PHYSICS ASSESSMENT OF A TWO FIELD PERIOD REACTOR

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High resolution NSTAB calculations show that the LHD stellarator is linearly unstable, but remains nonlinearly stable, at the β of 3.2% achieved experimentally. There is similar agreement between theory and experiment for the W7-AS. The correlation of computations with observations has been exploited to design a quasiaxially symmetric stellarator with two field periods. Transport is like that in a comparable tokamak. Coils for this reactor are under investigation at UCSD. There are other good configurations with three, four or five field periods.

$$\int\int\int[\frac{1}{2}B^2 - p(s)]\,dV \;\;=\;\; \text{minimum}$$

$$\mathbf{B} = \nabla s \times \nabla\theta = \nabla\phi - \zeta\nabla s\ , \quad \mathbf{J} = \nabla s \times \nabla\zeta$$

$$r + iz = e^{iu}\,\Sigma\,\Delta_{mn}\,e^{-imu+inv}$$

$$\frac{1}{B^2} = \Sigma\,B_{mn}\cos(m\theta - [n - \iota m]\phi)$$

$$\zeta = p'\,\Sigma\,\frac{B_{mn}}{n - \iota m}\sin(m\theta - [n - \iota m]\phi)$$

$$\mathbf{B} = \nabla \times \sum_{j=0}^{k}\;\delta s_j \int\int \frac{\nabla\zeta_j \times \mathbf{N}}{4\pi r}d\sigma$$

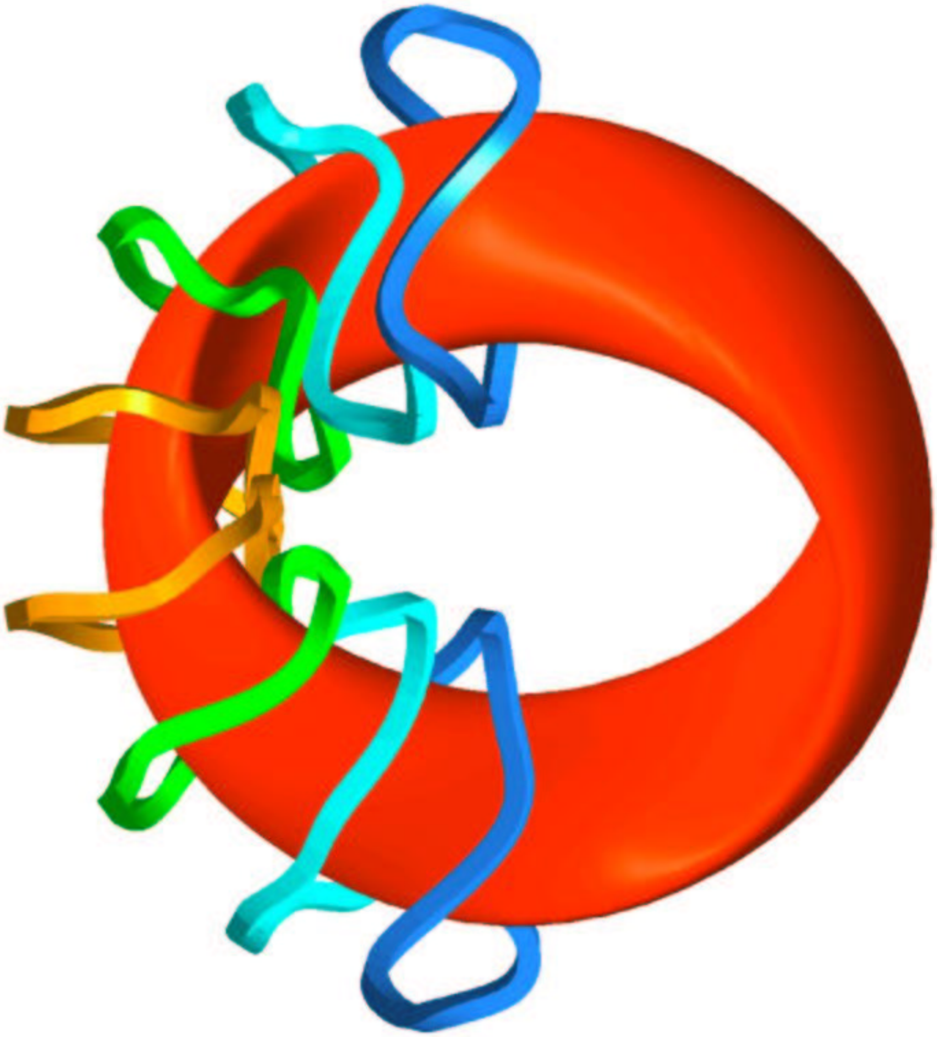
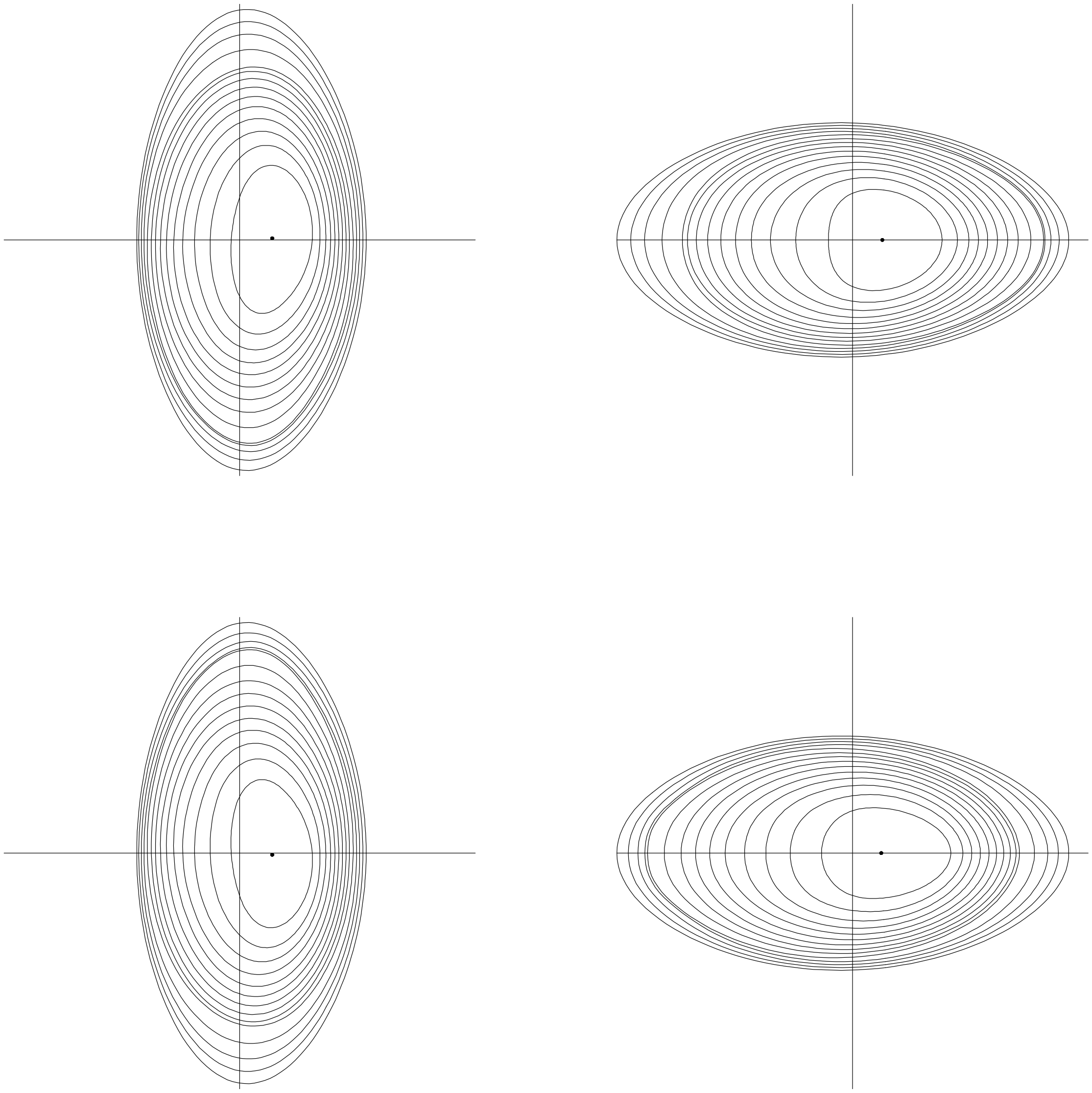
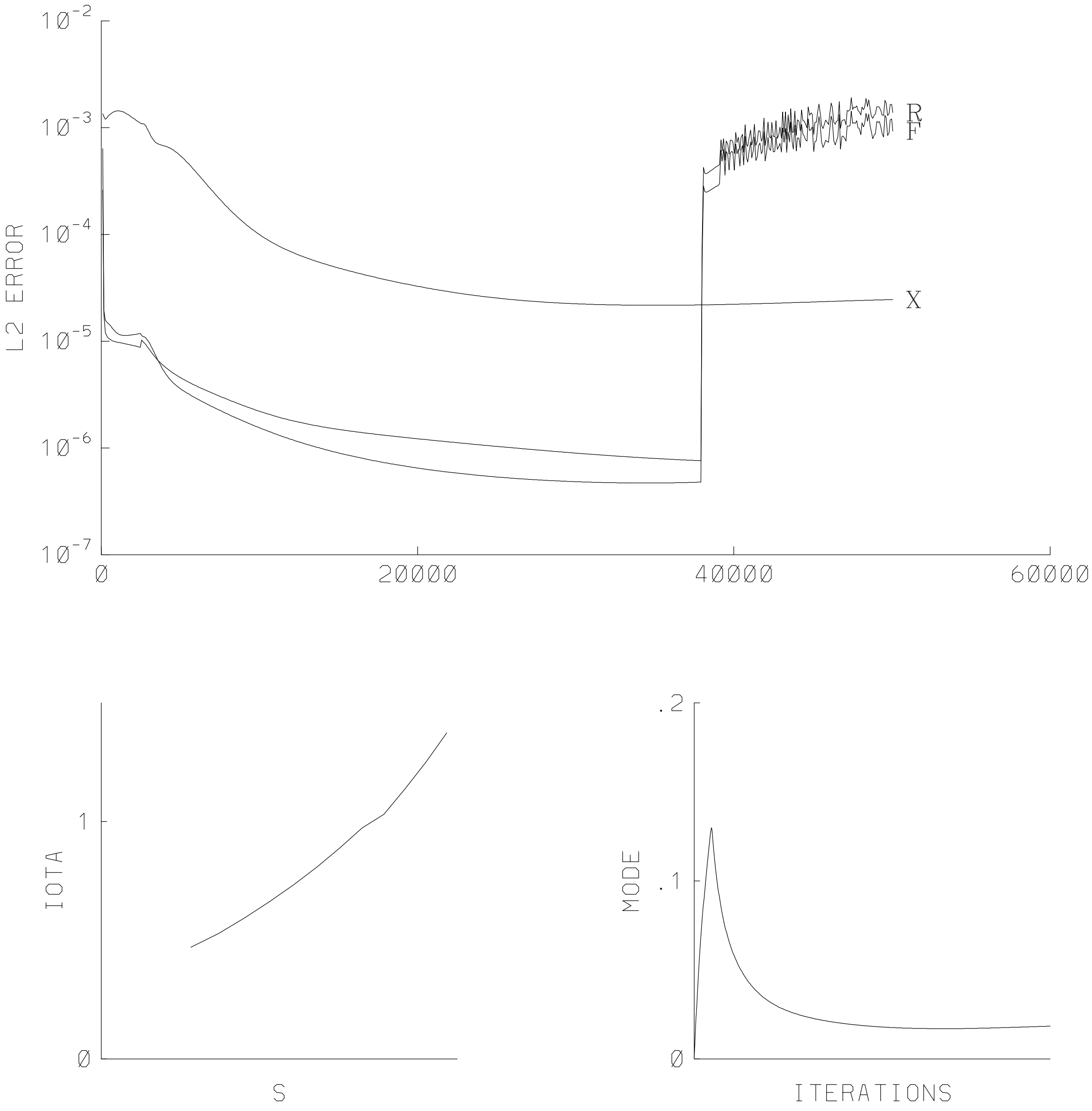


