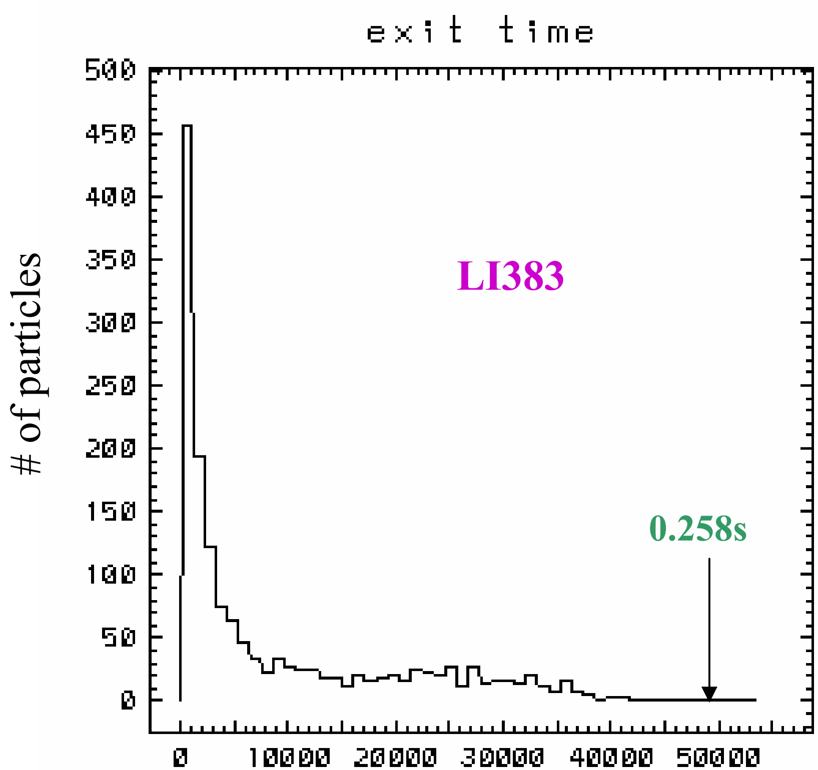
Mod B along field line on a surface at r/a=0.95.

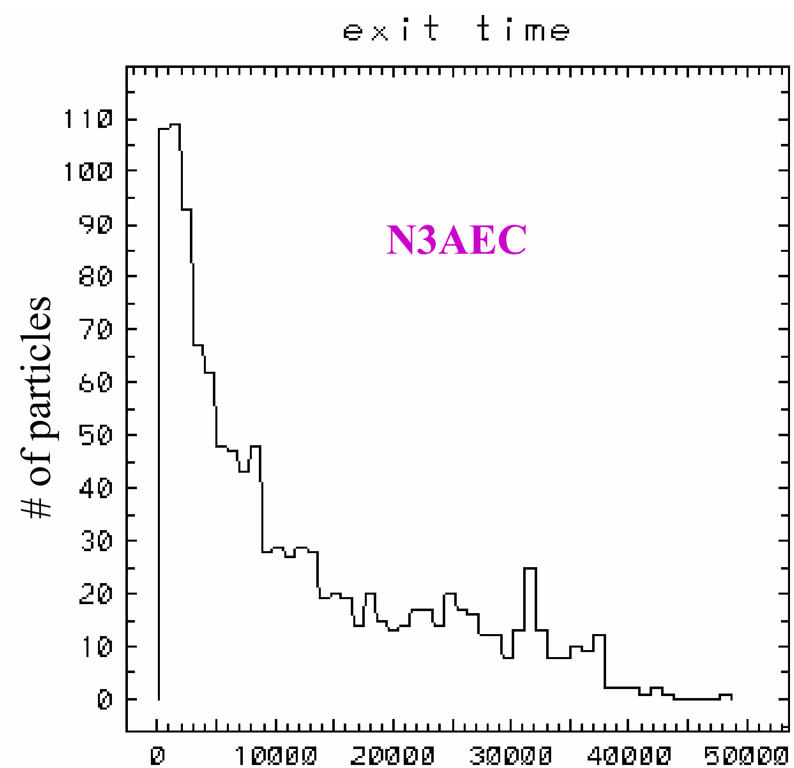
Again, |B| is equally noisy in LI383 and N3AEC.



