Assessment of Streaming Instabilities for Neutralized- Ballistic Transport: *A Progress Report*

D. V. Rose, T. C. Genoni, and D. R. Welch
Mission Research Corp.

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Overview:

- We are working towards an assessment of the possible growth and saturation of two-stream instabilities for heavy ion beams propagating in the reactor chamber.
- In particular, the neutralized-ballistic transport mode is being considered.
- Can we build on the previous of work on converging beams (e.g., P. Stroud [1]), or is a new assessment needed?
- Assess impact of instability saturation on net current generation – a possible connection with past simulation work showing net current generation near the beam focus [2].

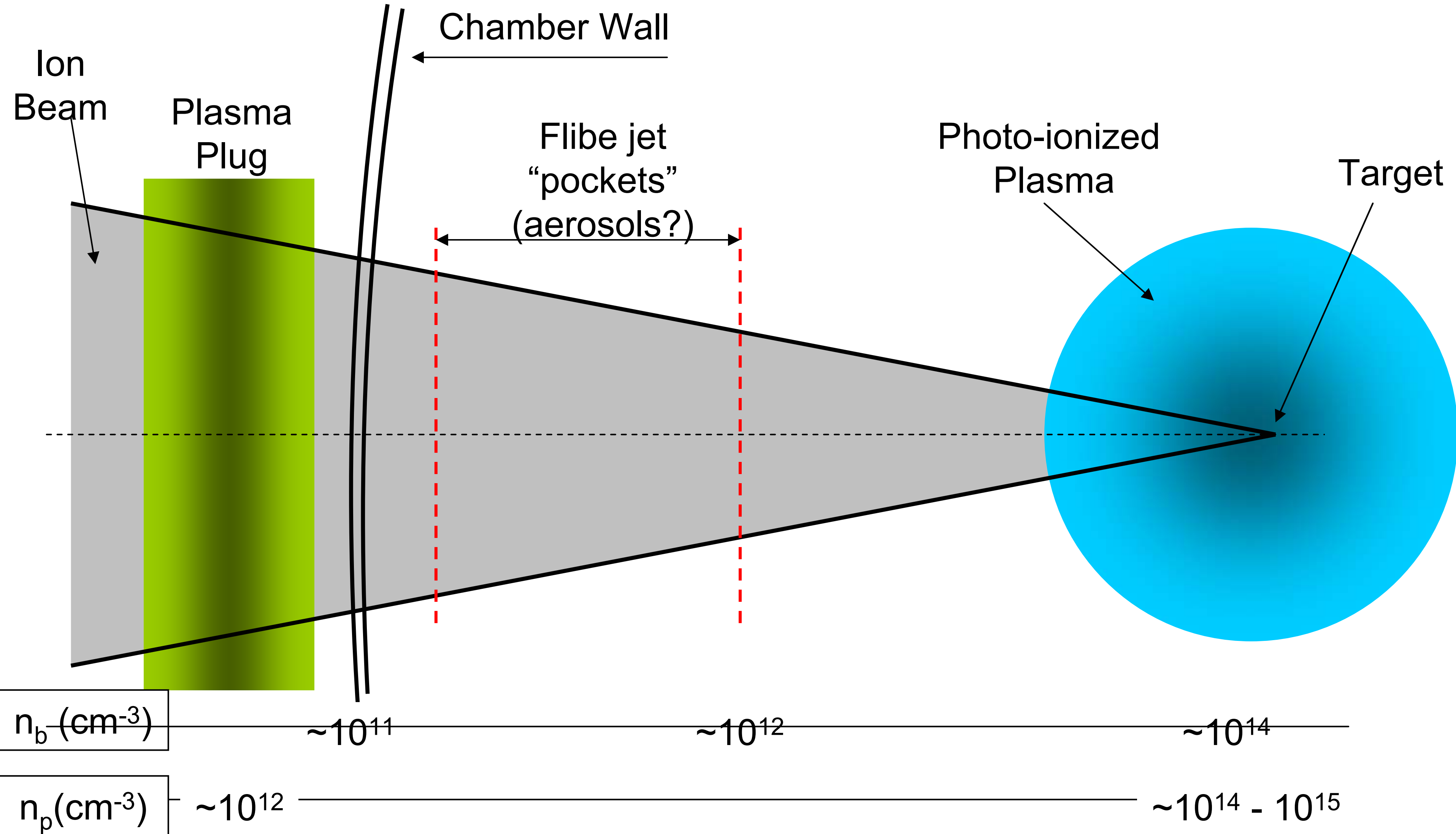
[1] P. Stroud, "Streaming modes in final beam transport for heavy ion beam fusion," Laser and Particle Beams **4**, 261 (1986).
[2] D. R. Welch, et al., Phys. Plasmas **9**, 2344 (2002).

Previous analysis [1] of streaming instabilities for a converging heavy ion beam assumed a different baseline parameter set than present "robust point design [3]."

- Ref. [1] assumed 10 GeV heavy ions propagating over 5 – 10 meters in background gas densities of 10^{11} – 10^{16} cm^{-3}.
- Results assumed two-stream growth rates based on 1-D dispersion analysis.
- Growth rate compared to rate of beam and plasma density evolution gives spatially dependent k_{max} (wave-number of fastest growing mode). If k_{max} is changing fast enough, then instability doesn't have time to fully develop.

[3] S. Yu, *et al*., "An Updated Point Design for Heavy Ion Fusion," submitted to Fusion Sci. and Technol. (2003).

For Neutralized Ballistic Transport, the ion beam passes through a <u>wide range</u> of background plasma and gas parameters.



1-D Studies:

- Benchmark LSP against standard dispersion relations for 2 and 3-species
- Include usage of “HIF” parameters in comparisons.
- 1-D modeling encompasses the “body” mode of the two-stream instability.

1-D Studies: Two-stream growth rates

Two-species dispersion relation (Buneman):

$$\frac{\omega_b^2}{(\omega - kv_b)^2} + \frac{\omega_p^2}{\omega^2} = 1$$

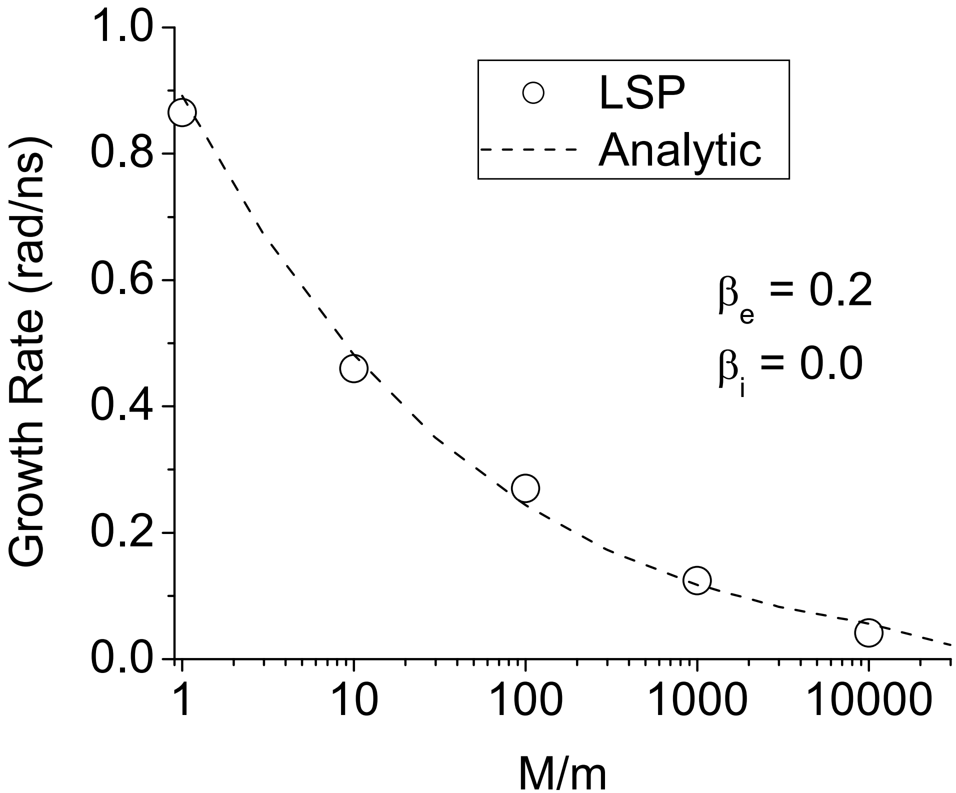
Three-species dispersion relation:

