NONLINEAR STABILITY AND COILS FOR FUSION REACTORS

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The NSTAB code has been applied to show that the LHD stellarator is linearly unstable, but remains nonlinearly stable, at levels of β below those achieved experimentally. Predictions of ballooning theory for the LHD are more pessimistic than estimates from bifurcated solutions calculated over 1, 2, 5 or 10 periods. Similar results have been obtained for a W7-AS configuration.

Correlation of computations with observations has been used to design a quasiaxially symmetric fusion reactor. The β limit can be estimated by performing long, accurate runs of the NSTAB code to find out whether wall stabilized ballooning structures appear in the solution. Correct stability analysis leads to coils that are only moderately twisted.

$$\int\int\int[\frac{1}{2}B^2 - p(s)]\,dV \;\;=\;\; \text{minimum}$$

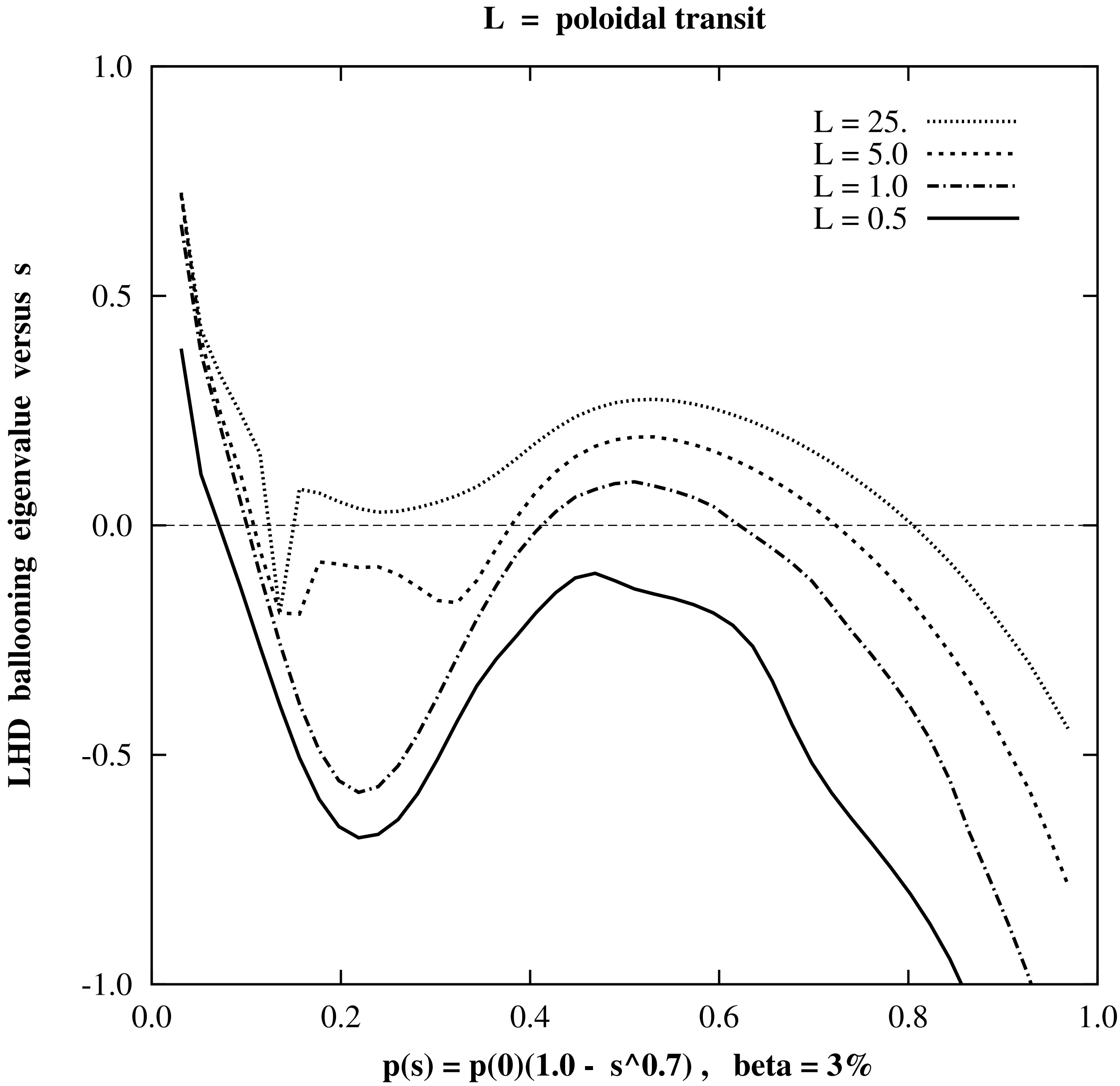
$$\mathbf{B} = \nabla s \times \nabla\theta = \nabla\phi - \zeta\nabla s\;, \quad \mathbf{J} = \nabla s \times \nabla\zeta$$

$$r + iz = e^{iu} \sum \Delta_{mn}\, e^{-imu+inv}$$

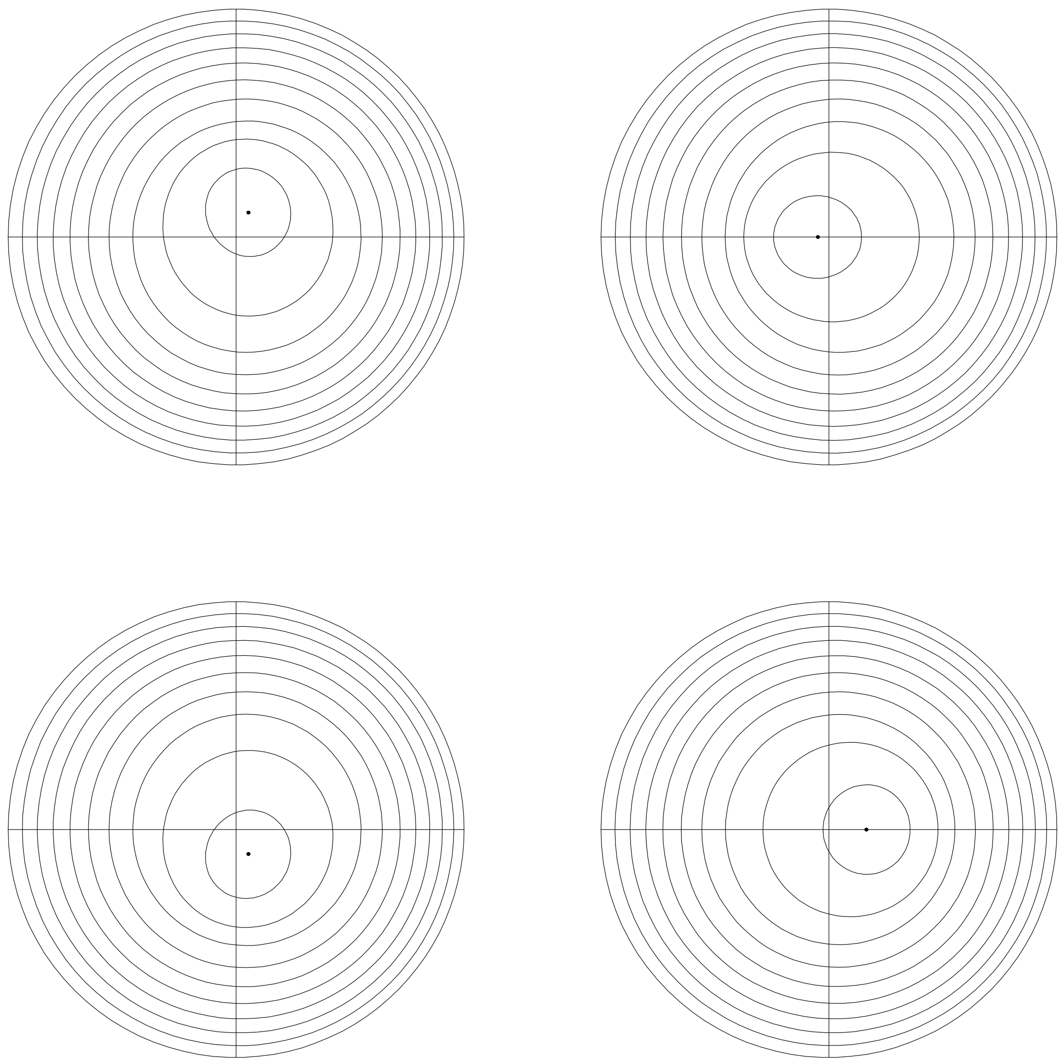
$$\frac{1}{B^2} = \sum B_{mn} \cos(m\theta - [n - \iota m]\phi)$$