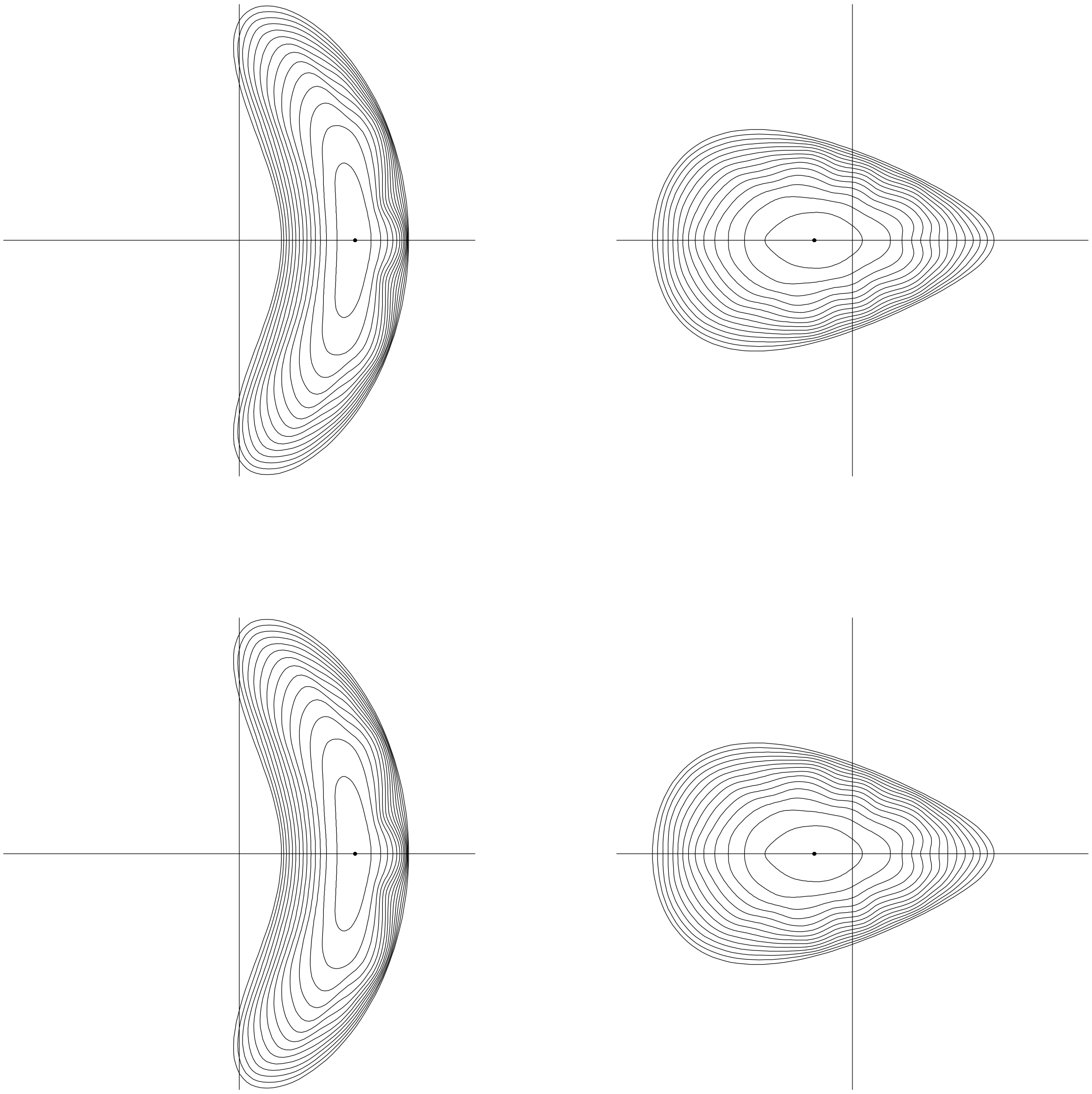
Diagram of an MHH2 stellarator with $A = 3.5$ designed by running the NSTAB equilibrium code. Only half of the twelve moderately twisted modular coils needed to produce the external field are shown. This configuration is a candidate for the stellarator reactor study being conducted at UCSD. Maintenance seems to be feasible through ports between each pair of coils. (Courtesy of Tak-Kuen Mau and Tsueren Wang.)



