ISSUES IN THE STABILITY ANALYSIS FOR THE ARIES COMPACT STELLARATOR DESIGN

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General Atomics

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GENERAL ATOMICS

SEVERAL IMPORTANT ISSUES ARISE IN EVALUATION OF EQUILIBRIUM TOOLS FOR COMPACT STELLARATOR DESIGN STUDIES

- **Fixed versus free boundary equilibrium:**
  - **Is the plasma boundary shape imposed or determined by coils?**

- **Full 3D versus 2D approximations:**
  - **Is the 'stellarator expansion' (large aspect ratio ordering) utilized?**
  - **Is helical symmetry assumed?**
    - ⇒ **infinite aspect ratio and infinite number of field periods**
  - **Is the geometry averaged over field periods? ⇒ axisymmetry**
  - **Is 'stellarator symmetry' assumed?**
    - ⇒ **At least one toroidal plane exists with up-down symmetry)**

- **Direct or inverse equilibrium:**
  - **Is $\psi(r,z,\phi)$ determined directly or $r(\psi,\chi,\phi)$ and $z(\psi,\chi,\phi)$ computed for a set of prescribed flux values ψ, poloidal angles χ and toroidal angles ϕ?**
  - **Are nested flux surfaces imposed or not?**
    - ⇒ **If nested flux surfaces are assumed, are they *simply nested*?**
    - ⇒ **If nested flux surfaces are not assumed:**
      - → **do well defined magnetic islands exist?**
      - → **do approximate surfaces exist? ('flux surface quality')**

SEVERAL RELATED ISSUES ARISE IN THE EVALUATION OF COMPUTATIONAL STABILITY TOOLS FOR COMPACT STELLARATOR STUDIES

- **Fixed versus free boundary stability:**
  - **Is the plasma bounded by a wall or by a vacuum region?**
    - **⇒ Can both internal and external modes be evaluated?**

- **Nested versus non-nested flux surfaces:**
  - **Are nested flux surfaces assumed for the equilibrium?**
  - **Are non-nested islands or ergodic regions allowed in the equilibrium?**

- **Full 3D versus 2D approximations:**
  - **Is an equilibrium expansion (e.g. 'stellarator expansion') used?**
  - **Is axisymmetry or helical symmetry assumed in the equilibrium?**
  - **Is 'stellarator symmetry' assumed in the underlying equilibrium?**
  - **Is 'field period' symmetry invoked in the selection of fourier modes? (Fourier modes are coupled within mutually decoupled families that depend on the number of field periods)**

SEVERAL ADDITIONAL ISSUES ARISE IN THE EVALUATION OF COMPUTATIONAL STABILITY TOOLS FOR COMPACT STELLARATOR STUDIES

- **Ideal versus resistive MHD stability:**
  - **Are plasma and field components incompressible or compressible?**
  - **Is the plasma resistivity finite?**
  - **Are other non-ideal 'Extended MHD' effects (kinetic effects, fast particles, two-fluid or multi-fluid effects) included?**

- **Primitive or derived MHD equations:**
  - **Solution of initial value, dynamic equations, or variational method**
  - **Primitive physical variables (δp, δx, δB) or derived quantities**

- **Linear versus nonlinear stability:**
  - **Is the underlying equilibrium topology fixed?**

- **Local stability or global stability:**
  - **Is the instability assumed to be localized to a flux surface or extended over a finite region?**
    - ⇒ **If localized, is Mercier or ballooning stability assumed?**
    - ⇒ **If global, is it assumed confined to a region (edge or core)?**

A SUITE OF DIRECT AND INVERSE FREE BOUNDARY AND FIXED BOUNDARY EQUILIBRIUM CODES EXISTS

- **VMEC (S. Hirshman, ORNL):**
  - **Fixed boundary, fully 3D, inverse equilibrium code with simply nested flux surfaces imposed**
- **HINST (K. Harafuji, NIFT):**
  - **Free boundary, fully 3D, direct equilibrium code**
- **PIES (A. Rieman, PPPL):**
  - **Free boundary, fully 3D direct equilibrium code**
- **NEAR (T. Hender, Culham):**
  - **3D MHD equilibrium code with simply nested flux surfaces imposed**
- **BETA (O. Betancourt, Courant Institute):**
  - **3D MHD finite difference equilibrium code**
- **RSTEQ (B. Carreras, ORNL):**
  - **2D equilibrium from averaging 3D equilibrium over field periods**