$$\zeta = p' \sum \frac{B_{mn}}{n - \iota m} \sin(m\theta - [n - \iota m]\phi)$$

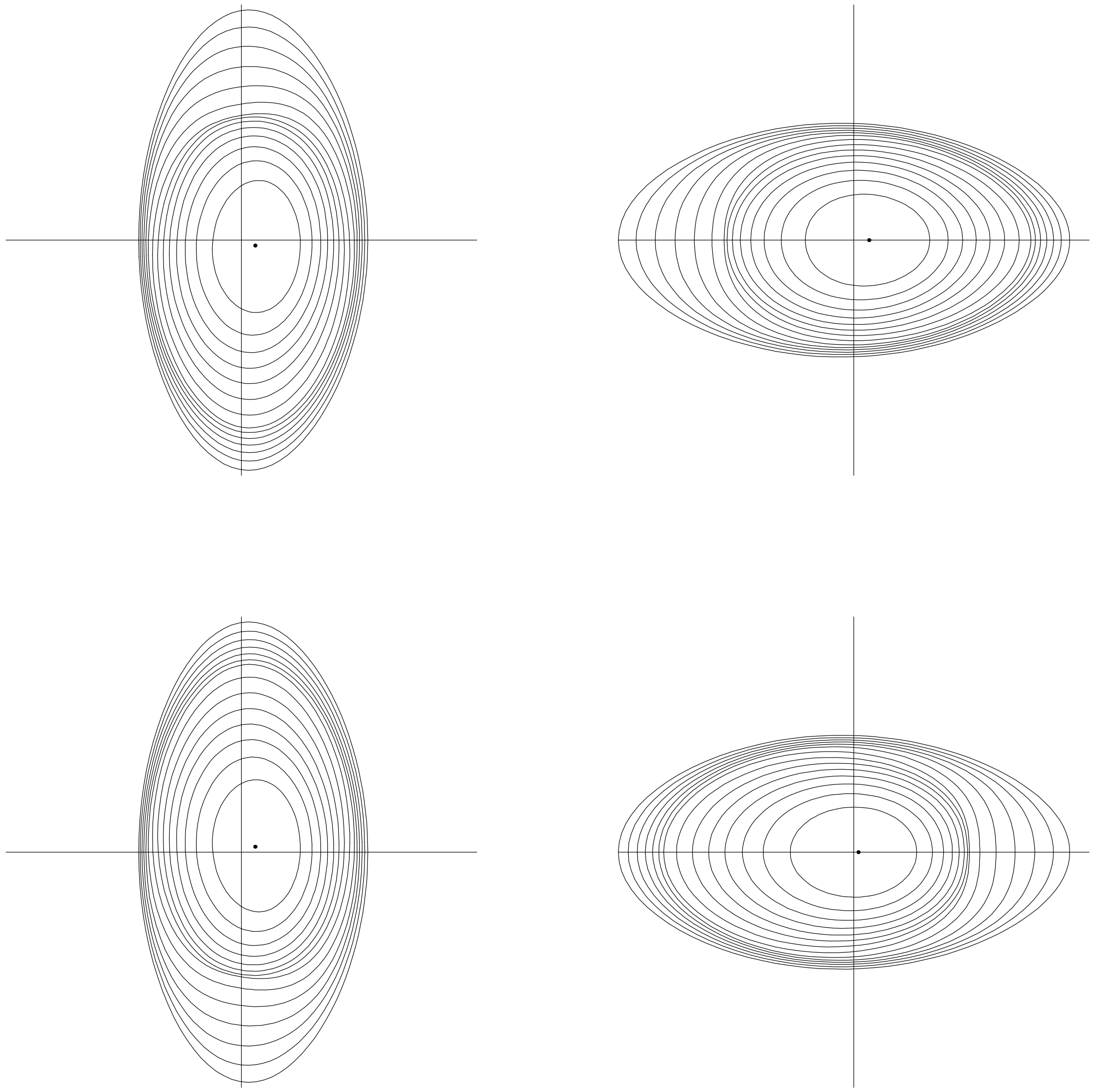
$$\mathbf{B} = \nabla \times \sum_{j=0}^{k} \;\delta s_j \int\int \frac{\nabla\zeta_j \times \mathbf{N}}{4\pi r} d\sigma$$



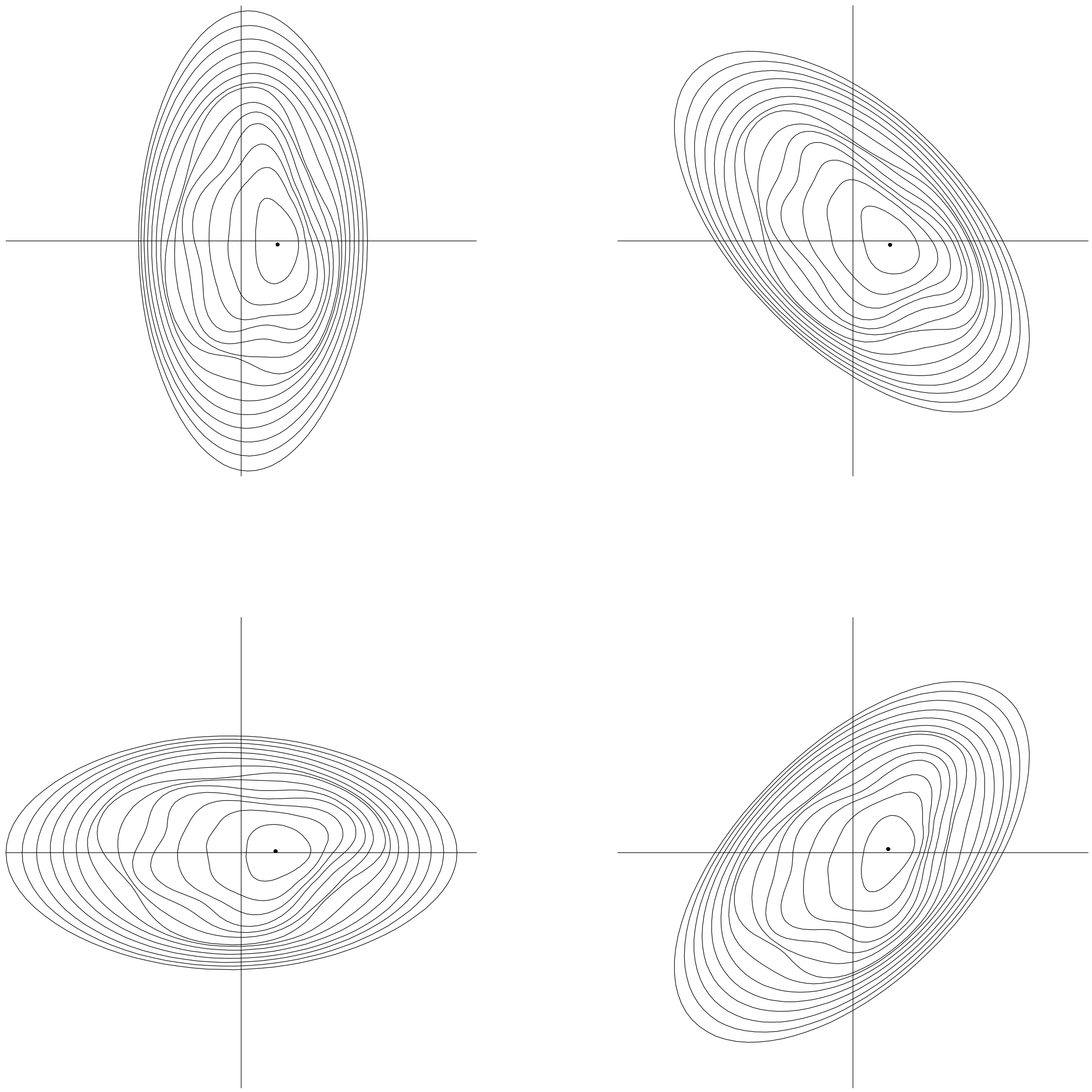
Numerical implementations of the local ballooning criterion depend on a choice of boundary conditions and give estimates of the β limit for the LHD that have been exceeded in the experiment by a significant margin.