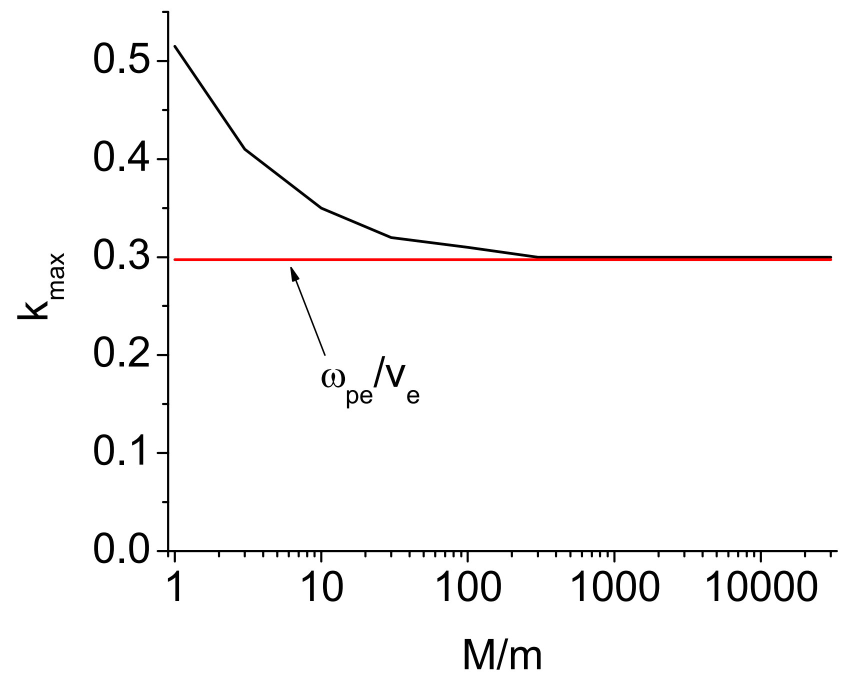
$$\frac{\omega_b^2}{(\omega - kv_b)^2} + \frac{\omega_e^2}{(\omega - kv_e)^2} + \frac{\omega_p^2}{\omega^2} = 1$$

Simulation Configuration:

- 1-D, electrostatic ADI solver
- Periodic boundaries with $\lambda = 2\pi/k_{max}$, with k_{max} determined from solutions to dispersion relations.
- Cold species in all cases
- Growth stimulated by small amplitude velocity perturbation applied to electron species ($\Delta v/v_b \sim 10^{-4}$)
- Nominal parameters: v_b=0.2c, n_p=10^9 cm^{-3}

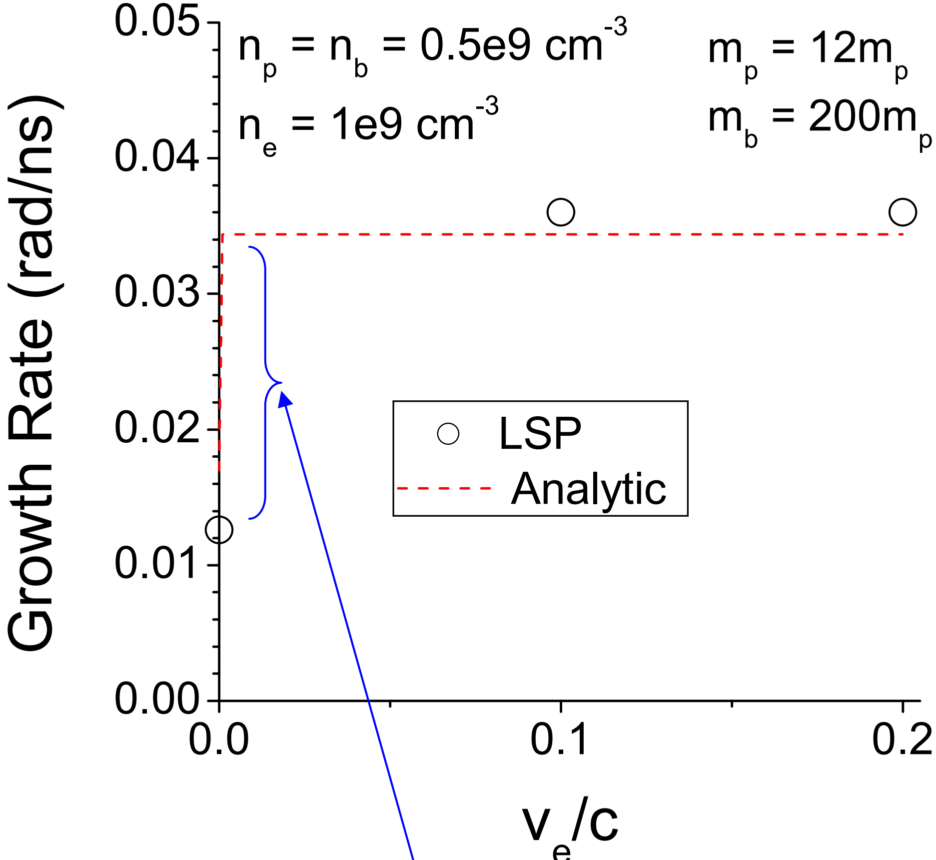
Two-species, 1-D results: LSP simulations accurately track the basic instability mechanism

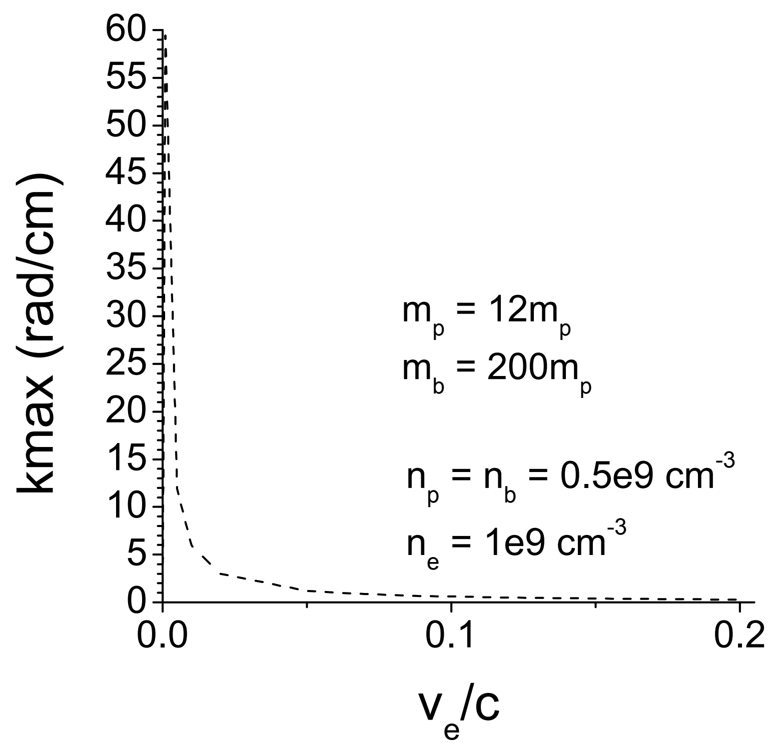




Classic (M/m)^1/3 scaling for M/m > 1000 (about a proton mass…)