VMEC IS THE INDUSTRY STANDARD INVERSE FIXED BOUNDARY EQUILIBRIUM CODE

- **Fixed boundary, full 3D, inverse equilibrium code:**
  - **3D plasma boundary surface shape is imposed**
  - **Simply nested flux surfaces:**
    $\Rightarrow$ **Calculates r(ψ,χ,ϕ) and z(ψ,χ,ϕ) for a set of flux surfaces ψ and angles χ and ϕ**
  - **No assumptions of symmetry except 'stellarator symmetry'**

- **Widely utilized stellarator equilibrium code:**
  - **Used extensively since 1984**
  - **Computes equilibrium input for the major global stability codes**

- **Variational moment method to find stationary equilibrium states:**
  - **High accuracy suitable for stability studies**
  - **Computationally efficient**

A CHOICE OF DIRECT FREE BOUNDARY EQUILIBRIUM CODES IS AVAILABLE

- **Free boundary, fully 3D, direct equilibrium codes:**
  - **Two codes are available and used extensively: HINST and PIES**
  - **Plasma boundary surface shape is defined by coils**
    - ⇒ **Plasma surface is the outermost surface with confined field lines**
  - **Calculate ψ (r,z,ϕ) irrespective of whether ψ forms nested surfaces**
    - ⇒ **Islands and ergodic regions can be treated**
  - **No assumptions of symmetry except 'stellarator symmetry'**
- **HINST (K. Harafuji NIFS):**
  - **Iterates to an equilibrium by relaxation of dissipative MHD equations to steady state**
    - ⇒ **Enhanced dissipation used to accelerate relaxation to equilibrium**
  - **Code is extremely time consuming**
  - **Not widely utilized outside of Japan**
- **PIES (A. Rieman, PPPL):**
  - **Integrates magnetic differential equation div(j) = 0 with $j_\perp$ from grad(p)= $j_\perp$xB**
  - **Use Amperes Law curl(B) = μ_0j to determine field line structure**
  - **Time consuming but routinely used for NCSX**

A SUITE OF LINEAR GLOBAL MHD STABILITY TOOLS IS AVAILABLE FOR STELLARATORS

- **HERA (R. Gruber, EPFL):**
  - **Helically symmetric 2D linear MHD stability code**
- **TWIST (S. Medvedev, Keldysh Inst.):**
  - **Linear ideal MHD stability code with 3D equilibrium averaged over field periods**
- **RST (B. Carreras, ORNL):**
  - **Linear ideal MHD stability code with 3D equilibrium from RSTEQ averaged over field periods**
- **TERPSICHORE (W.A. Cooper, CRPP/EPF-Lausanne):**
  - **Free boundary, fully 3D, global, linear *ideal* MHD stability code with simply nested flux surfaces assumed**
- **CAS-3D (C. Nuhrenberg, Max Planck Institute):**
  - **Free boundary, fully 3D, global, linear *ideal* MHD stability code with simply nested flux surfaces assumed**
- **SPECTOR-3D (R.G. Storer, Flinders University):**
  - **Free boundary, fully 3D, global, linear *resistive* MHD stability code with simply nested flux surfaces assumed**

TERPSICHORE AND CAS-3D ARE STATE OF THE ART FREE BOUNDARY FULLY 3D LINEAR IDEAL MHD STABILITY CODES

- **Comprehensive free boundary, fully 3D, linear ideal MHD stability:**
  - **Equilibrium plasma boundary surface shape is bounded by a vacuum region and can be perturbed**
    - ⇒ **external modes can be treated**
  - **No expansions or assumptions of symmetry in underlying equilibrium except 'stellarator symmetry'**
  - **Advantage taken of 'field period' symmetry**
  - **Variational energy principle formulation**
  - **Compressible and incompressible ideal MHD plasma versions exist**

- **Restricted to simply nested inverse equilibria:**
  - **Input equilibrium from VMEC**

- **Extensively benchmarked:**
  - **Against each other for 3D LHD cases**
  - **Against 2D linear MHD stability codes for tokamak cases**

TERPSICHORE AND CAS-3D ARE ESSENTIALLY EQUIVALENT CODES

- **TERPSICHORE (W.A. Cooper, CRPP):**
  - **Robust and widely utilized 3D stability code**
    - ⇒ **extensively used in design studies for LHD and NCSX, and in exploratory stability studies**
  - **The vacuum formulation (so-called 'pseudo-vacuum) has numerical problems except in LHD and quasi-symmetric cases**
  - **Code runs on NEC-SX5 and recently ported to SGI Octane (GA)**
    - ⇒ **porting to Pentium-4 Linux system looks promising**