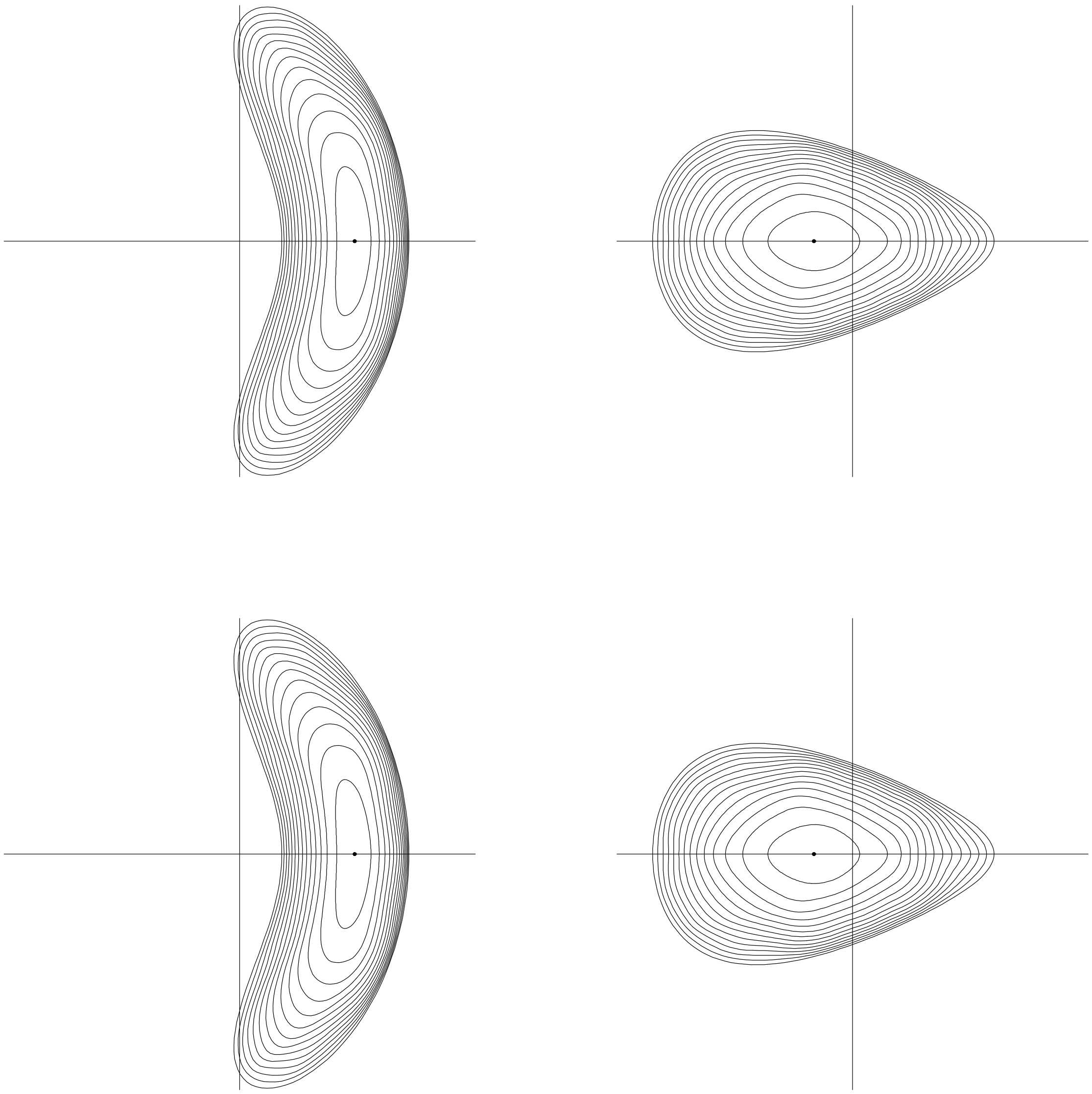
Poincaré map of the flux surfaces at four cross sections over the full torus of a bifurcated LHD equilibrium at $\beta = 0.032$ with the magnetic axis at a position with plasma radius $R = 3.6$ m. For a standard pressure profile $p = p_0(1 - s)$ the global $m = 1,\ n = 1$ mode of this solution is linearly unstable, but nonlinearly stable. This NSTAB calculation was performed using harmonics of degree up to 24 in the poloidal angle and up to 120 in the toroidal angle.