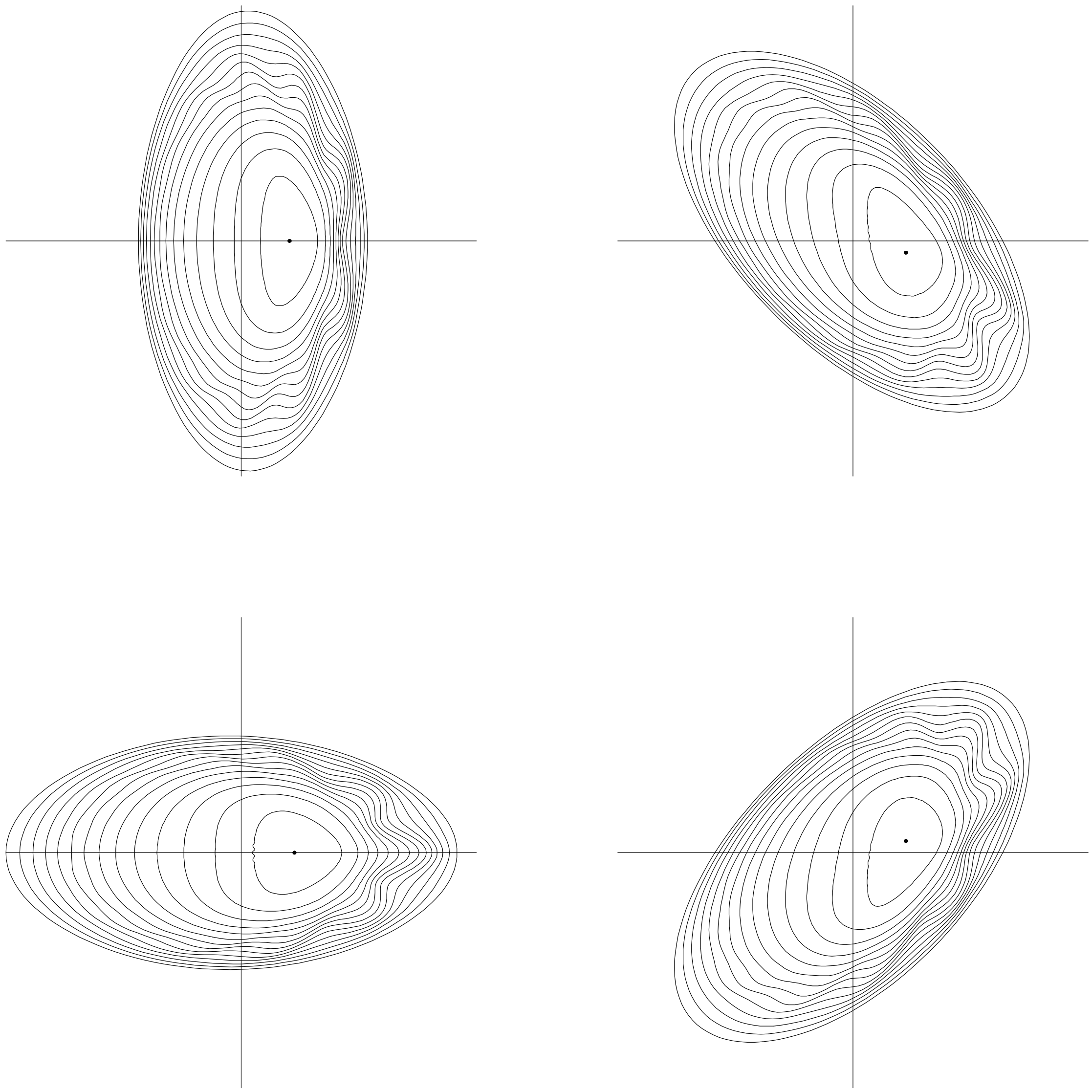
Magnetic surfaces of the TFTR tokamak computed by the NSTAB code showing how bifurcated equilibria can appear that have asymmetries, like islands, that can alter the two-dimensional solution enough to affect transport calculations. The helical excursion of the magnetic axis is associated with an MHD instability at the rational surface $\iota = 1$.



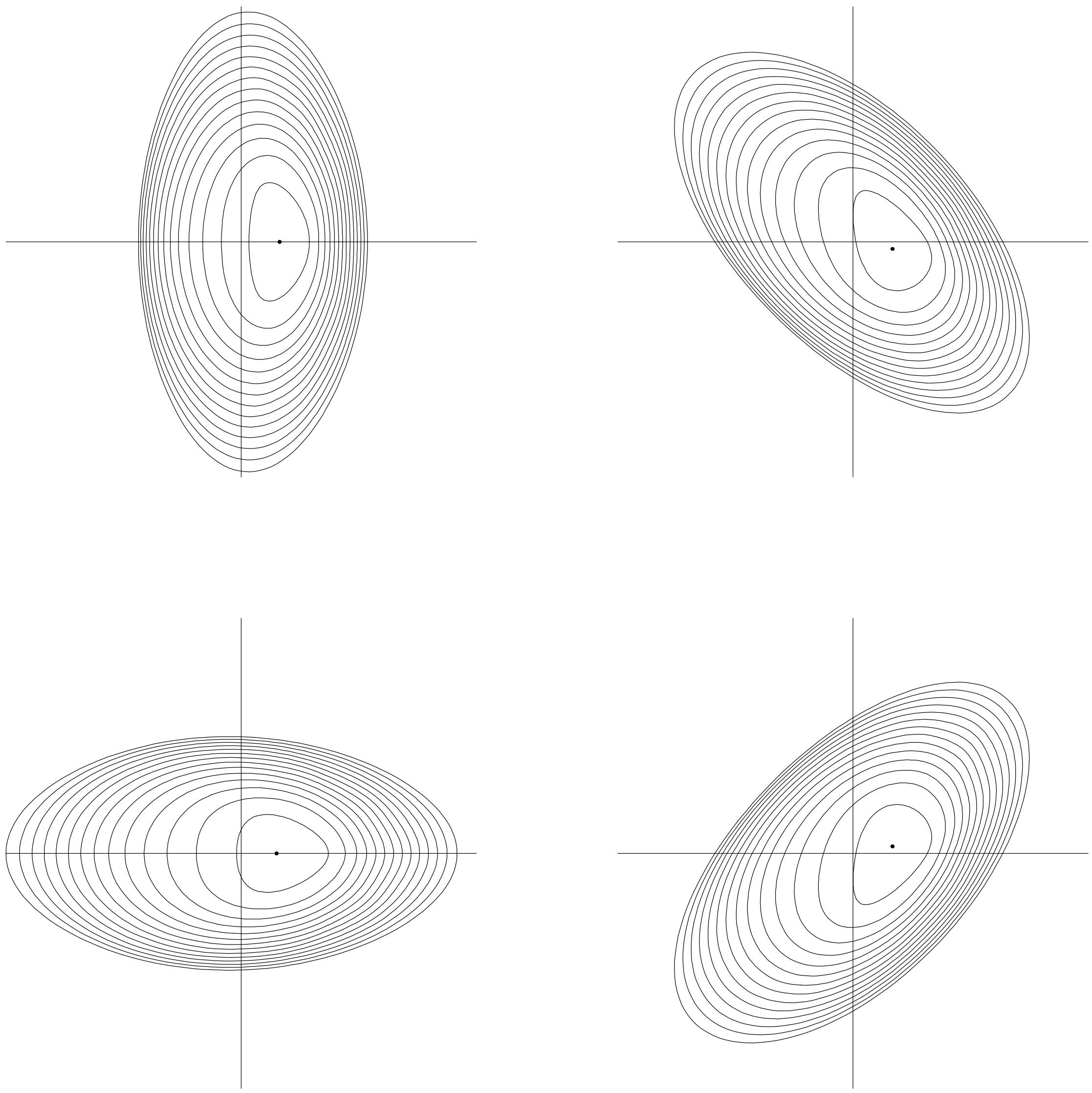
Flux surfaces at four cross sections over the full torus of a bifurcated LHD equilibrium at $\beta = 0.025$ with the magnetic axis located at major radius $R = 3.6$ m. For a pressure profile $p = p_0(1-s)$, and with bootstrap current, the global $m = 1, n = 1$ mode of this solution is linearly unstable, but nonlinearly stable. Harmonics of degree 40 in the toroidal angle were used in the computation.