- **CAS-3D (C. Nuhrenberg, MPI):**
  - **Used extensively in the design of W7AS and W7X**
  - **Code consists of a double family of codes:**
    - ⇒ **Compressible and incompressible ideal MHD plasma and compressible and incompressible toroidal field options**
    - ⇒ **Explicit extraction of high toroidal and poloidal mode numbers**
  - **Vacuum formulation is not fully benchmarked and tested**
  - **Runs on several platforms in Europe and at PPPL**

SPECTOR-3D IS THE ONLY EXISTING LINEAR GLOBAL RESISTIVE MHD STABILITY CODE

- **Free boundary, fully 3D, global, resistive MHD stability code:**
  - **Equilibrium plasma boundary surface shape is imposed and is assumed bounded by a perfectly conducting wall**
  - **No expansions or assumptions of symmetry in underlying equilibrium except 'stellarator symmetry'**
  - **Advantage taken of 'field period' symmetry**

- **Restricted to simply nested inverse equilibria:**
  - **Input equilibrium from VMEC**
  - **Initial value code formulation in terms of magnetic and velocity potentials**
  - **An option with a vacuum does not appear to be available (i.e.**
    $\Rightarrow$ **internal global modes only**

- **Not yet widely utilized code:**
  - **Code has difficulties with accuracy of VMEC equilibria near the axis**
  - **Code is presently incompressible but plans are underway for a compressible option**

IT IS NOT CLEAR THAT LOCALIZED MHD STABILITY IS RELEVANT FOR STELLARATORS

- **Stellarator experiments have substantially exceeded the stability limits predicted from local Mercier and ballooning code calculations:**
  - **LHD and W7AS have exceeded the predicted β limits by a factor two**

- **Localized instabilities predicted by these calculations well below the limit predicted by global MHD codes should be strongly stabilized by finite orbit effects:**
  - **In tokamaks, finite toroidal mode number n corrections to ballooning and Mercier stability are generally small**
    - ⇒ **the infinite n calculation accurately reflects the real limit**
  - **In stellarators, the global stability codes in principle incorporate the high n localized modes with low and intermediate n**
    - ⇒ **In practice the high n modes are numerically excluded**

- **Global calculations are much closer to experimental stability limits:**
  - **By excluding the high n modes that in practice are stabilized by finite orbit effects the global codes are more closely reflecting the physics**
    - ⇒ **More realistic to ignore localized Mercier and ballooning limits and just use low and intermediate n global calculations**
    - ⇒ **In the global calculations the range of n needs to be terminated at the limit where finite orbit effects become important**

3D GLOBAL EXTENDED MHD STABILITY CODE TOOLS ARE AVAILABLE

- **M3D (W. Park, PPPL):**
  - **Full 3D nonlinear extended MHD code has recently been applied to compact stellarator equilibria**
  - **Presumably assumes nested (but not necessarily simply nested) flux surfaces, at least to a high approximation:**
    - ⇒ **Otherwise numerical problems arise in accurately resolving perturbations parallel and perpendicular to the equilibrium field**

- **NIMROD (National Nimrod Team):**
  - **Full 3D nonlinear extended MHD code can also be applied in principle to compact stellarator equilibria**
  - **Presumably assumes nested (but not necessarily simply nested) flux surfaces, at least to a high approximation:**
    - ⇒ **Otherwise numerical problems arise in accurately resolving perturbations parallel and perpendicular to the equilibrium field**