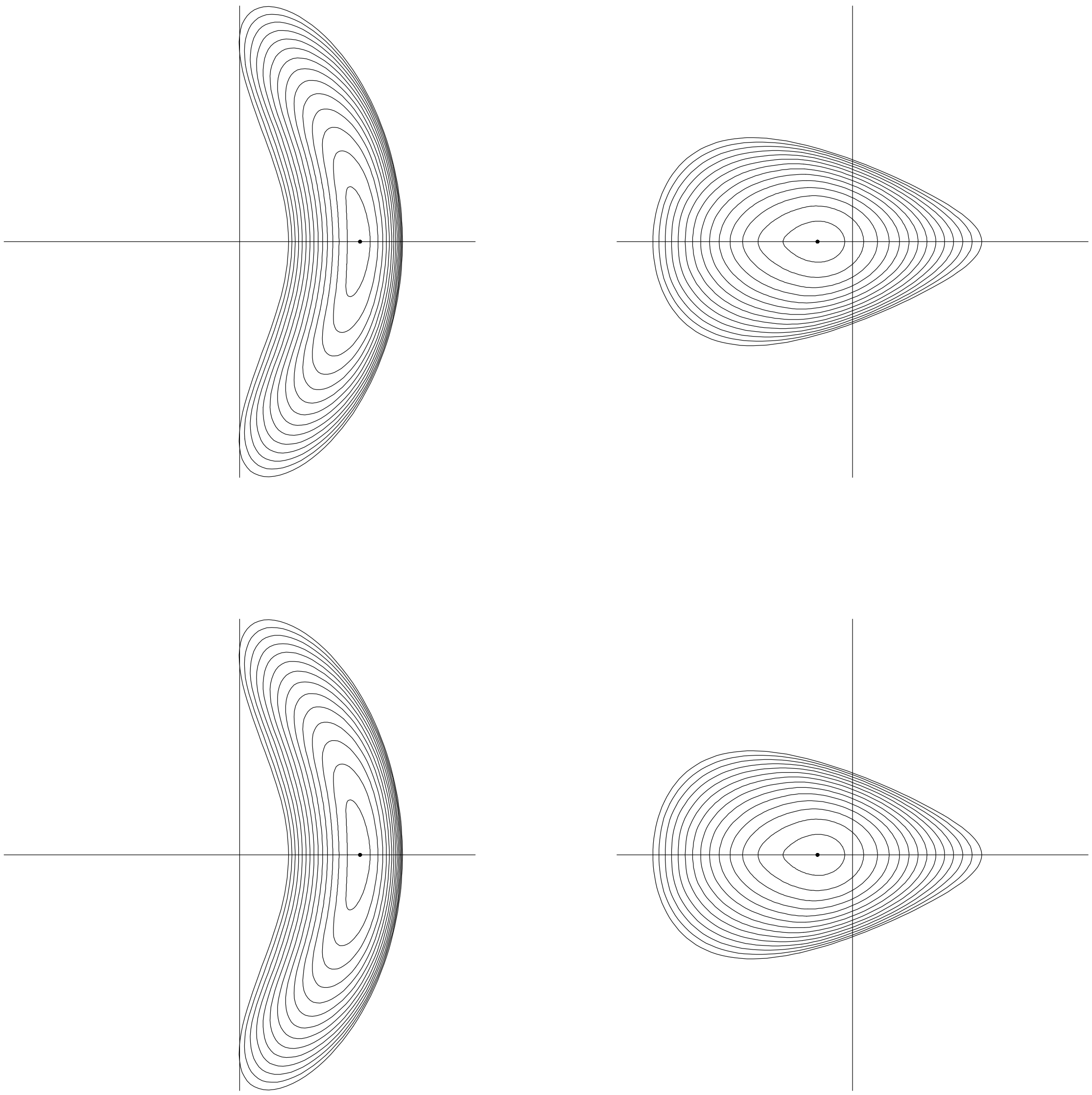
Convergence of NSTAB computations over ten field periods for a bifurcated LHD equilibrium at $\beta = 0.032$. Difficulties occur when the Jacobian becomes very large at the current sheet modeling an island where $\iota = 1$. Because of field errors this island is always present in the experiment. Earlier versions of the results about nonlinear stability were published in the book Magnetic Equilibrium and Stability of Stellarators, Springer, 1984.



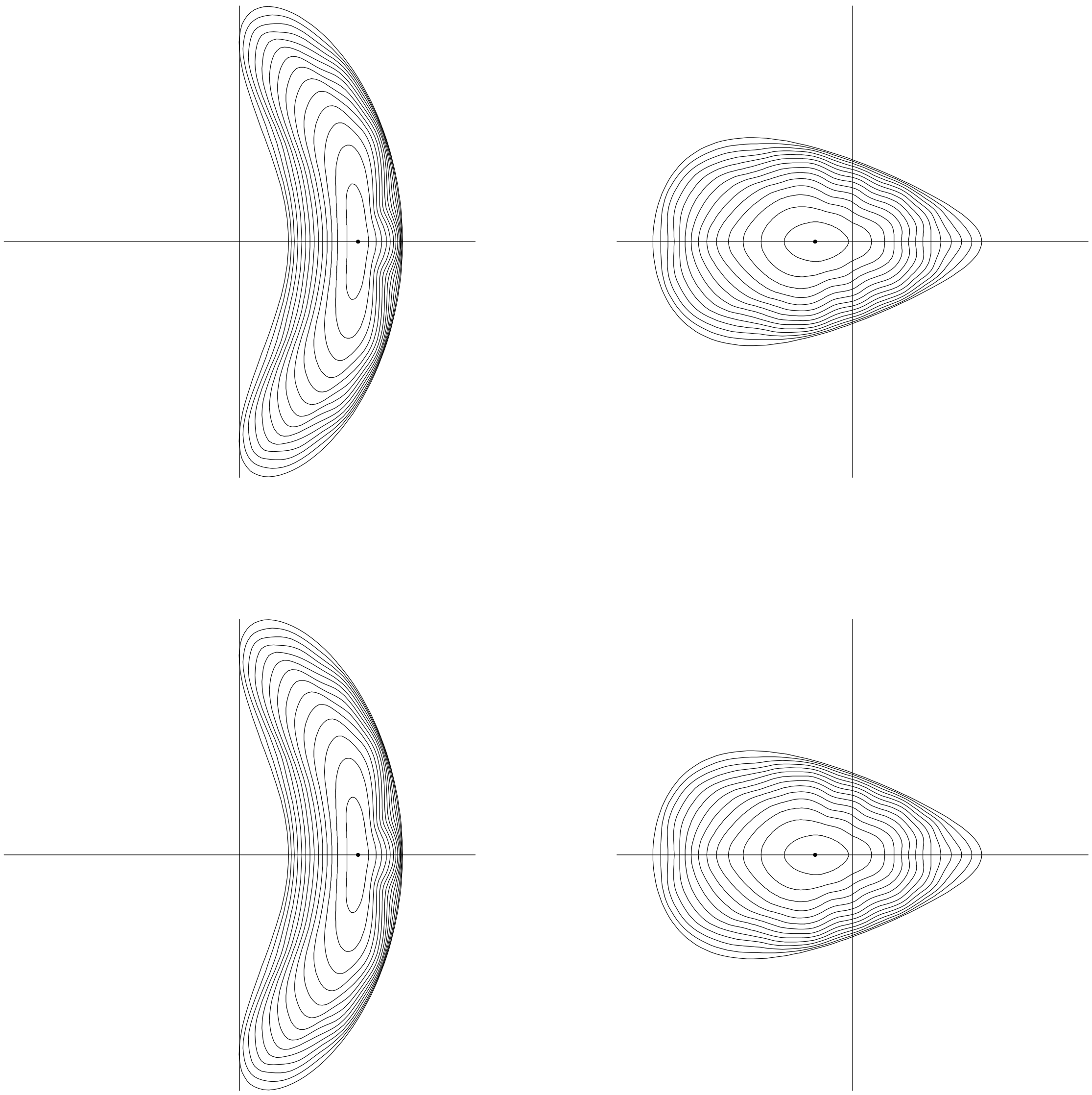
Four cross sections of the flux surfaces over two field periods of a wall stabilized MHH2 equilibrium at $\beta = 0.045$ with pressure $p = p_0(1 - s^{1.5})^{1.5}$ and with hybrid net current bringing the rotational transform into the interval $0.61 > \iota > 0.51$. The ballooning mode that has become visible in the solution after 500,000 cycles of an accelerated iteration scheme shows that a β limit has been reached. The structure of the mode does not change much when the mesh is refined.