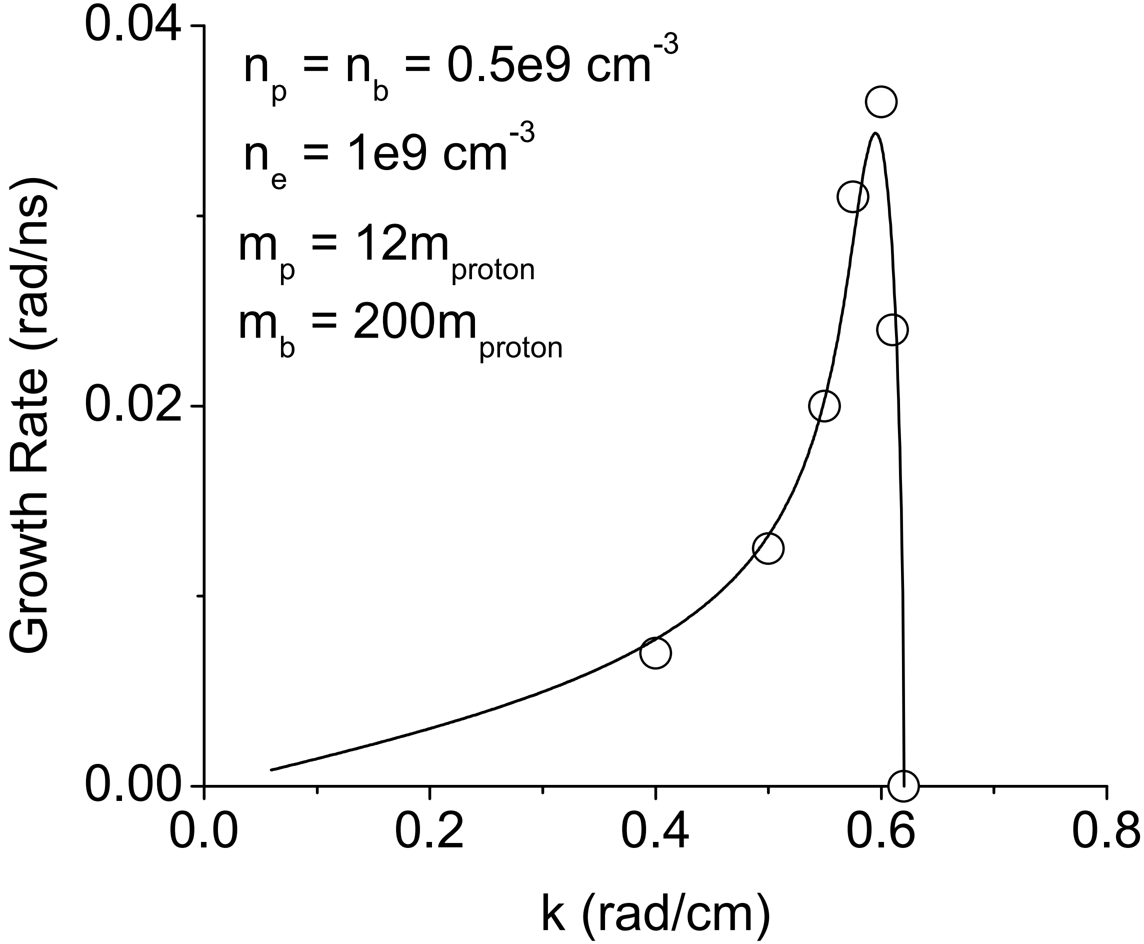
LSP results in good agreement with 3-species dispersion relation analysis.





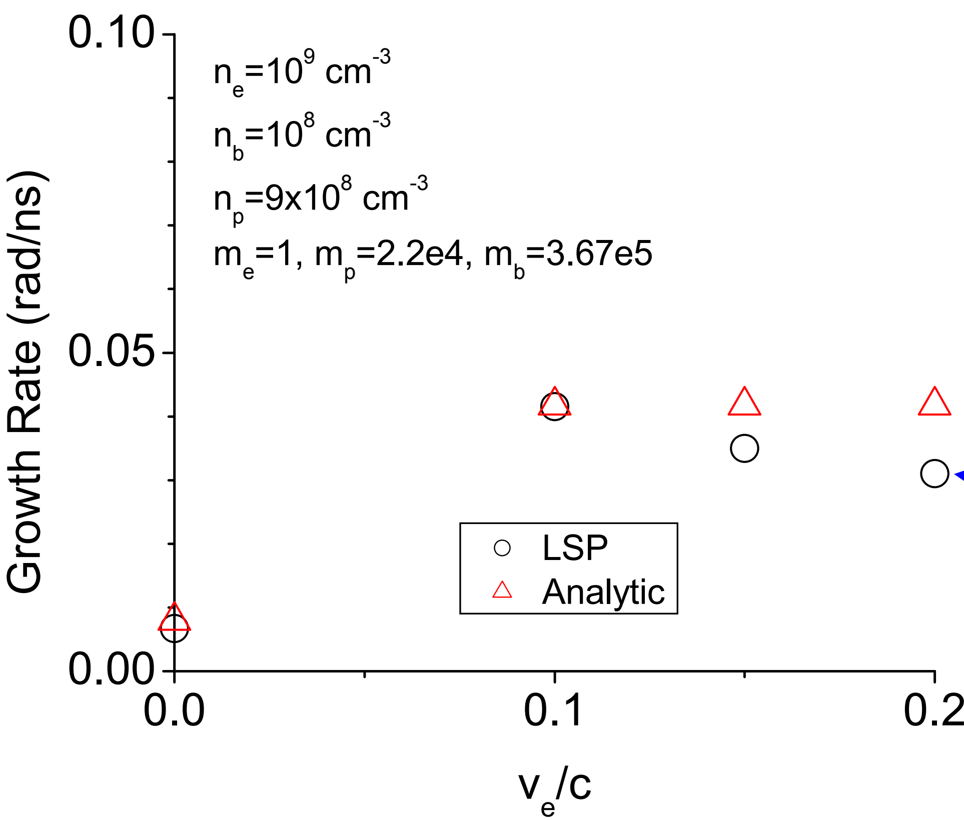
The sudden drop in growth rate at $v_e = 0$ is because electrons are now only streaming against the more massive beam ions rather than the plasma ions…

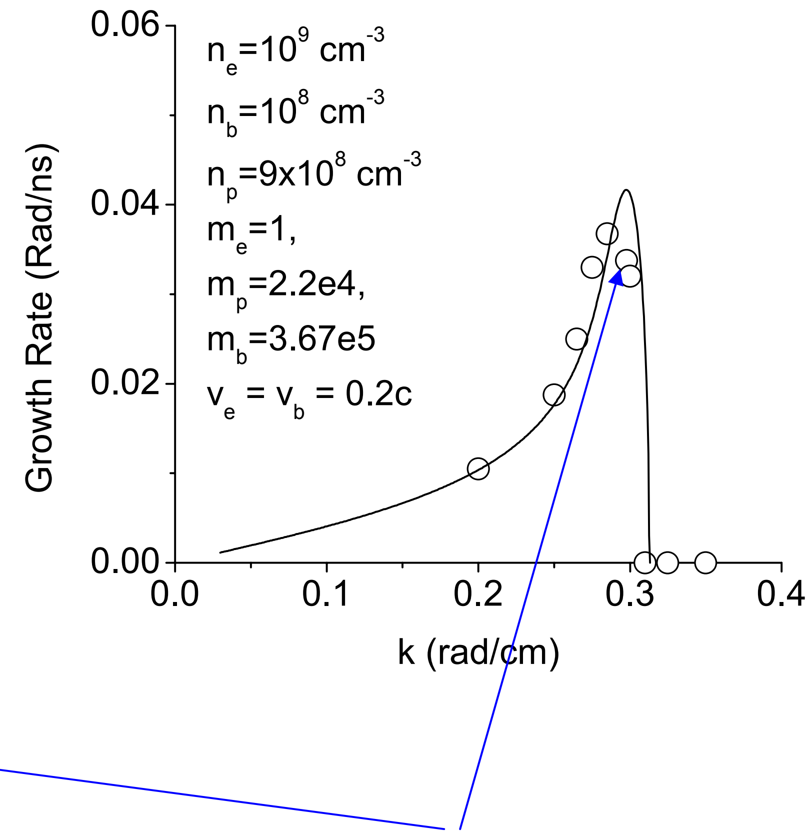
LSP simulation scans in *k*-space also correctly follow growth rates and real frequencies.