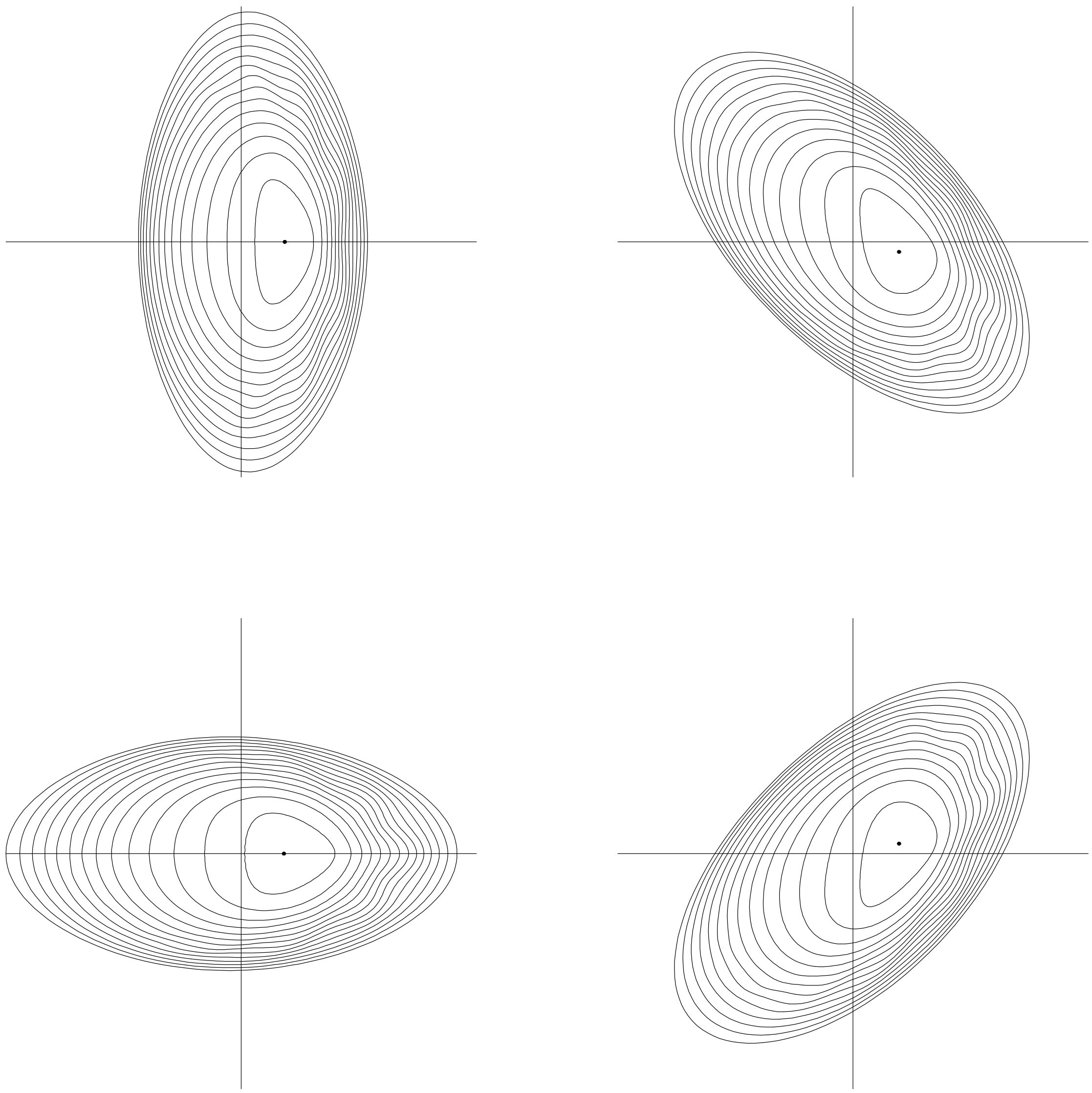
Flux surfaces at four cross sections over five field periods of a bifurcated LHD equilibrium at $\beta = 0.02$ with the magnetic axis located at major radius $R = 3.6$. For a triangular pressure profile $p = p_0(1 - s^{0.7})$ like observed values of the electron temperature, the solution is linearly unstable but nonlinearly stable. The existence of several solutions implies linear instability because there are saddle points in the energy landscape.



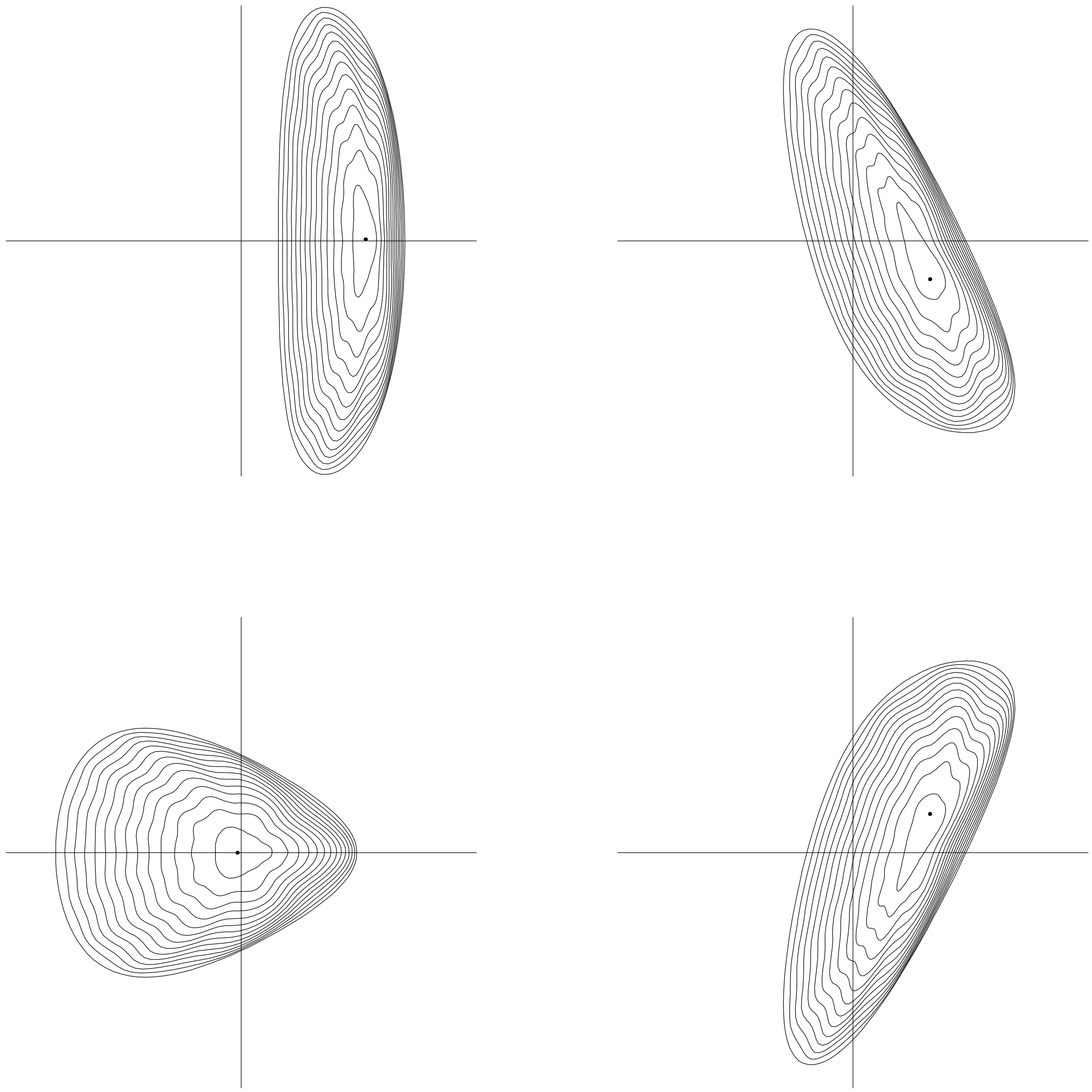
Poincaré map of the flux surfaces at four cross sections over one field period of a wall stabilized LHD equilibrium at $\beta = 0.04$ with the magnetic axis shifted inward to a position with major radius $R = 3.6$. For a standard pressure profile $p = p_0(1 - s)$, this exceptionally accurate solution developed a ballooning mode that shows it has become nonlinearly unstable. The mode was stable at $\beta = 0.032$.