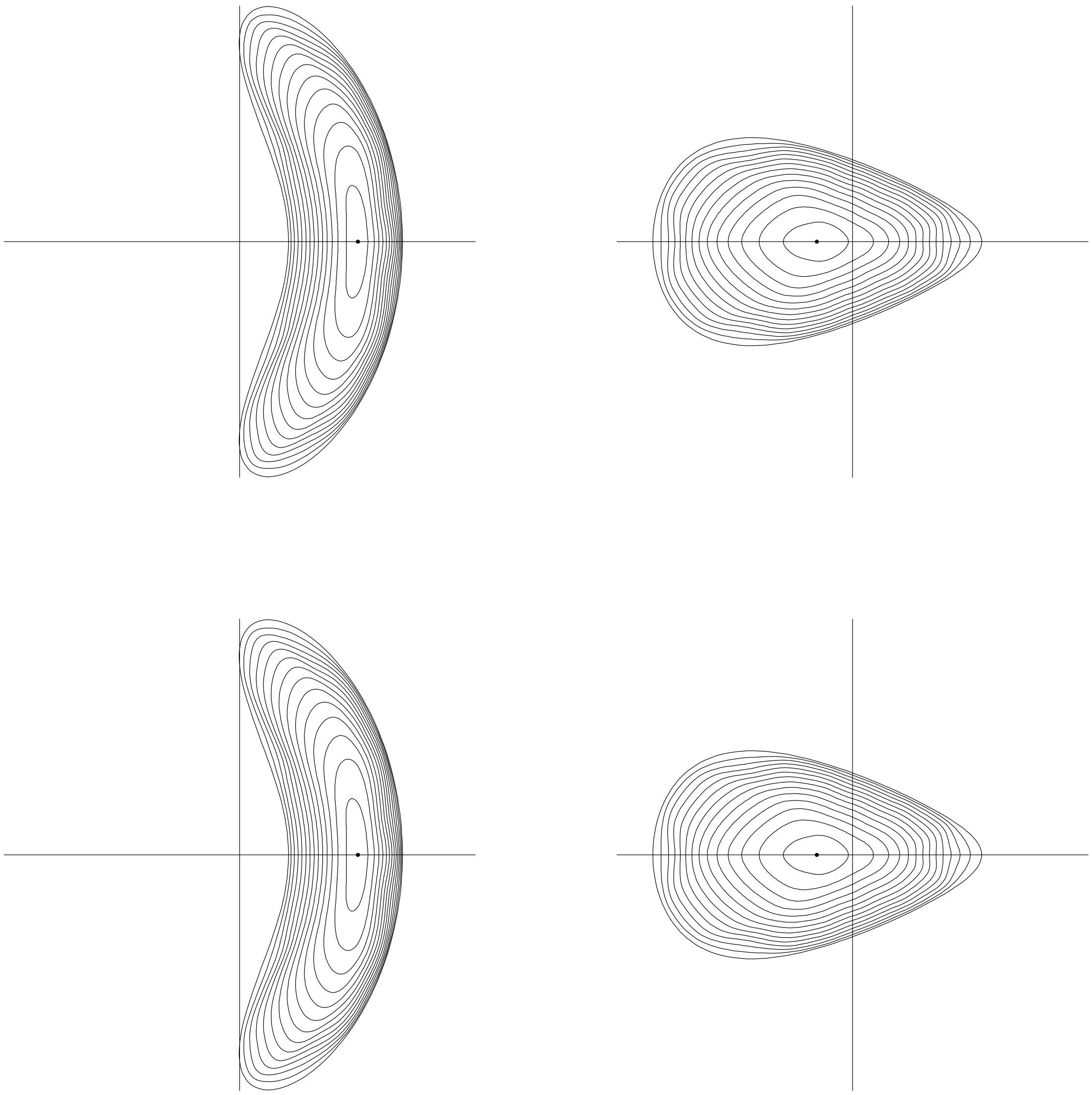
Four cross sections of the flux surfaces over two field periods of a stable MHH2 equilibrium at $\beta = 0.04$ with a standard pressure profile and with bootstrap current such that $0.51 < \iota < 0.55$. After 200000 iterative cycles of the NSTAB code the flux surfaces still look physically reasonable. No evidence of MHD instability when $\beta < 0.04$ has been found in a convergence study of these calculations.



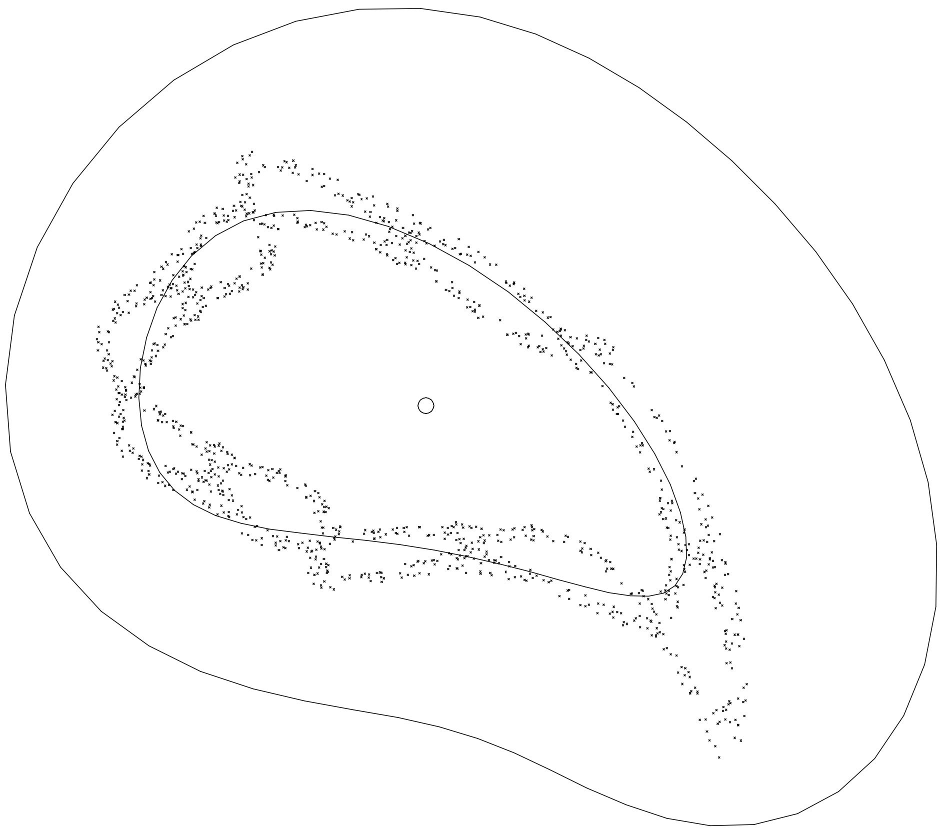
Four cross sections of the flux surfaces over two field periods of an MHH2 equilibrium at $\beta = 0.06$ with pressure $p = p_0(1 - s^{1.1})^{1.1}$ and with bootstrap current bringing the rotational transform into the interval $0.51 < \iota < 0.55$. After only 2000 iterative cycles of the NSTAB code the magnetic surfaces look acceptable and the solution does not appear to have reached an equilibrium limit on β. However, after so few cycles the resolution of the computation is inadequate.



Four cross sections of the flux surfaces over two field periods of an MHH2 equilibrium at $\beta = 0.06$ with pressure $p = p_0(1 - s^{1.1})^{1.1}$ and with bootstrap current bringing the rotational transform into the interval $0.55 > \iota > 0.53$. After 100000 iterative cycles of the NSTAB code a ballooning mode appears in the solution and the flux surfaces show that there may be an $m = 4,\ n = 2$ island near the separatrix. It is doubtful whether such a high value of β can be reached in an experiment.