Ve = 0.2c

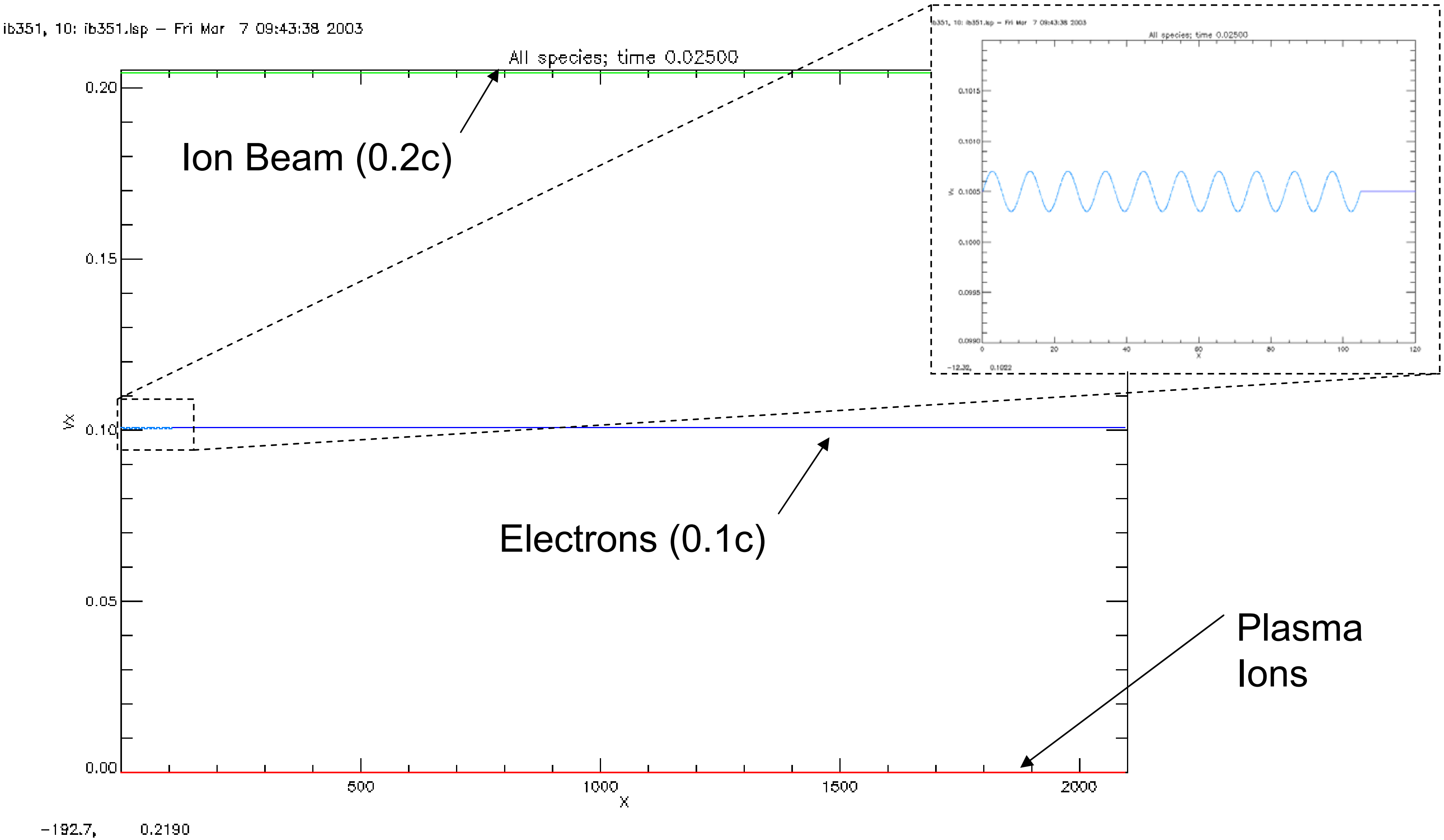
1-D, 3-species with "dense" plasma (n_p/n_b=9):