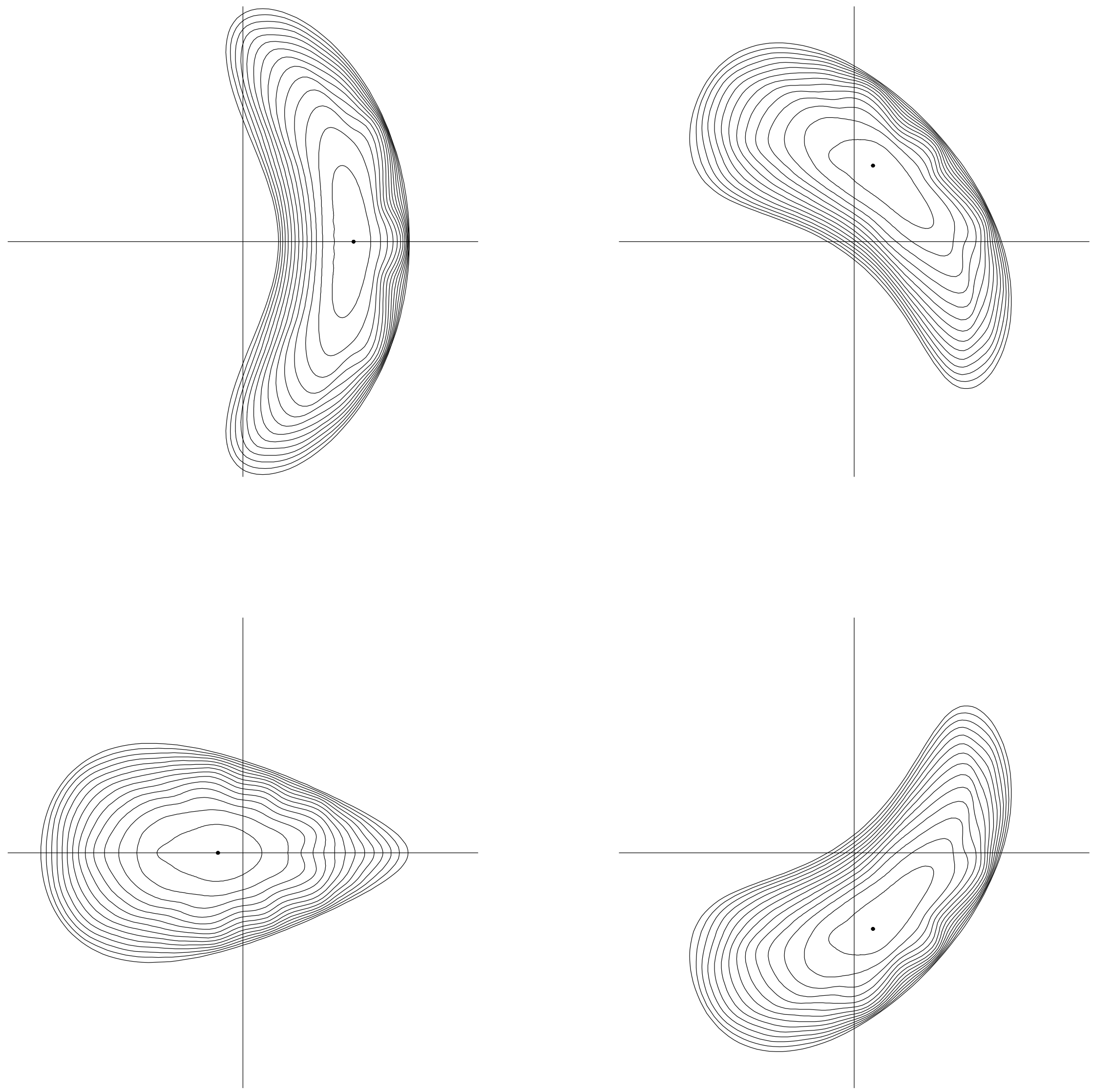
Poincaré map of the flux surfaces at four cross sections over one field period of a stable LHD equilibrium at the maximum value $\beta = 0.032$ observed in the experiment. This calculation helps to validate the NSTAB test for ballooning stability of actual stellarators in the laboratory.



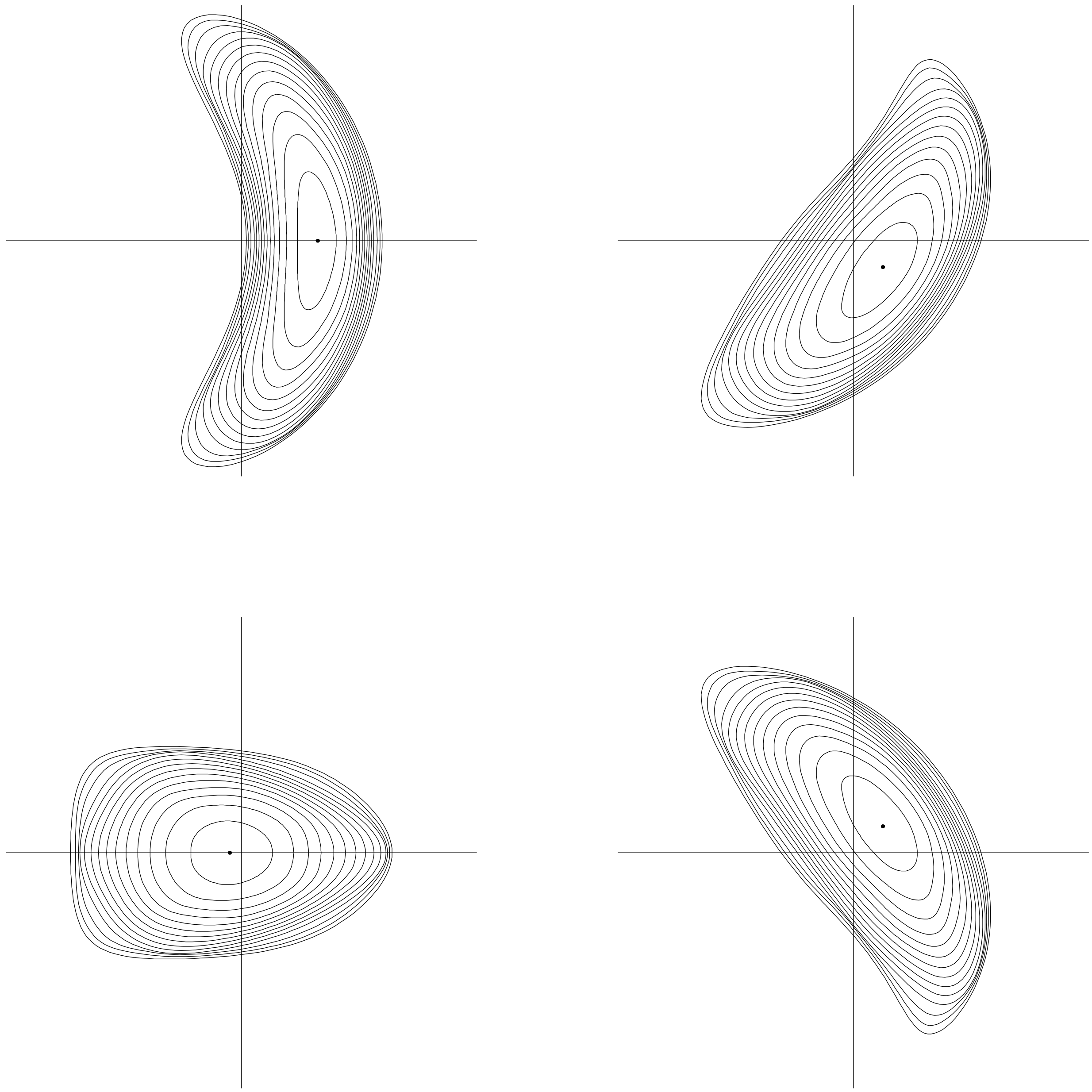
Poincaré map of the flux surfaces at four cross sections over one field period of a marginally stable LHD equilibrium at the critical value $\beta = 0.035$ predicted by the NSTAB test to be the limit for ballooning stability. In this theory it is not altogether clear how to decide precisely when the ripple of the magnetic surfaces has become unacceptably large.



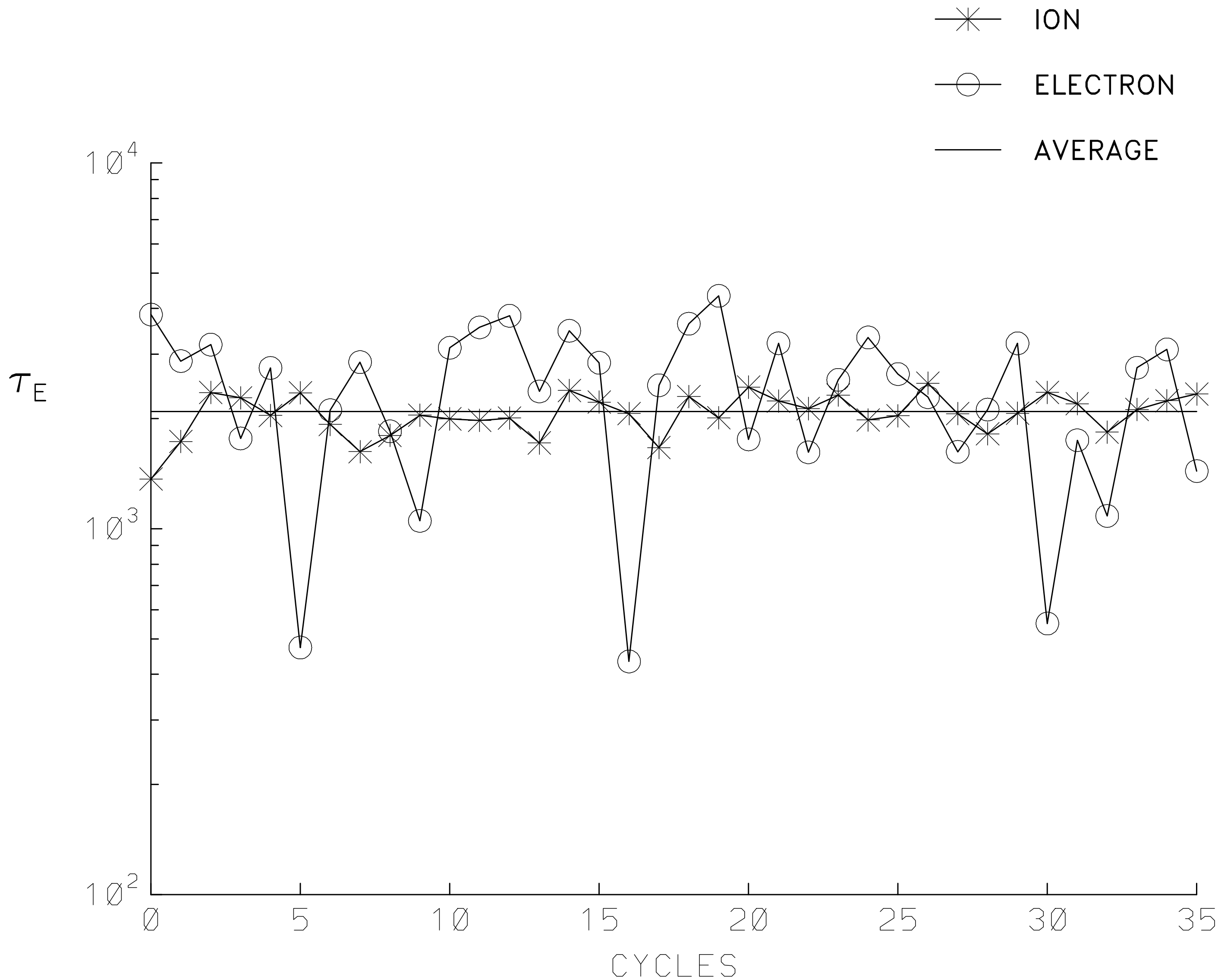
Cross sections of the flux surfaces over one field period of a bifurcated W7-AS equilibrium at $\beta = 0.03$ with net current bringing the rotational transform into the interval $0.6 > \iota > 0.5$. An MHD mode with ballooning structure appears in the solution, which became stable after some of the net current was removed. These predictions were confirmed by observations of $\beta = 0.033$ in the experiment.