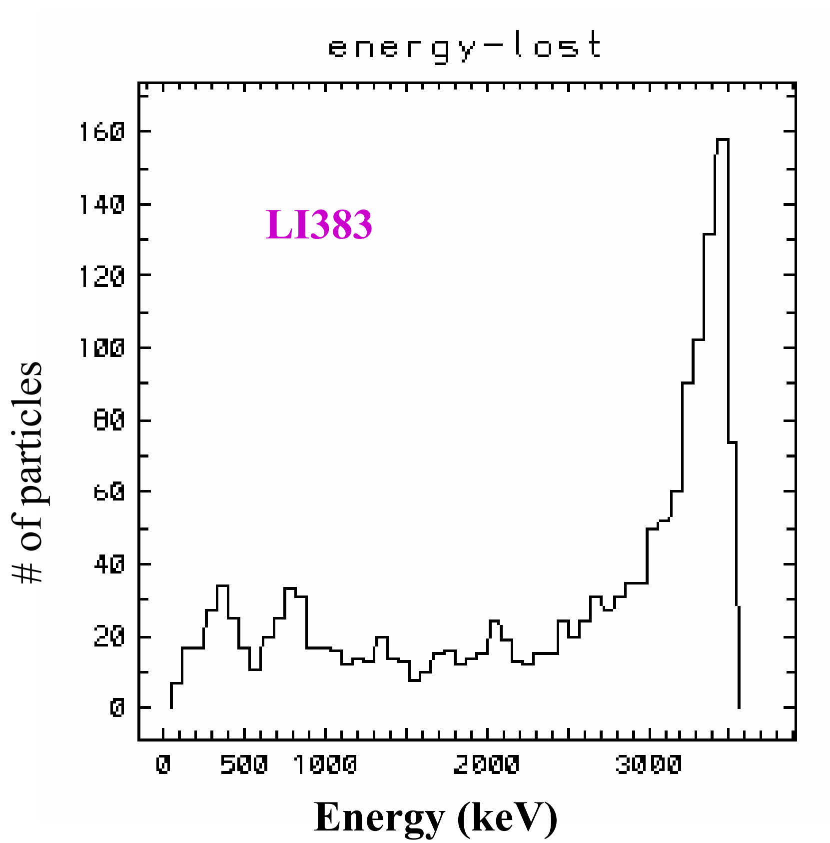


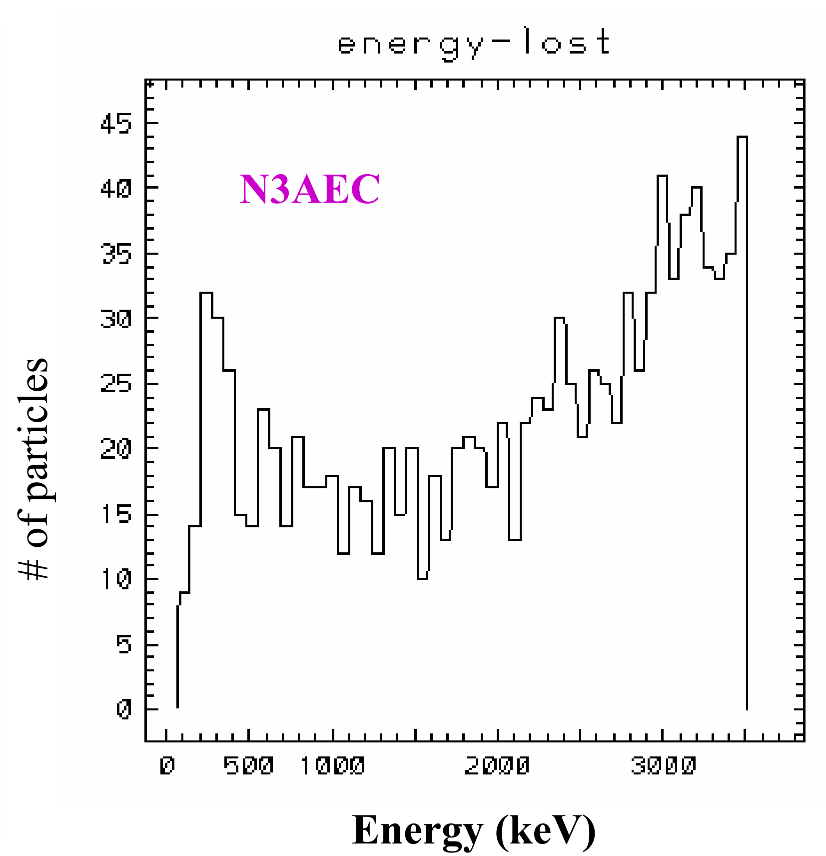
D.5. Comparison of α loss characteristics: Initial loss in LI383 is much more severe than N3AEC. Once the exit time is successfully prolonged in the optimization, the penalty function that minimizes the initial collision-less losses becomes less and less effective.