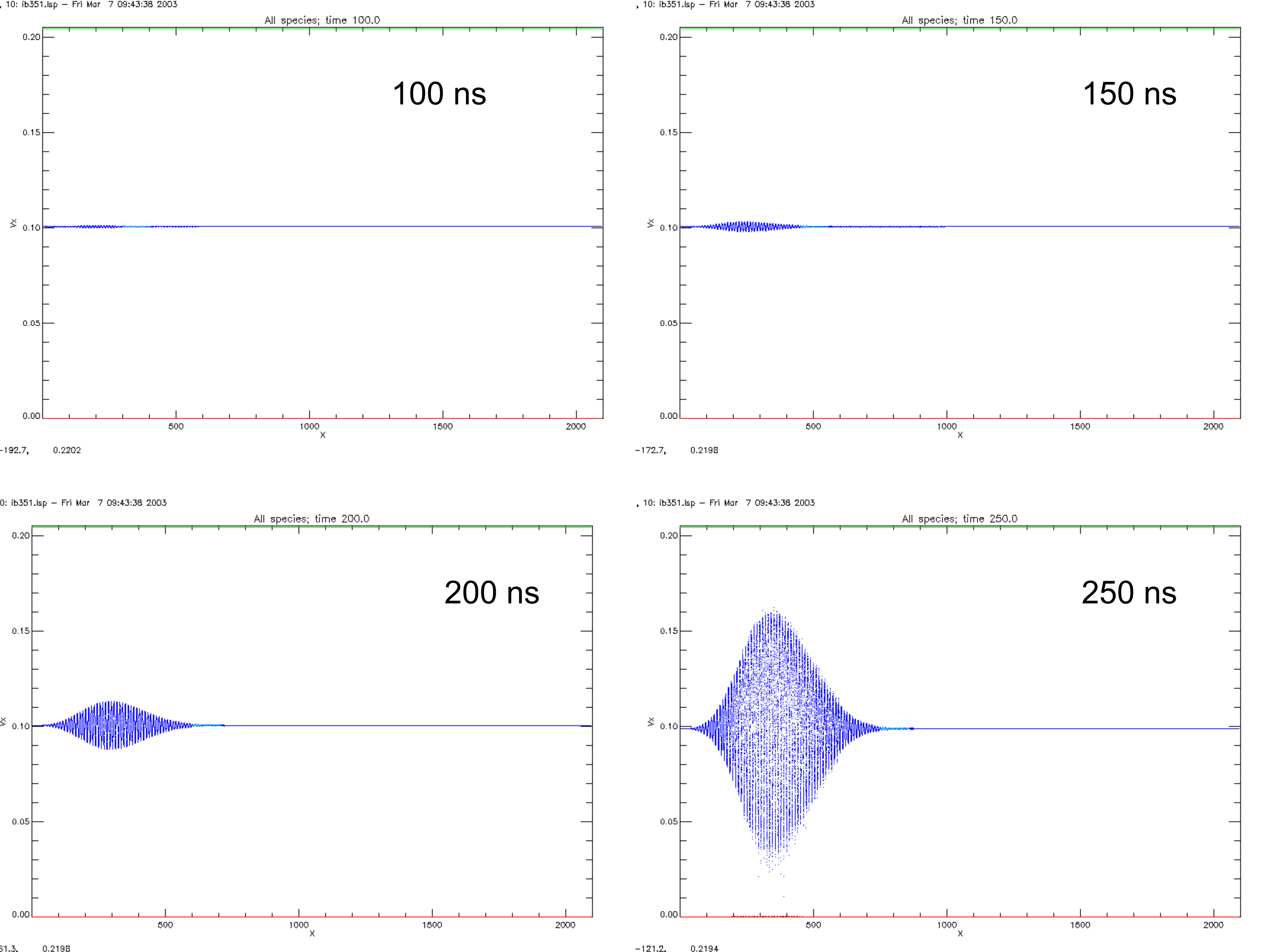




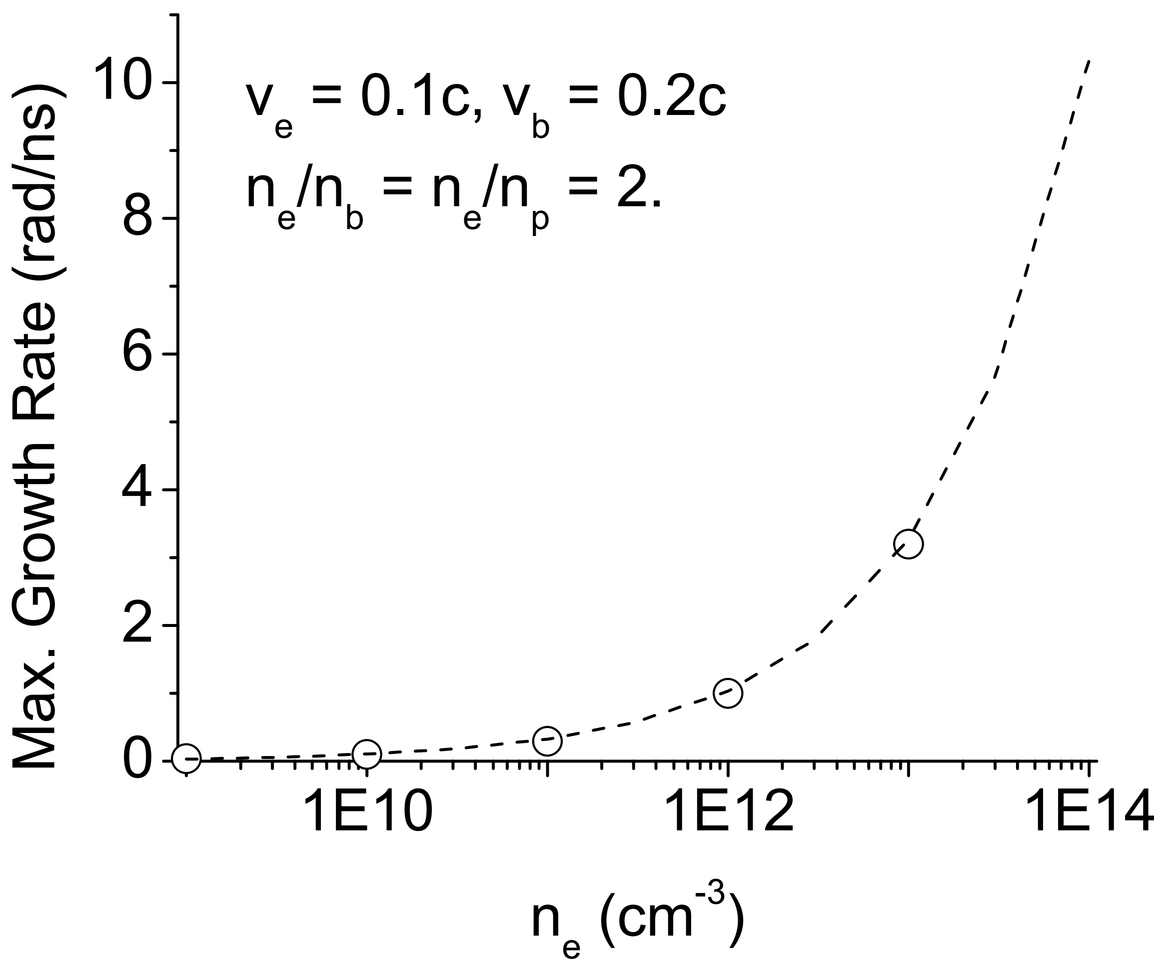
Simulations varying k for this case show slightly different k_{max} values.

1-D simulation (periodic) with L=200λ_{max}, initial perturbation region is 10λ_{max} :




, 10: ib351.lsp — Fri Mar 7 09:43:38 2003
All species; time 100.0
100 ns
Vx
X
-192.7, 0.2202
, 10: ib351.lsp — Fri Mar 7 09:43:38 2003
All species; time 150.0
150 ns
Vx
X
-172.7, 0.2198
0: ib351.lsp — Fri Mar 7 09:43:38 2003
All species; time 200.0
200 ns
Vx
X
61.3, 0.2198
, 10: ib351.lsp — Fri Mar 7 09:43:38 2003
All species; time 250.0
250 ns
Vx
X
-121.2, 0.2194

1-D density scan illustrates density scaling (growth $\propto n_e^{1/2}$).



2-D (r,z) two-stream growth for radially bounded ion-beam-plasma systems:

- Dispersion relation developed for 2-D "hard-edge" beam system.
- Numerical solution in time for system of equations for "soft" beam profiles.
- Direct comparisons with 2-D LSP simulations (electrostatic).
- 2-D LSP EM simulations giving finite net current fractions – in progress.

2-D dispersion analysis for "hard-edge" ion beam in plasma:

Assumptions:

Charge Neutral: $$n_e(r) = n_b(r) + n_p$$

Current Neutral: $$v_e(r) = \frac{n_b(r)}{n_b(r) + n_p} v_b$$

Linearized Equations (single wave-number, $f \sim e^{ikz}$):

$$\frac{\partial v_z}{\partial t} = -ikV_o v_z - v_r \frac{\partial V_o}{\partial r} + \frac{q}{m} e_z$$

$$\frac{\partial v_r}{\partial t} = -ikV_o v_r + \frac{q}{m} e_r$$

<- Momentum

2-D dispersion analysis for “hard-edge” ion beam in plasma (*cont.*):

$$\frac{\partial n}{\partial t} = -ik\left(N_o v_z + nV_o\right) - \frac{1}{r}\frac{\partial}{\partial r}\left(rN_o v_r\right)$$

<- Continuity

$$ike_r = \frac{\partial e_z}{\partial r}$$

<- Fields

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial e_z}{\partial r}\right) - k^2 e_z = -4\pi eik\left(n_b + n_p - n_e\right)$$

For $f \sim e^{i\omega t}$

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\varepsilon\frac{\partial e_z}{\partial r}\right) - \varepsilon k^2 e_z = 0$$

$$\varepsilon = 1 - \sum_i \frac{\omega_i^2}{\left(\omega - kV_{oi}\right)^2}$$

$$e_z(r) = \begin{cases} AI_o(kr), & 0 \le r \le r_b \\ B\left[I_o(kR_w)K_o(kr) - I_o(kr)K_o(kR_w)\right], & r_b \le r \le R_W \end{cases}$$

2-D dispersion analysis for "hard-edge" ion beam in plasma (*cont*.):