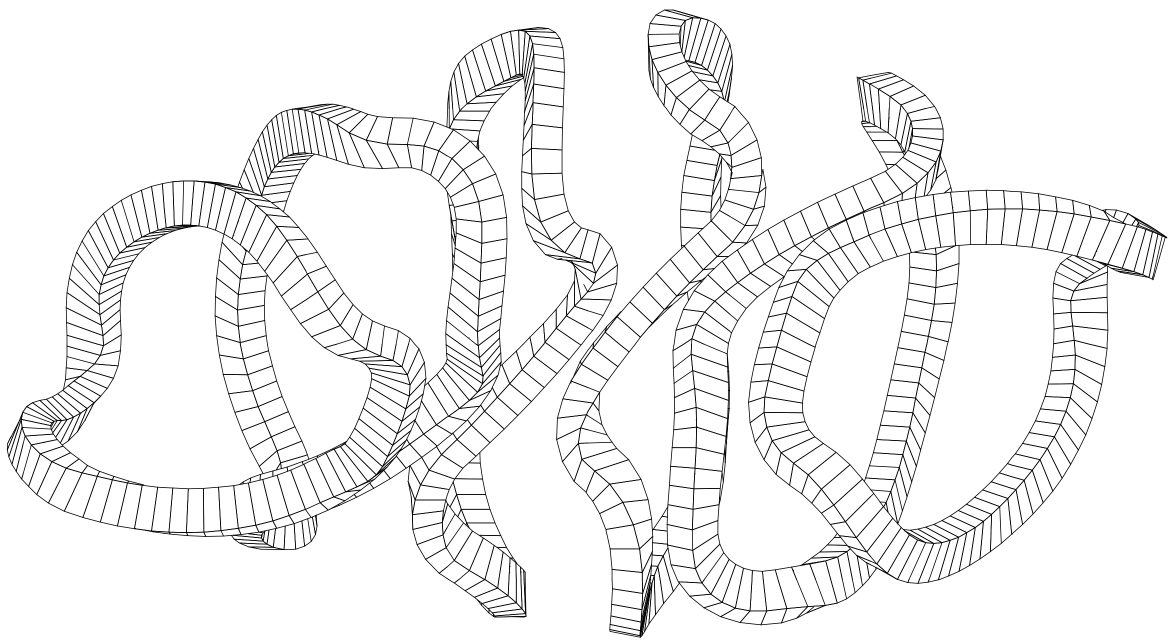
Four cross sections of the flux surfaces over one field period of a wall stabilized MHH2 equilibrium at $\beta = 0.05$ with pressure $p = p_0(1 - s^{1.5})^{1.5}$ and with hybrid net current bringing the rotational transform into the interval $0.58 > \iota > 0.52$. The ballooning mode that has become visible in the solution after 200,000 cycles of an accelerated iteration scheme shows that a β limit has been reached.



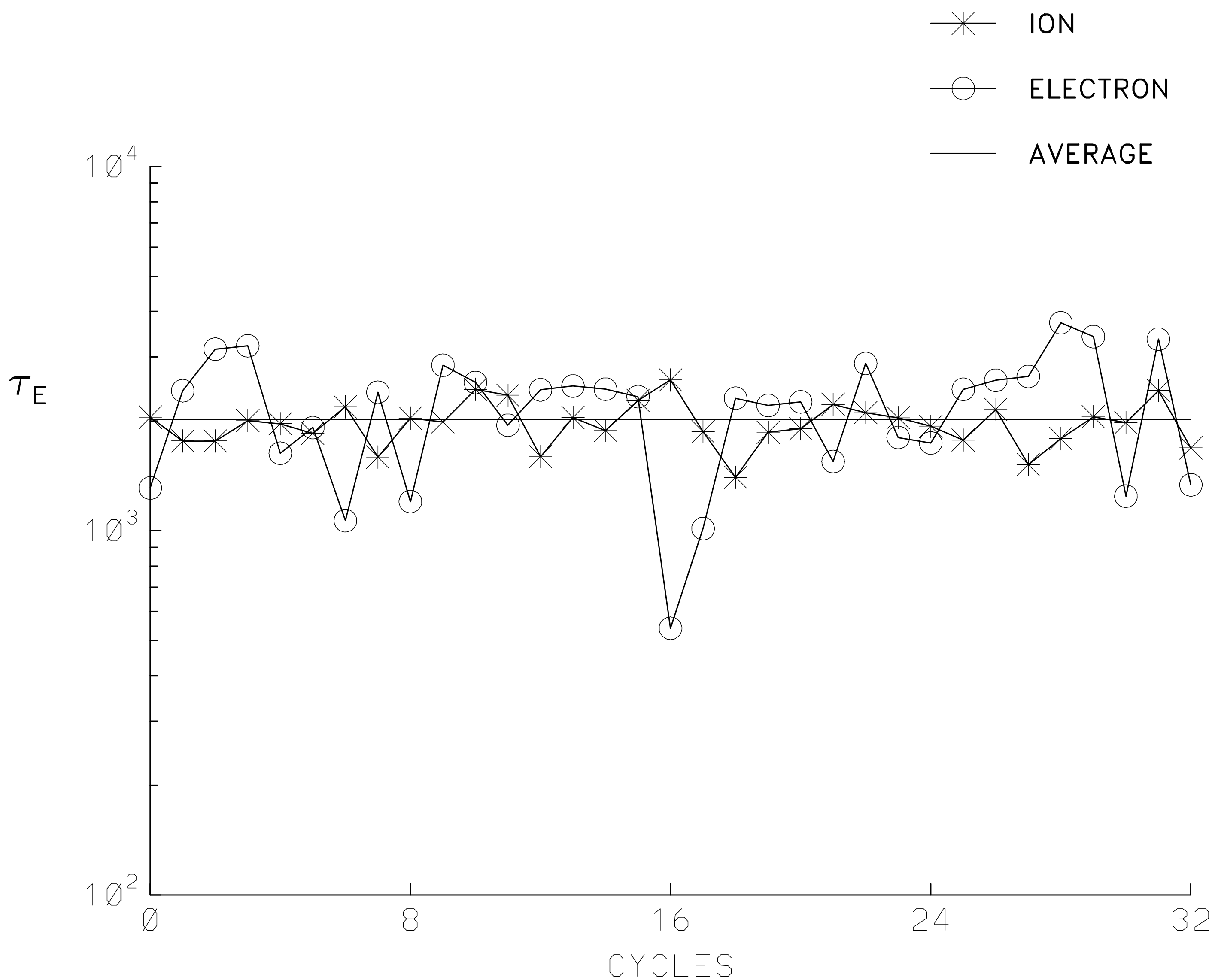
Four cross sections of the flux surfaces over the full torus of a wall stabilized PG3 (simulated LI383) equilibrium operated as a hybrid at $\beta = 0.05$ with a pressure profile $p = p_0(1 - s^{1.2})^{1.2}$ and with net current bringing the rotational transform into an interval between 0.55 and 0.80. A visible $m = 4,\ n = 3$ island in the solution shows that an equilibrium limit on β may have been reached.