Four cross sections of the flux surfaces over two field periods of an MHH2 equilibrium at $\beta = 0.05$ with pressure $p = p_0(1 - s^{1.1})^{1.1}$ and with bootstrap current bringing the rotational transform into the interval $0.51 > \iota > 0.49$. After 100000 iterative cycles of the NSTAB code there is only marginal evidence of an $m = 4,\ n = 2$ island at $\iota = 2/4$. Further optimization of the MHH2 configuration could be undertaken to meet new requirements that may come up during the ARIES reactor study.



Poincaré section of a line tracing calculation for the MHH2 stellarator displaying the control surface for the coils, the known shape of the plasma at $\beta = 0$, a filament that simulates current in the plasma, and magnetic lines computed at finite β in the scrape-off layer of the solution, where the rotational transform is crossing the resonant value $\iota = 2/5$. The extent of the magnetic lines outside the separatrix shows there are good possibilities for the construction of a divertor.

```
     ns1   npsi1   nphi1   nmag1  nekin1   tend1   radl1   freq1
       1       4       4       4       1    2.00 0.00150   0.250

     P00    del1   sizel   alpha       B   title
   -2.00   1.000 0.01500      2.    5.00    MHH2


     Pmn       m       n   italg       m       n           bmn
   0.030       1       0       1       0       0     0.997E+00
   0.000       0       1       1       1       0     0.367E+00
   0.000       2       2       1       2       0     0.116E+00
                                       3       0     0.529E-01
                                       0       1     0.545E-01
                                       4       0     0.223E-01
                                       1       1     0.200E-01
                                       1       2     0.186E-01
                                       2       2     0.184E-01
                                       3       2     0.166E-01


    DEL1    P10    P01    P22    TAUi    TAUe   <TAU>  Error

   1.000  0.030  0.000  0.000   34915   44004   39197  0.002
   1.023  0.031  0.002  0.000   35580   53785   42174  0.002
   1.059  0.031  -.001  0.002   42132  104659   52920  0.004
   1.126  0.023  -.001  0.000  115413    7680   42043  0.006
   1.012  0.012  0.000  -.001   20223  140052   45480  0.001
   1.034  0.027  -.004  0.001   49872   77375   49718  0.001
   1.087  0.023  -.001  0.002   20983   35561   42804  0.002
   1.153  0.018  -.001  0.001   51879   18460   39827  0.001
   1.081  0.025  -.001  -.002   32282   46154   39579  0.003
```

TRAN MONTE CARLO CALCULATION OF ION AND ELECTRON CONFINEMENT IN THE MHH2 STELLARATOR AT REACTOR CONDITIONS. ITERATIONS ARE PERFORMED ON THE MIRROR TERM b01 IN THE SPECTRUM TO ACHIEVE QUASINEUTRALITY.

```
     ns1   npsi1   nphi1   nmag1  nekin1   tend1   radl1   freq1
       1      32      32       8       1    2.00 0.00170   0.200

     P00    del1   sizel   alpha       B   title
   -2.00   1.000 0.02400      1.    5.00 TOKAMAK
```

```
     Pmn       m       n   italg       m       n           bmn
   0.030       1       0       1       0       0     0.100E+01
   0.010       2       0       1       1       0     0.390E+00
   0.000       0       1       1       2       0     0.877E-01
                                       3       0     0.355E-01
                                       1       6     0.256E-01
```

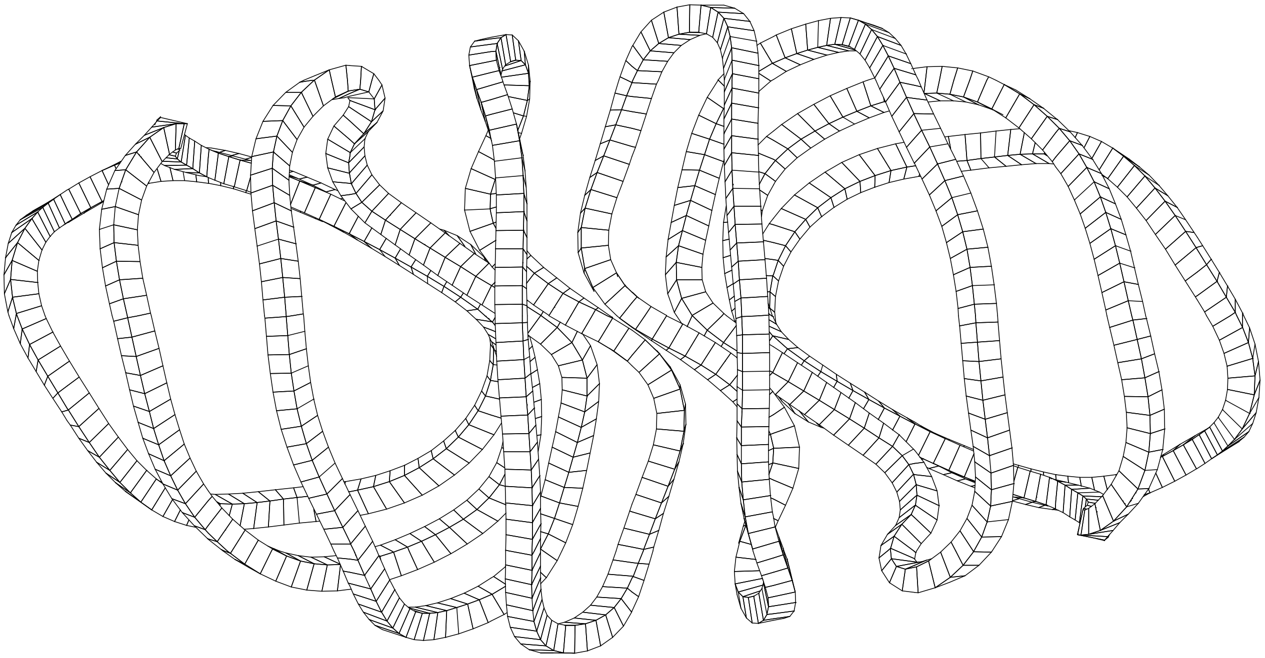
```
    DEL1    P10    P20    P01    TAUi    TAUe   <TAU>  Error

   1.000  0.030  0.010  0.000   12492   15979   14128  0.011
```