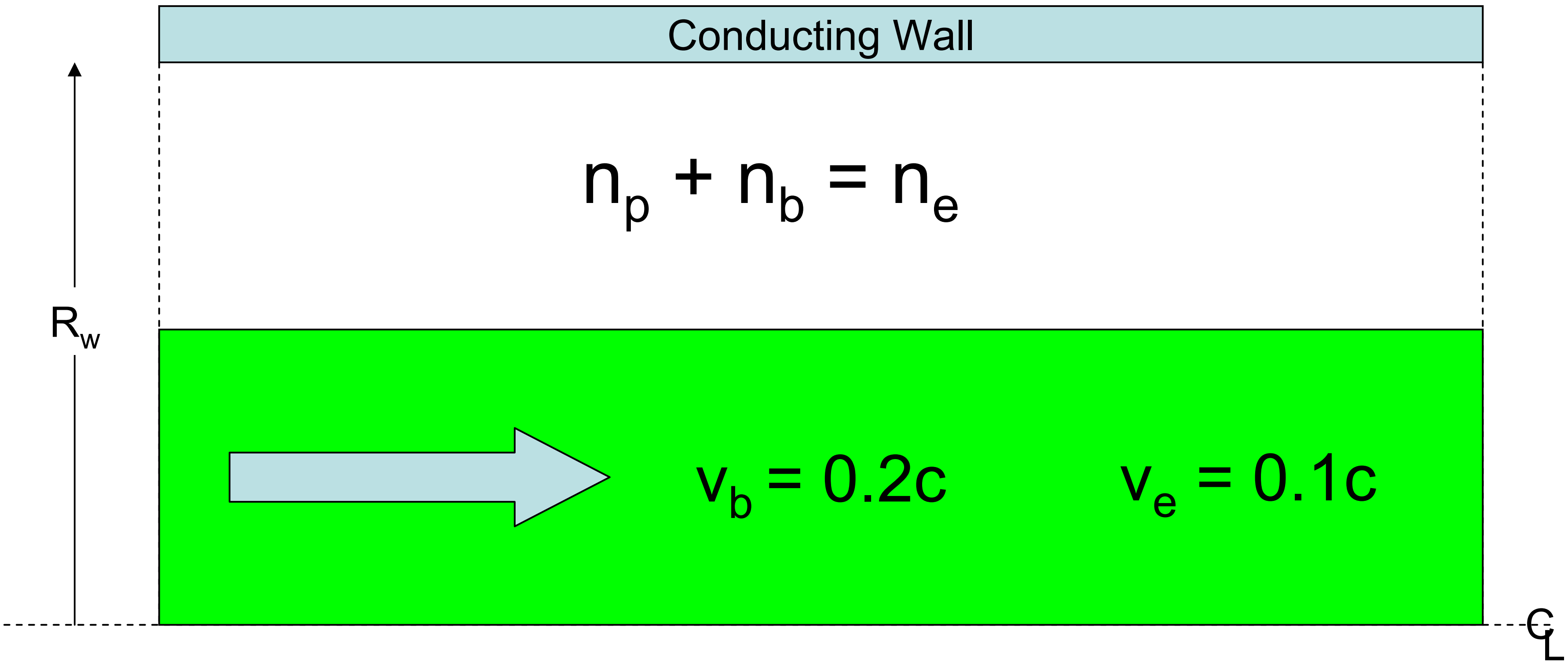
Matching e_z and jump condition on $\varepsilon\partial e_z/\partial r$ at r_b gives:

$$1-\frac{\omega_{e1}^2}{\left(\omega-kV_e\right)^2}-\frac{\omega_b^2}{\left(\omega-kV_b\right)^2}-\frac{\omega_p^2}{\omega^2}=-\alpha\left[1-\frac{\omega_{e2}^2}{\omega^2}-\frac{\omega_p^2}{\omega^2}\right]$$

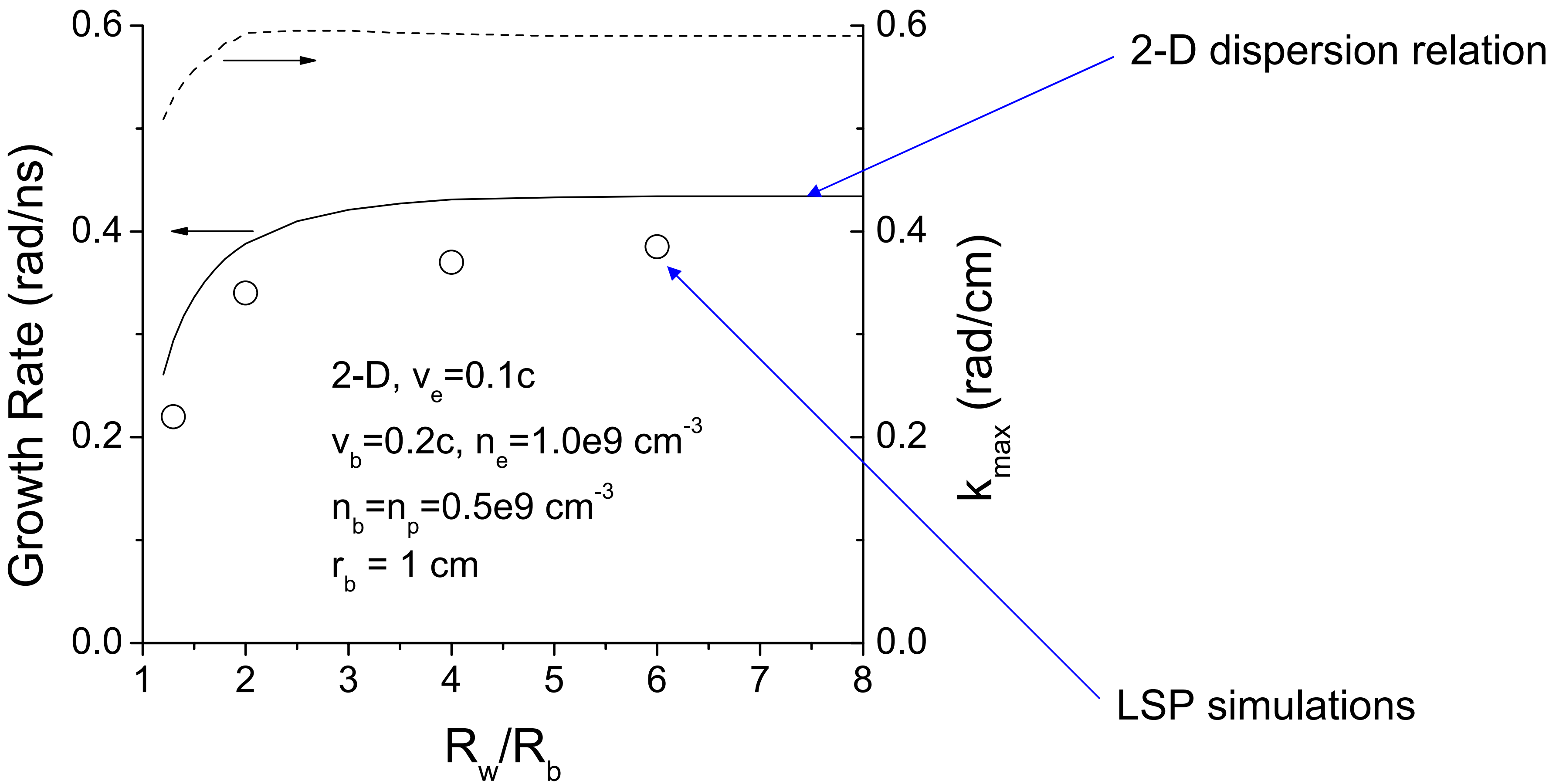
$$\alpha=\frac{I_o\left(kr_b\right)\left[I_o\left(kR_w\right)K_1\left(kr_b\right)+I_1\left(kr_b\right)K_o\left(kR_w\right)\right]}{I_1\left(kr_b\right)\left[I_o\left(kR_w\right)K_o\left(kr_b\right)-I_o\left(kr_b\right)K_o\left(kR_w\right)\right]}$$

This dispersion relation describes the growth of surface waves on a beam/plasma column.