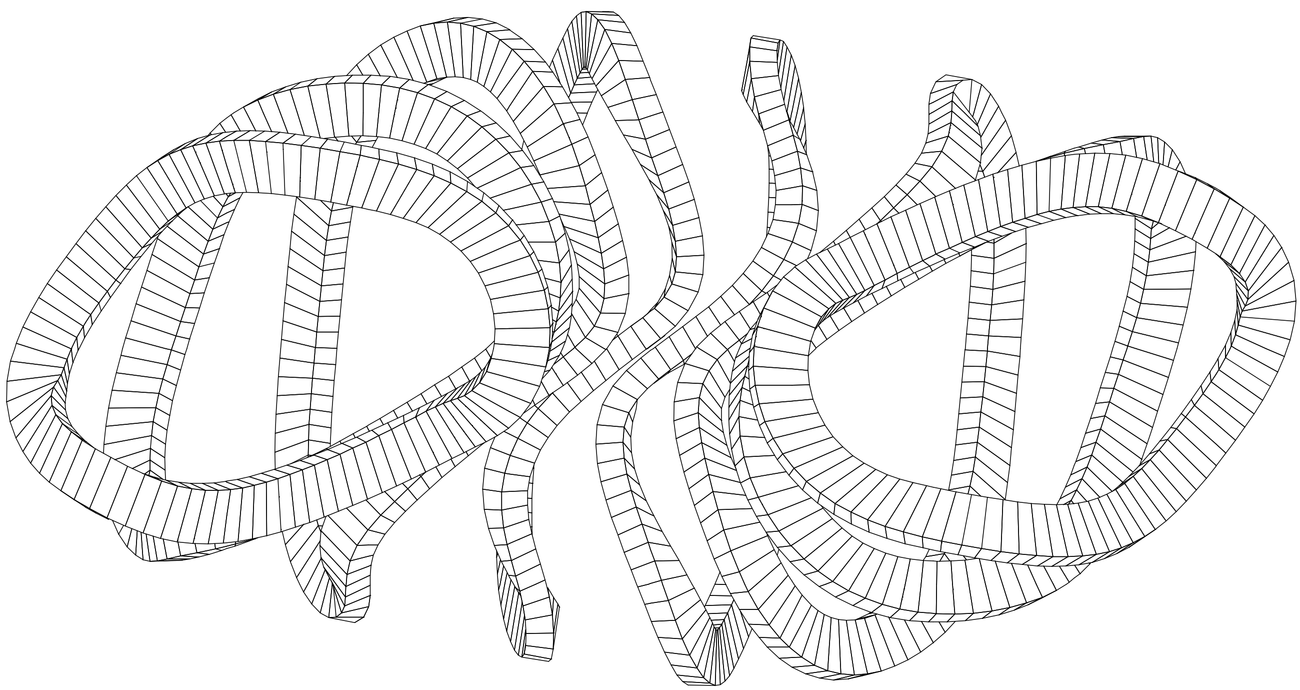
Iterations to quasineutrality in a Monte Carlo computation of the energy confinement time τ_E, measured in milliseconds, for a PG3 stellarator with major radius 10.5 m and plasma radius 2.3 m at reactor conditions with average $T = 15\,\text{keV}$, $n = 11 \times 10^{13}\,\text{cm}^{-3}$, and $B = 5\,\text{tesla}$. The magnetic spectrum has good quasiaxial symmetry, and the radial electric field rises to a potential level three times as big as the temperature. A better reactor can be designed if $B = 6.5\,\text{tesla}$.



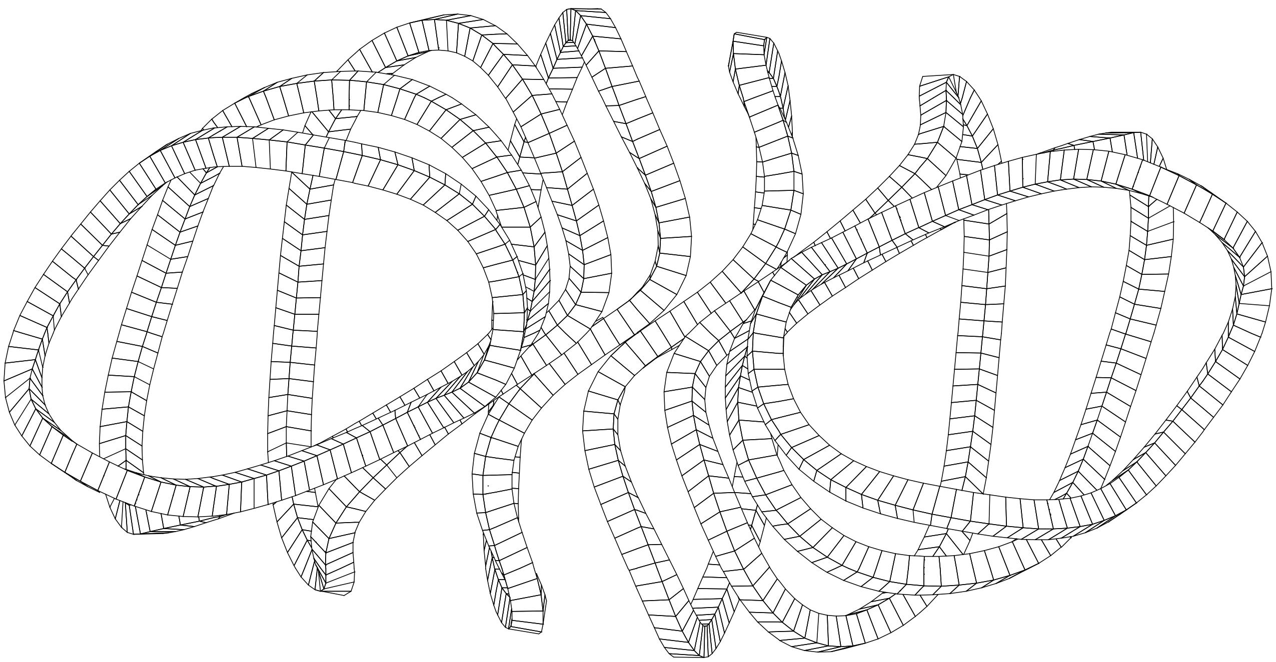
Asymmetric view of 6 out of 18 modular coils for the PG3 stellarator in a vacuum magnetic field given by the Biot-Savart law. Parameters have been adjusted to provide ample space around each coil, and the aspect ratio of the plasma is 4.5. The low number of coils generates significant ripple in the magnetic field.



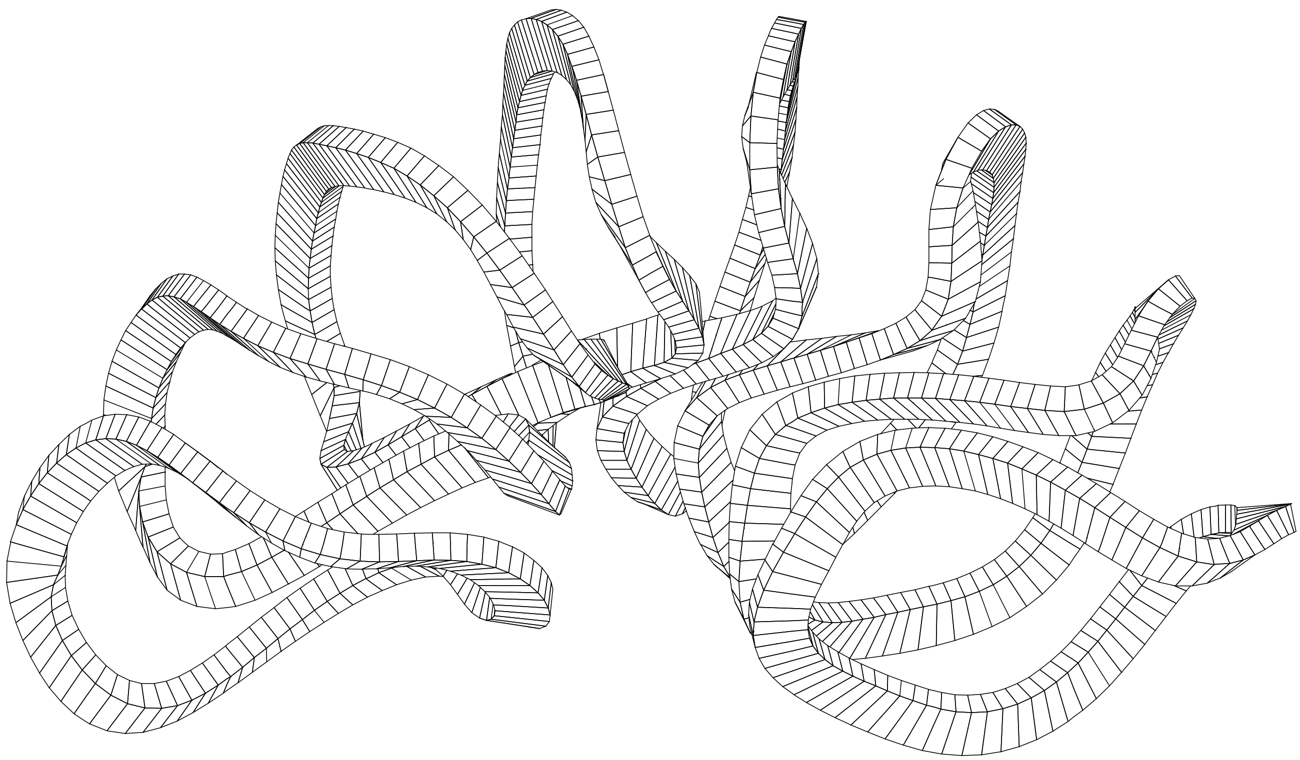
Iterations to quasineutrality in a Monte Carlo computation of the energy confinement time τ_E, measured in milliseconds, for an MHH2 stellarator with major radius 7.5 m and plasma radius 2.2 m at reactor conditions with average $T = 16\,\mathrm{keV}$, $n = 10^{14}\,\mathrm{cm}^{-3}$, and $B = 5\,\mathrm{tesla}$. The magnetic spectrum has excellent quasiaxial symmetry, and the radial electric field rises to a potential level two times as big as the temperature. This MHH2 reactor is designed to deliver 1 GW of electric power.