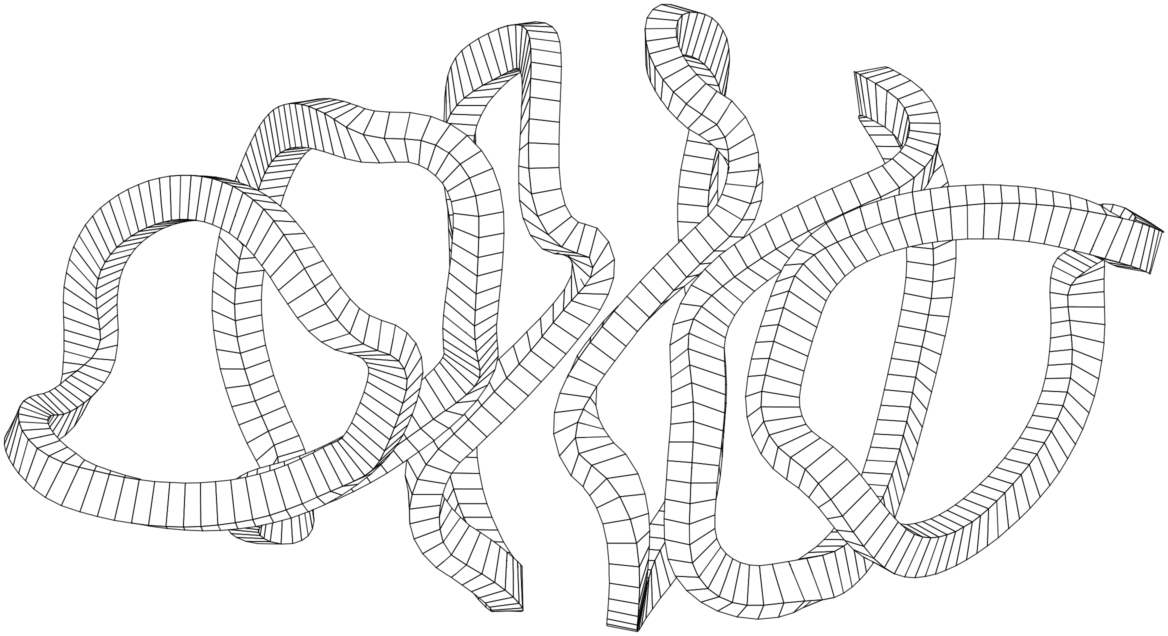
TRAN MONTE CARLO COMPUTATION OF ION AND ELECTRON CONFINEMENT

FOR A TOKAMAK COMPARABLE TO THE MHH2 STELLARATOR WITH A 3D TERM

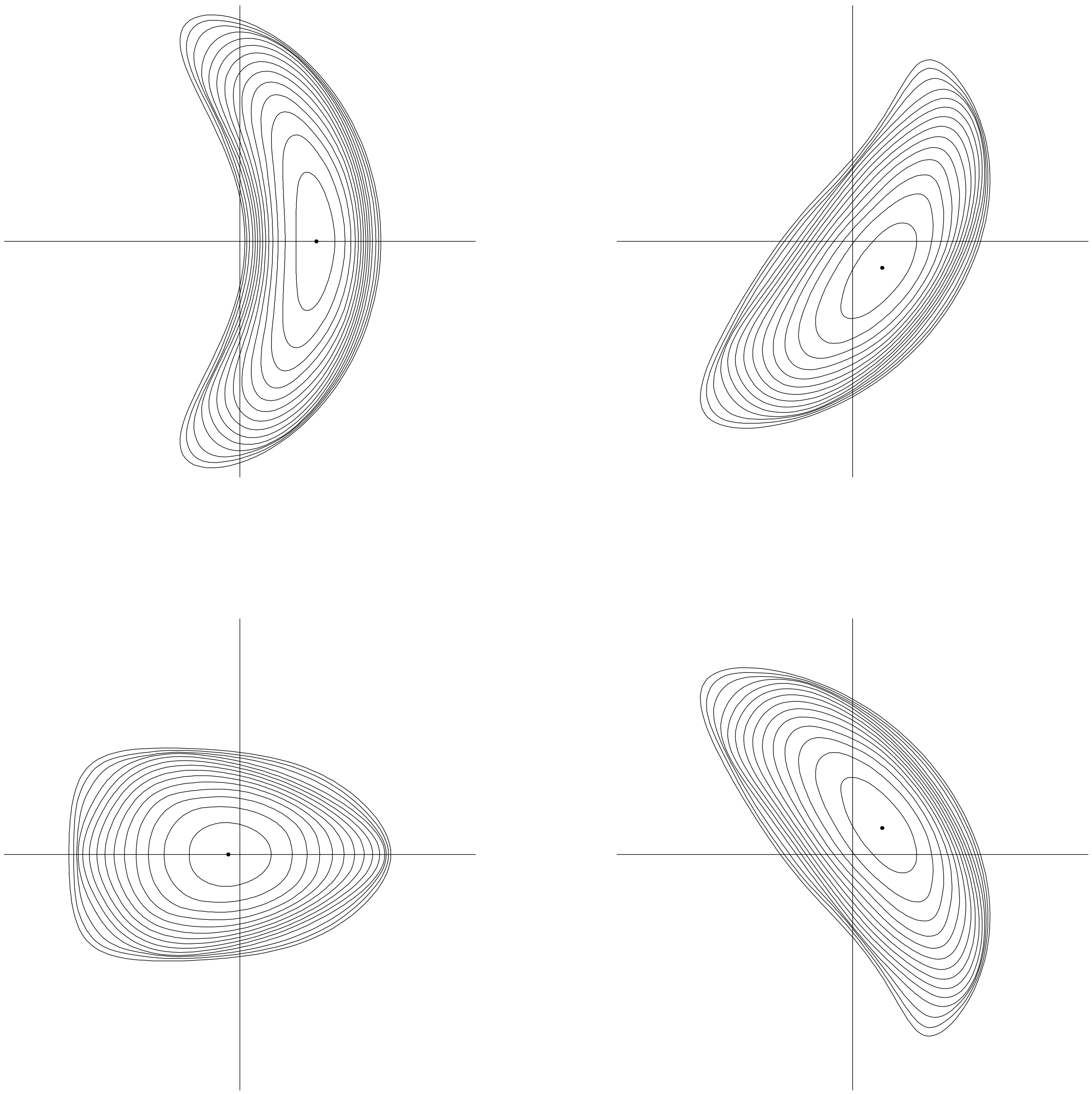
b16 IN THE SPECTRUM DUE TO RIPPLE FROM 12 TOROIDAL FIELD COILS.