MISCELLANEOUS COMMENTS: EQUILIBRIUM

- **Equilibrium stability and transport are not separable in stellarators:**
  - **Existence of nested flux surface equilibrium can be considered as either an equilibrium or a stability problem**
  - **Transport is strongly dependent on underlying equilibrium magnetic topology and in turn determines the possible equilibrium profiles**
- **Equilibrium codes can therefore be considered stability codes:**
  - **An equilibrium computed under certain constraints must be stable unless those constraints can be avoided by a physically valid motion:**
  - **VMEC imposes fixed boundary and simply nested flux surfaces:**
    - $\Rightarrow$ **Stable to all fixed boundary topology preserving instabilities**
  - **PIES and HINST have essentially no constraints on the equilibrium:**
    - $\Rightarrow$ **Stable to all MHD instabilities preserving 'stellarator symmetry'**
- **VMEC is interfaced to TERPSICHORE CAS-3D and SPECTOR-3D:**
  - **PIES or HINST are necessary to determine to what extent a nested flux surface configuration actually exists with a specific finite coil set**
- **The NCSX Group has developed a procedure for optimizing stellarator design using reverse engineering (S. Hudson, PPPL):**
  - **Design the configuration for specific physics properties using VMEC**
  - **Find a coil set to give a nearby similar free boundary equilibrium**
  - **Heal remaining bad surfaces by shifts in coil positions or trim coils**

MISCELLANEOUS COMMENTS: STABILITY

- **CAS-3D and TERPSICHORE are almost equivalent**
  - **TERPSICHORE is apparently easier to use and does have a working vacuum boundary condition for some cases at least**
  - **CAS-3D has a number of versions within the code family, some of which greatly reduce the computation time**

- **SPECTOR-3D is still under development but is used for the H-1 Heliac at ANU in Canberra**
  - **There is no other comparable code and like most resistive MHD codes it tends to be restricted to lower magnetic Reynolds numbers than are typical of experiments**

- **Local stability criteria are routinely computed by the major global linear stability codes TERPSICHORE and CAS-3D**
  - **Local stability criteria can be ignored when significantly different from global stability limits calculated with sufficiently high mode number**

SUMMARY OF EQUILIBRIUM AND STABILITY ISSUES

- **Equilibrium codes:**
  - **Situation is adequate with VMEC for accurate and fast equilibrium calculations suitable for stability analysis and**
  - **HINST and PIES for evaluating the 'realizability' of the equilibria**
    - ⇒ **Major deficiency: Lack of a 3D Stellarator equilibrium fitting code analogous to EFIT for 2D equilibria**
- **Global linear stability codes:**
  - **Situation is almost adequate for linear ideal stability:**
    - ⇒ **A completely robust free boundary and vacuum formulation does not yet exist**
    - ⇒ **Major deficiency: There is no linear stability code capable of handling islands or ergodic regions (except in the sense in which PIES and HINST can guarantee some stability)**
  - **Situation is still inadequate for linear resistive stability:**
    - ⇒ **SPECTOR-3D presently suffers from numerical problems when mapping from VMEC near the axis**
    - ⇒ **Major deficiency: No linear code exists based on the asymptotic matching method suitable for high S**

SUMMARY OF EQUILIBRIUM AND STABILITY ISSUES

- **Localized linear stability codes:**
  - **Situation is probably adequate for localized ideal stability:**
    - ⇒ **There is a consensus developing that predictions from the infinite n limit are irrelevant and misleading**
    - ⇒ **Global linear codes are better suited to optimizing the stability limits if a sufficiently high range in n is covered**
    - ⇒ **Major deficiency: No formulation of finite n corrections in the high n stability limit**

- **Global nonlinear stability codes:**
  - **Situation is almost adequate for nonlinear extended MHD stability**
  - **The M3D code can be used in limited studies and NIMROD can also handle stellarator geometry in principle**
    - ⇒ **Major deficiency: Nonlinear simulations are extremely time consuming**

STABILITY ANALYSIS PLANS FOR 2003

- **Port TERPSICHORE to local GA Linux workstation:**
  - **Assistance from W.A. Cooper with non source code libraries**
  - **Assistance from G.Y. Fu (PPPL) with interpretations**
  - **Obtain equilibrium from VMEC (M.C. Zarnstorff PPPL):**
    - **Stability analysis using TERPSICHORE for significant range of toroidal mode number n**
    - **Monitor high n stability but ignore for optimization**
    - **Increase β or estimate modifications to plasma parameters required to stabilize**
  - **Modify VMEC equilibrium accordingly (M.C. Zarnstorff):**
    - **Iterate with stability analysis using TERPSICHORE**
    - **Converge on final equilibrium when β limit is reached and simple adjustments to other parameters are ineffective**
- **Reconstruct free boundary equilibrium using PIES (M.C. Zarnstorff):**
  - **Modification to coils and plasma parameters to reproduce a reasonable set of free boundary nested surfaces**
- **Iterate with VMEC and TERPSICHORE calculations:**
  - **Linear resistive and nonlinear MHD calculation of final equilibria**