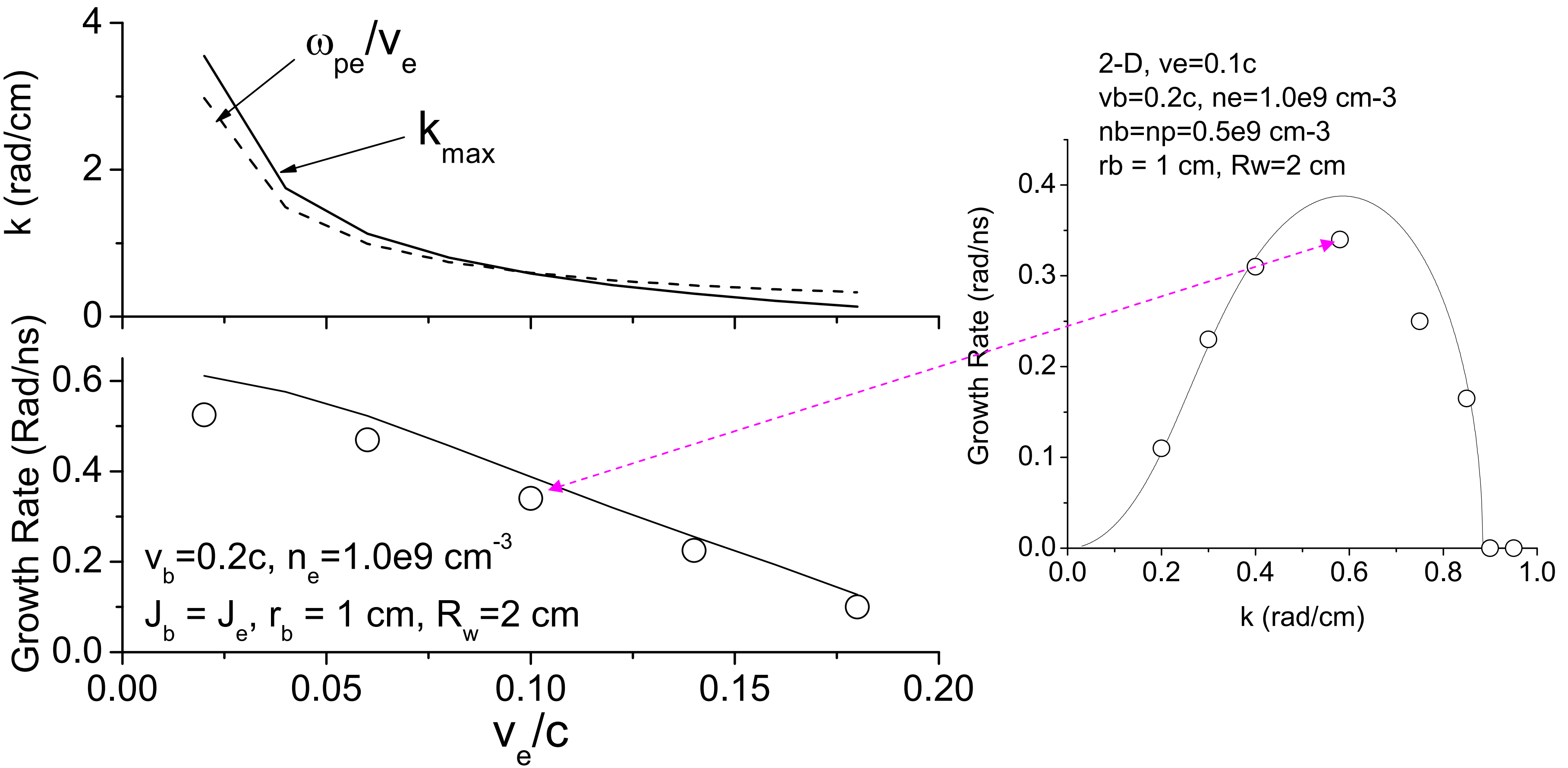
2-D Problem Geometry



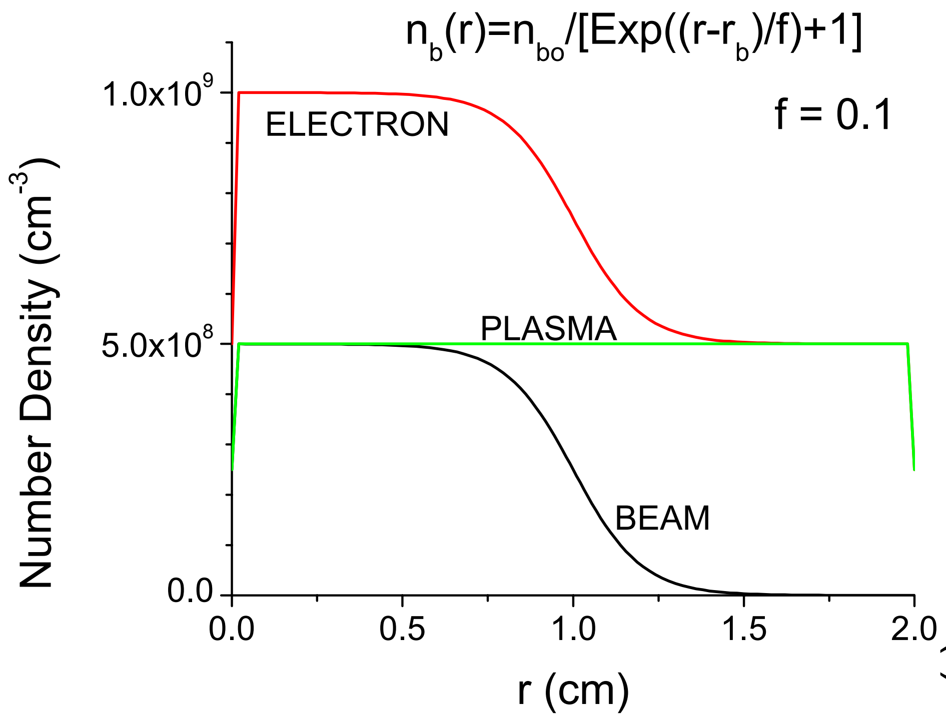
Hard-edge-profile ion beam simulation and theory results: influence of radial boundary minimal at modest values of R_w/r_b.

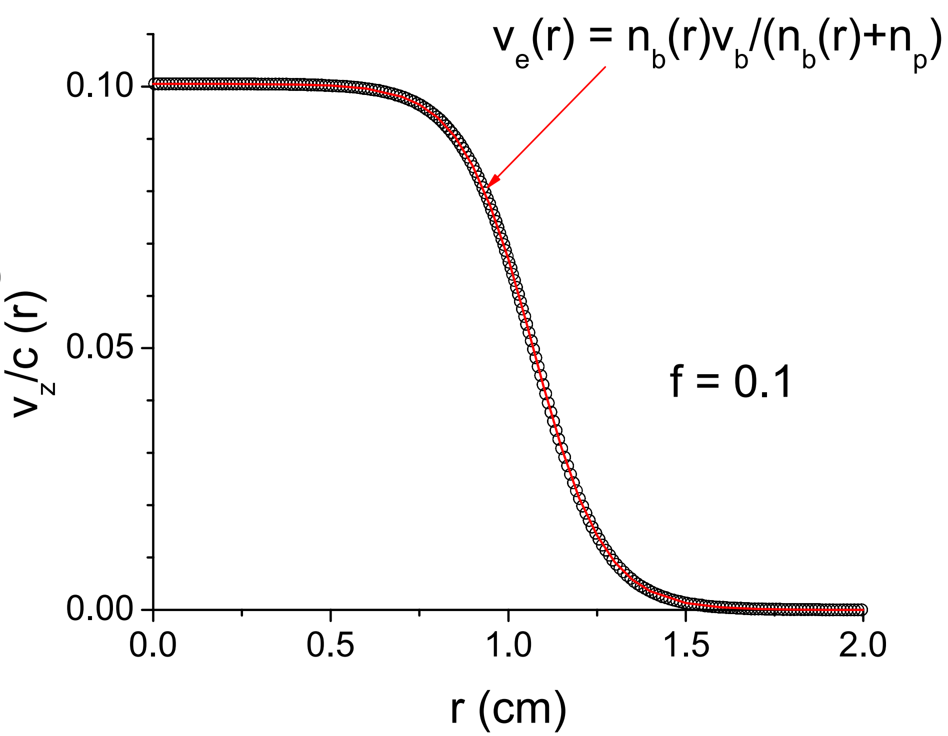


Scan in electron velocity for 2-D “hard-edge” beam profile shows a broader k-range around peak growth values compared to 1-D dispersion analysis:



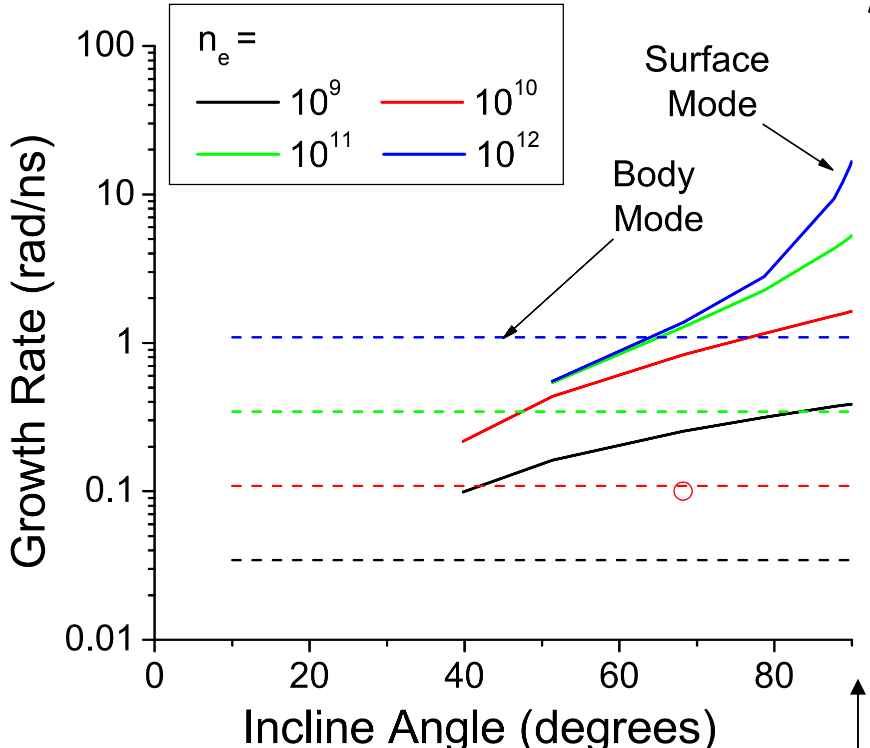
Simulations are compared directly the 2-D model using a radial profile (charge and current neutral):

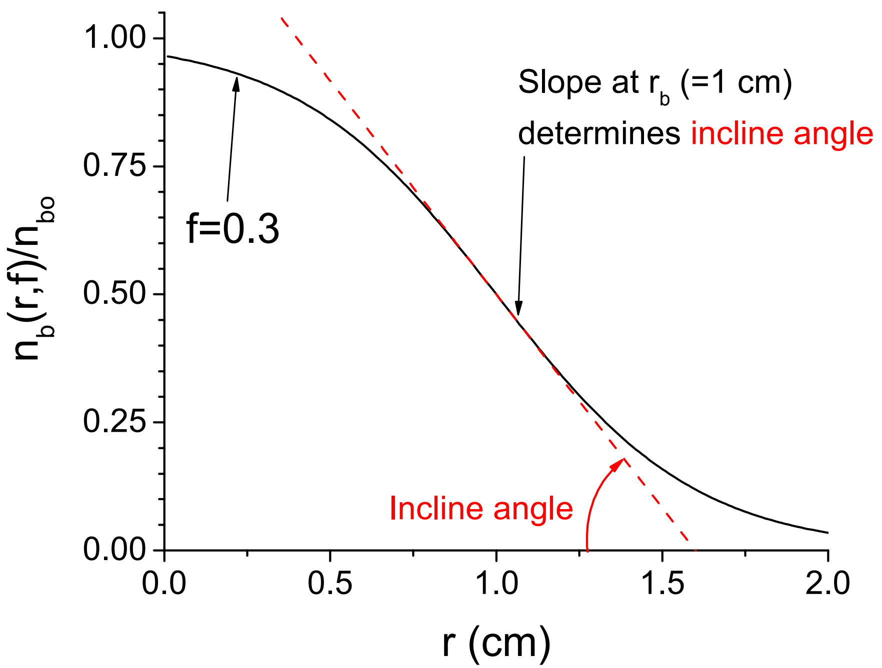




For the range of beam/plasma densities of interest, body (1-D) and surface (2-D) modes can have comparable growth rates:

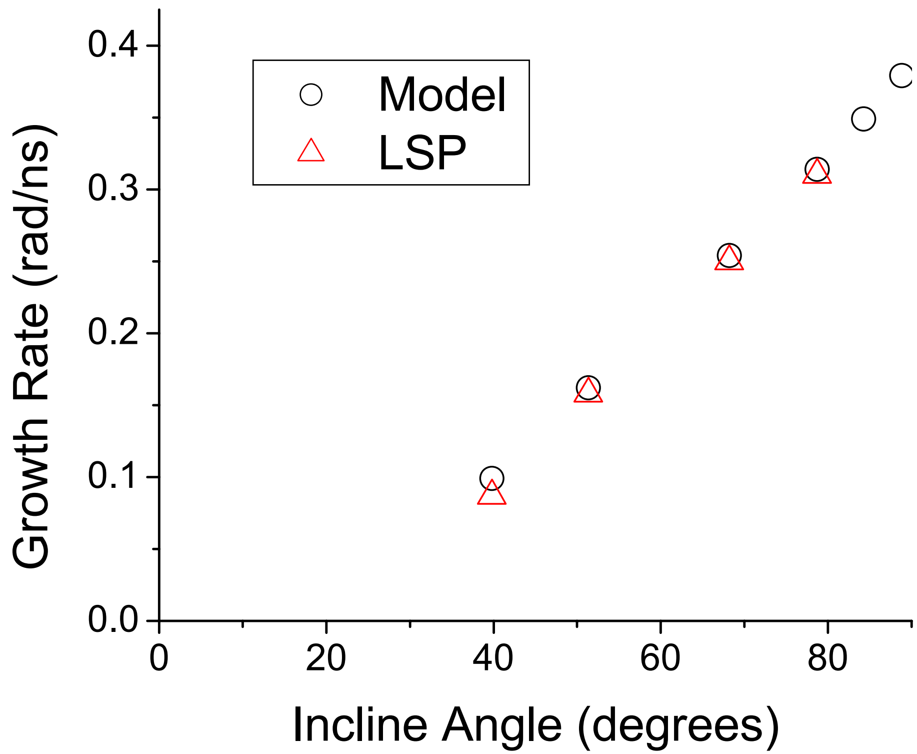
$$\frac{n_e}{n_b} = \frac{n_e}{n_p} = 2$$

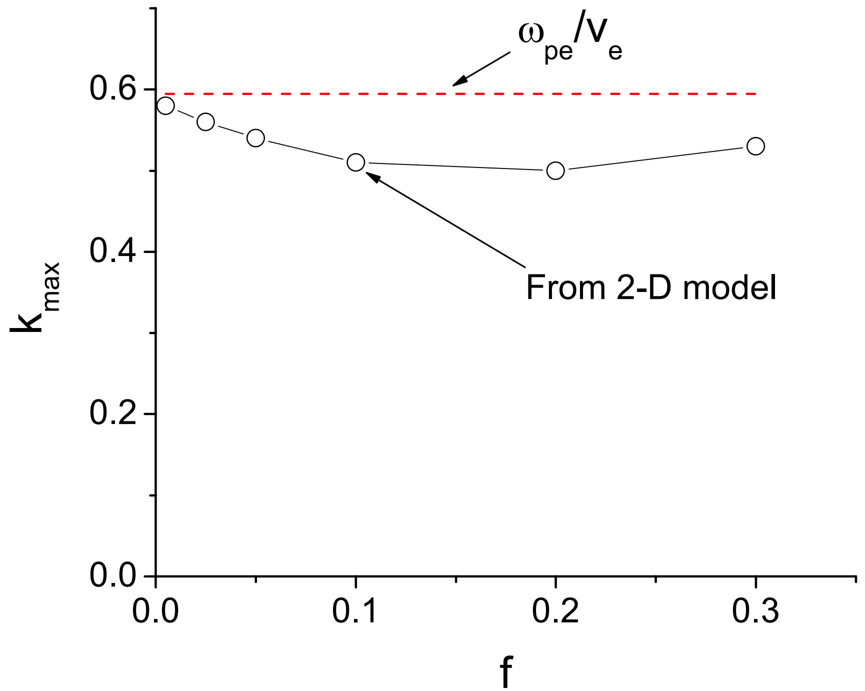




"hard-edge" beam profile

Simulations and theory in good agreement over a wide range of beam-edge profiles





Status:

- Initial two-stream studies have provided basic growth-rate scaling and important benchmarks for PIC.
- Converging ion beam studies are underway in “idealized” limit (collisionless, uniform background plasmas, etc.)
- EM studies of net current evolution are also planned as part of this analysis.