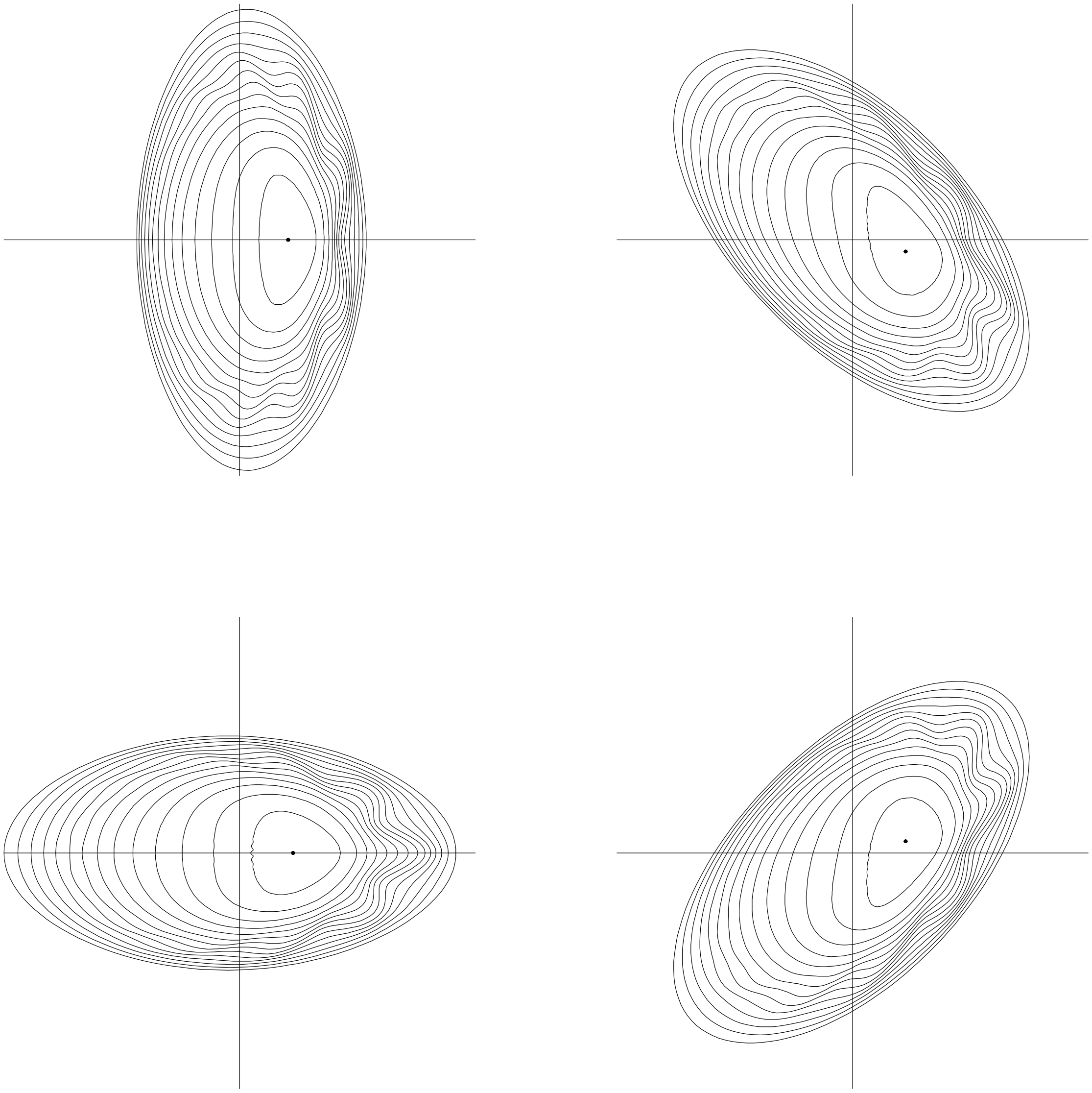
Exterior view of 8 out of 16 modular coils of the MHH2 stellarator in a magnetic field given by the Biot-Savart law. There is ample room between each pair of coils to allow for ports to take care of maintenance from the outside, where the geometry is similar to that in a tokamak. At the isolated point where they touch the coils can be separated a little by reshaping them. Smooth coils produce robust flux surfaces that do not deteriorate when changes are made in the vertical and toroidal fields.



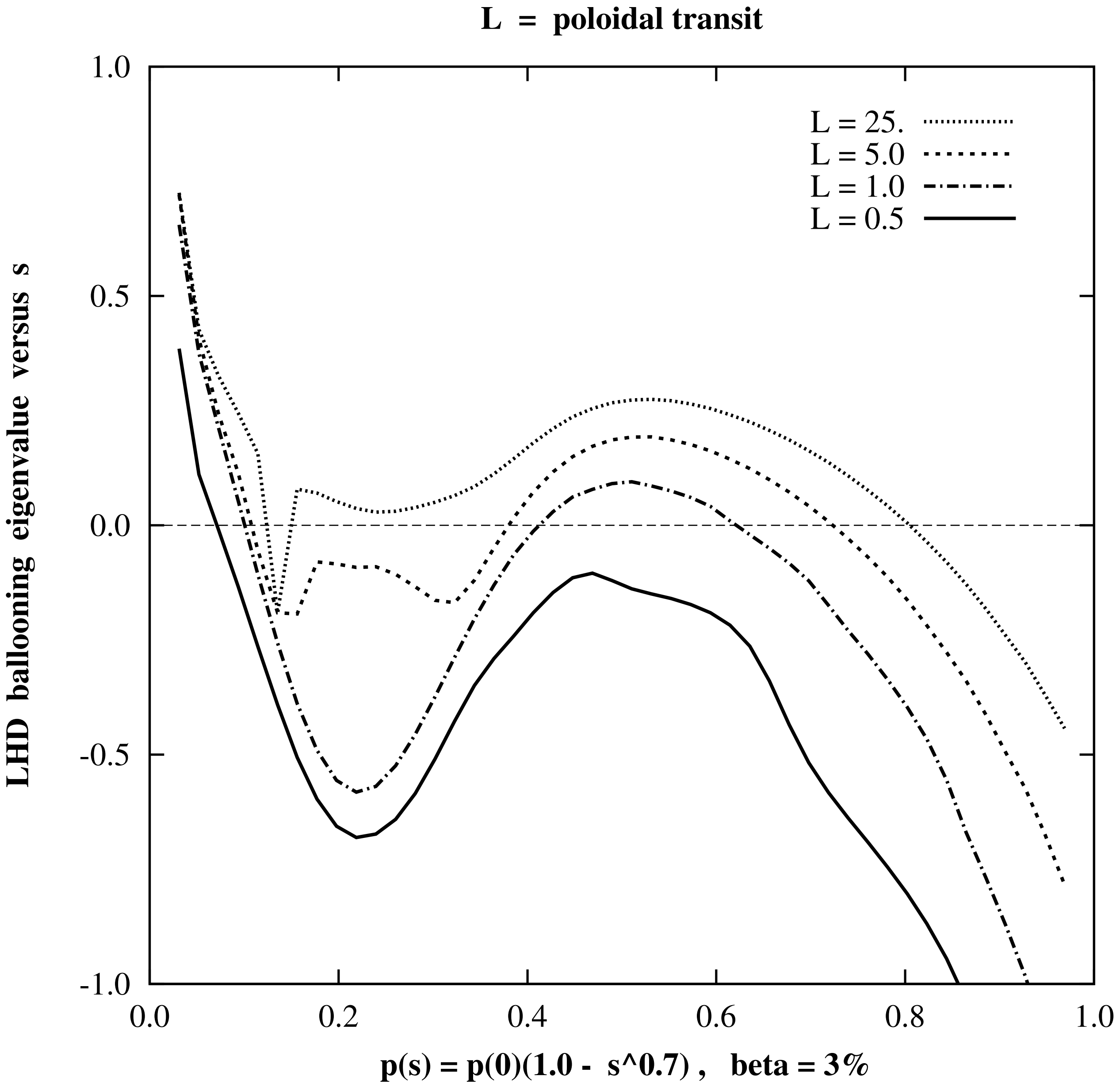
Asymmetric view of 6 out of 18 modular coils for the PG3 stellarator in a vacuum magnetic field given by the Biot-Savart law. Parameters have been adjusted to provide ample space around each coil, and the aspect ratio of the plasma is 4.5. The coils are smooth because they are relatively close to the plasma, but that means the corresponding reactor has to have a large major radius.



Four cross sections of the flux surfaces over the full torus of a wall stabilized PG3 (simulated LI383) equilibrium operated as a hybrid at $\beta = 0.05$ with a pressure profile $p = p_0(1 - s^{1.2})^{1.2}$ and with net current bringing the rotational transform into an interval between 0.55 and 0.80. A visible $m = 4$, $n = 3$ island in the solution shows that an equilibrium limit on β may have been reached. This calculation is sensitive to the choice of profiles.



Poincaré map of the flux surfaces at four cross sections over one field period of a wall stabilized LHD equilibrium at $\beta = 0.04$ with the magnetic axis shifted inward to a position with major radius $R = 3.6$ m. For a standard pressure profile $p = p_0(1 - s)$, this exceptionally accurate solution developed a ballooning mode that shows it has become nonlinearly unstable. The mode was stable at $\beta = 0.032$.



Numerical implementations of the local ballooning criterion depend on a choice of boundary conditions and give estimates of the β limit for the LHD that have been exceeded in the experiment by a significant margin.