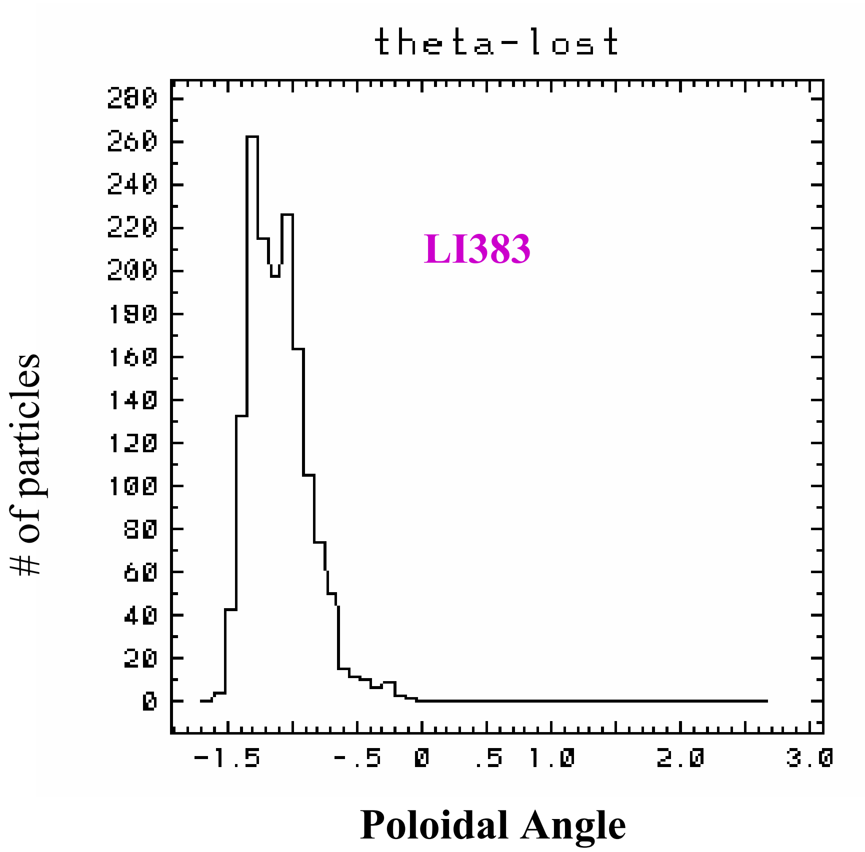


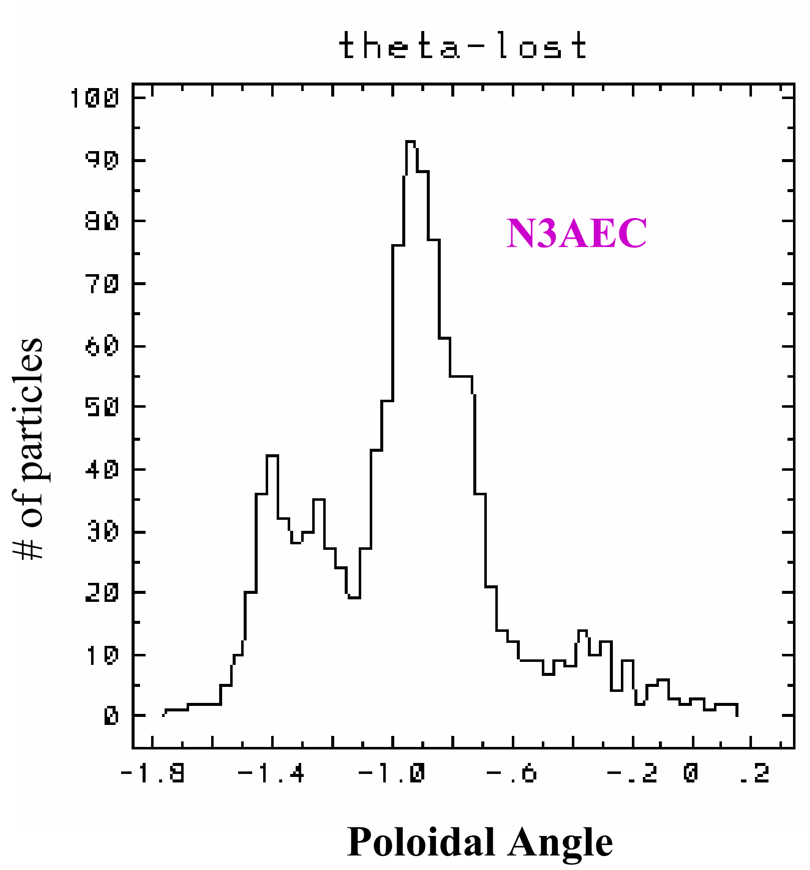
Along with the longer resident time, energy distribution of the lost α's in N3AEC is also broader.





The distribution of poloidal angle where the α’s exit is also different; again it is broader in N3AEC.





Summary of the present work

- A correlation study indicated that eliminating initial bad orbits probably is most effective in configuration search to improve α confinement; it has limits, however, when collision-less bad orbits are gotten rid of.

- Using direct initial orbit optimization, we developed a new class of three-field period configurations. Comparison of properties indicates that there may be an optimal aspect ratio for CS reactors where alpha losses maybe minimized while plasma volume is maximized (or reactor size is minimized for a given power)

What's next?

- Reactor configuration:
  - Raise β to > 5%, What is reactor tradeoff of β vs A?
  - Find pressure/current profiles that give least constraint on getting good QA/α confinement, yet they provide minimum required MHD stability margin,
  - Broaden the search space, looking for other ι's, particularly, in view of the increased plasma current as β is increased,
  - Broaden the search space, looking for other plasma shaping (hence the magnetic field structure) that maybe attractive (e.g., simpler shape for simple coils). Can we incorporate effects of Δmin on shaping?

What's next (cont.)?

- Reactor physics:
  - The role of B(0,1), and possibly B(1,1). Why they are there? How much does a good configuration need?
  - Mechanism of delayed α loss. Can we find an effective and efficient means in configuration optimization to reduce such losses?
  - What MHD stability margins deem acceptable? (e.g., high-n ballooning calculations indicate that the infinite-n solution may be too pessimistic.)