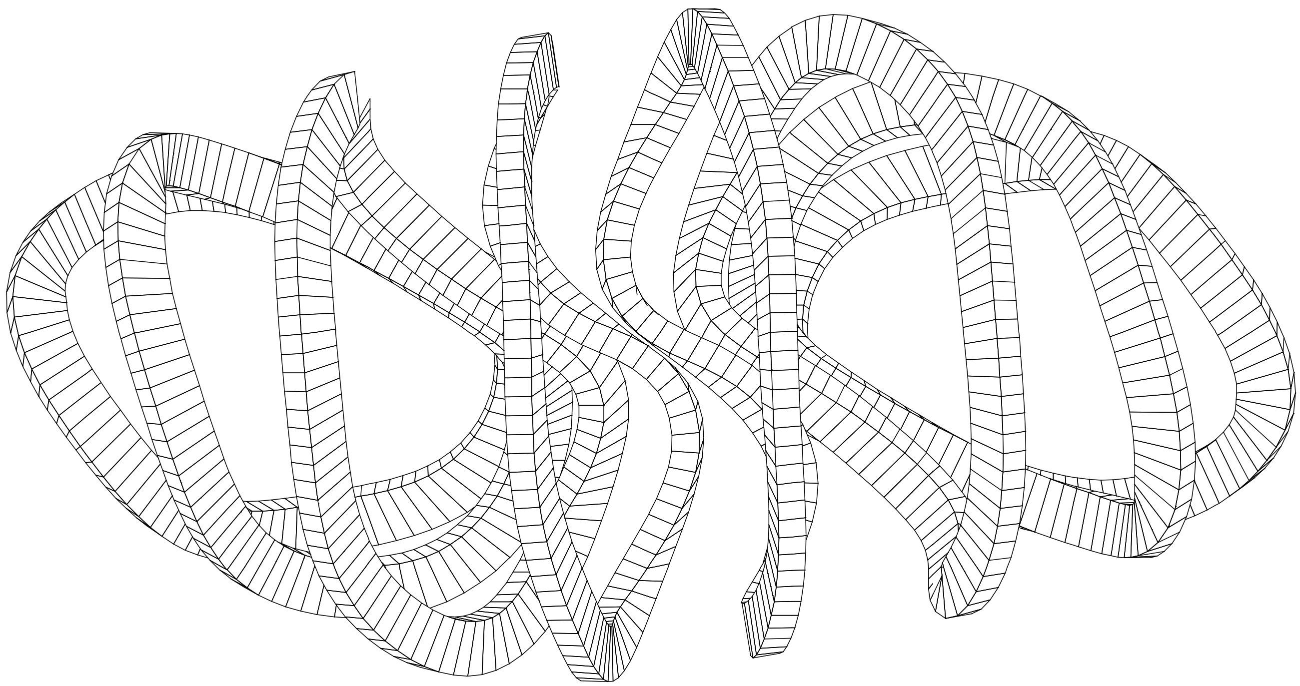
Symmetric view of 8 out of 16 modular coils of the MHH2 stellarator in a magnetic field given by the Biot-Savart law. Judicious filtering of the Fourier series used to calculate filaments specifying the geometry of the configuration defines shapes that are not excessively twisted. The coil packs have been rotated locally around their filaments to provide adequate space between them, and the aspect ratio of the plasma is 3.5.



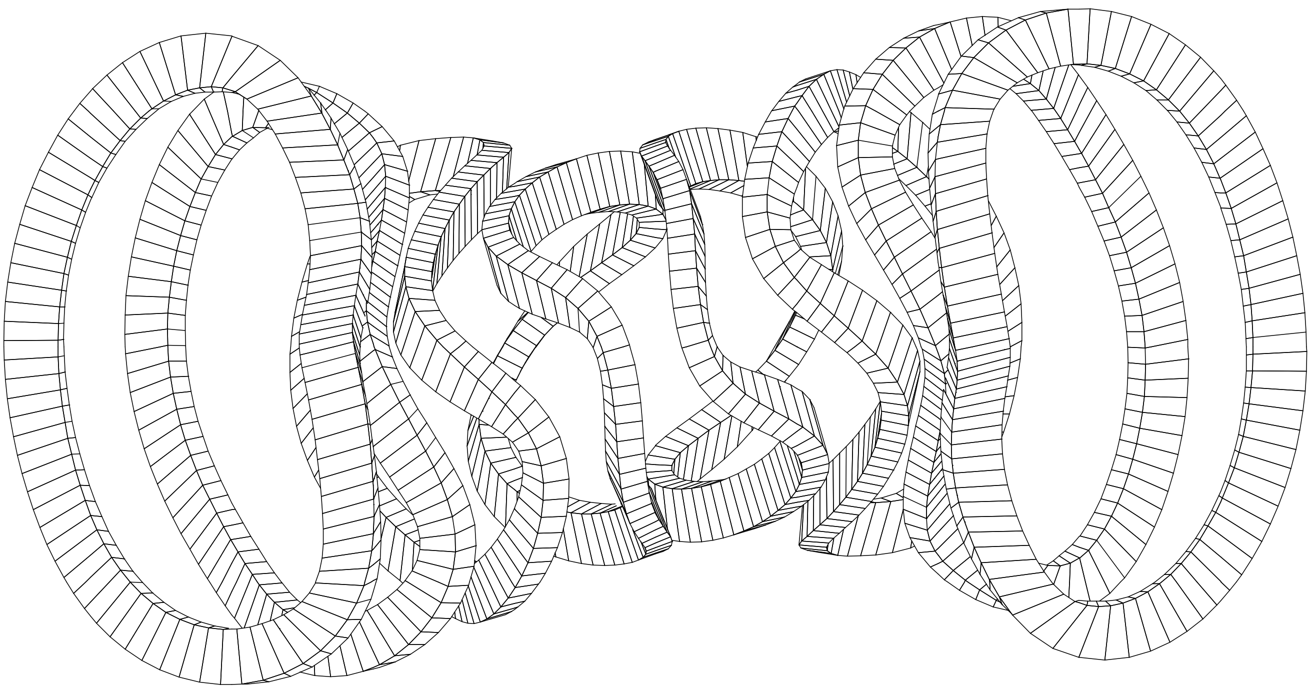
Symmetric view of 8 out of 16 modular coils of the MHH2 stellarator in a magnetic field given by the Biot-Savart law. Square coil packs have been used here with sides that are 28% of the plasma radius. In a practical case this is about 62 cm. It is possible to reshape the coils so they become fatter.



Rotated view of 8 out of 16 modular coils of the MHH2 stellarator in a vacuum magnetic field given by the Biot-Savart law. Parameters have been adjusted to provide ample space around each coil, and the long and short sides of the rectangular coil packs have lengths equal to one half and one quarter of the plasma radius, respectively.



Exterior view of 8 out of 16 modular coils of the MHH2 stellarator in a magnetic field given by the Biot-Savart law. There is ample space between the coils to allow for ports to take care of maintenance from the outside, where the geometry is similar to that in a tokamak.



Side view of 8 out of 16 modular coils of the MHH2 stellarator in a vacuum magnetic field given by the Biot-Savart law. Again ample space is visible between the coils to allow for ports to take care of maintenance. A good way to operate this configuration as a magnetic fusion reactor would be to make it a stellarator-tokamak hybrid.