Temperature Gradient Limits for Liquid-Protected Divertors

S. I. Abdel-Khalik, S. Shin, and M. Yoda

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G. W. Woodruff School of Mechanical Engineering
Atlanta, GA 30332–0405 USA

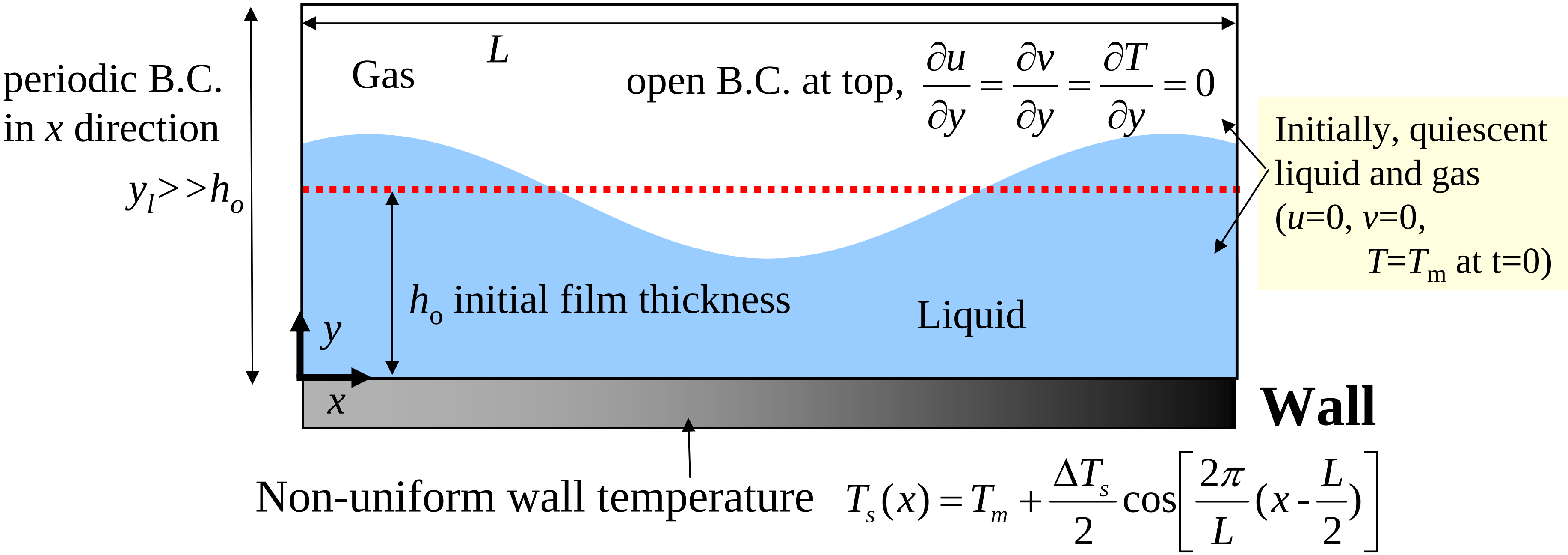
Problem Definition

- Work on Liquid Surface Plasma Facing Components and Plasma Surface Interactions has been performed by the ALPS and APEX Programs
- Operating Temperature Windows have been established for different liquids based on allowable limits for Plasma impurities and Power Cycle efficiency requirements
- This work is aimed at establishing limits for the maximum allowable temperature gradients (i.e. heat flux gradients) to prevent film rupture due to thermocapillary effects

Problem Definition

- Spatial Variations in the wall and Liquid Surface Temperatures are expected due to variations in the wall loading
- Thermocapillary forces created by such temperature gradients can lead to film rupture and dry spot formation in regions of elevated local temperatures
- Initial Attention focused on Plasma Facing Components protected by a “non-flowing” thin liquid film (e.g. porous wetted wall)

Problem Definition



- Two Dimensional Cartesian (x-y) Model (assume no variations in toroidal direction)
- Two Dimensional Cylindrical (r-z) Model has also been developed (local "hot spot" modeling)

Variables definition

- Non-dimensional variables

$$a = \frac{h_o}{L} \qquad y' = \frac{y}{h_o} \qquad x' = \frac{x}{L} = \frac{ax}{h_o}$$

$$u' = \frac{u}{(\mu_L / \rho_L L)} \qquad v' = \frac{v}{(a\mu_L / \rho_L L)} \qquad t' = \frac{t}{(\rho_L L^2 / \mu_L)}$$

$$T' = \frac{T - T_m}{\Delta T_s} \qquad V_g = \frac{\mu_L}{\rho_L h_o}$$

$$\mathrm{We} = \frac{\rho_L V_g^2 h_o}{\sigma_o} = \frac{\mu_L^2}{\rho_L \sigma_o h_o}$$

$$\mathrm{Fr} = \frac{V_g^2}{g h_o} = \frac{\mu_L^2}{g \rho_L^2 h_o^3}$$

$$\mathrm{Pr} = \frac{\mu_L c_L}{k_L} \qquad \mathrm{M} = \frac{\gamma_o \Delta T_s h_o}{\mu_L \alpha_L}$$

Governing Equations

- Conservation of Mass $$\frac{\partial u'}{\partial x'}+\frac{\partial v'}{\partial y'}=0$$
- Momentum

$$\sigma' = 1/\mathrm{We}-(\mathrm{M}/\mathrm{Pr})T'$$

$$a^2\rho^+\left[\frac{\partial u'}{\partial t'}+u'\frac{\partial u'}{\partial x'}+v'\frac{\partial u'}{\partial y'}\right]$$
$$=-\frac{\partial p'}{\partial x'}+a^2\frac{\partial}{\partial x'}\left(2\mu^+\frac{\partial u'}{\partial x'}\right)+\frac{\partial}{\partial y'}\left(\mu^+\frac{\partial u'}{\partial y'}\right)+a^2\frac{\partial}{\partial y'}\left(\mu^+\frac{\partial v'}{\partial x'}\right)+\int\left(\sigma'\kappa\mathbf{n}+\frac{\partial\sigma'}{\partial s}\mathbf{t}\right)\delta ds\cdot\hat{\mathbf{i}}$$

$$a^4\rho^+\left[\frac{\partial v'}{\partial t'}+u'\frac{\partial v'}{\partial x'}+v'\frac{\partial v'}{\partial y'}\right]$$
$$=-\frac{\partial p'}{\partial y'}+\frac{\rho^+}{\mathrm{Fr}}+a^4\frac{\partial}{\partial x'}\left(\mu^+\frac{\partial v'}{\partial x'}\right)+a^2\frac{\partial}{\partial x'}\left(\mu^+\frac{\partial u'}{\partial y'}\right)+a^2\frac{\partial}{\partial y'}\left(2\mu^+\frac{\partial v'}{\partial y'}\right)+a\int\left(\sigma'\kappa\mathbf{n}+\frac{\partial\sigma'}{\partial s}\mathbf{t}\right)\delta ds\cdot\hat{\mathbf{j}}$$

- Energy $$a^2\rho^+\left[\frac{\partial c^+T'}{\partial t'}+u'\frac{\partial c^+T'}{\partial x'}+v'\frac{\partial c^+T'}{\partial y'}\right]=\frac{a^2}{\mathrm{Pr}}\frac{\partial}{\partial x'}\left(k^+\frac{\partial T'}{\partial x'}\right)+\frac{1}{\mathrm{Pr}}\frac{\partial}{\partial y'}\left(k^+\frac{\partial T'}{\partial y'}\right)$$

Asymptotic Solution

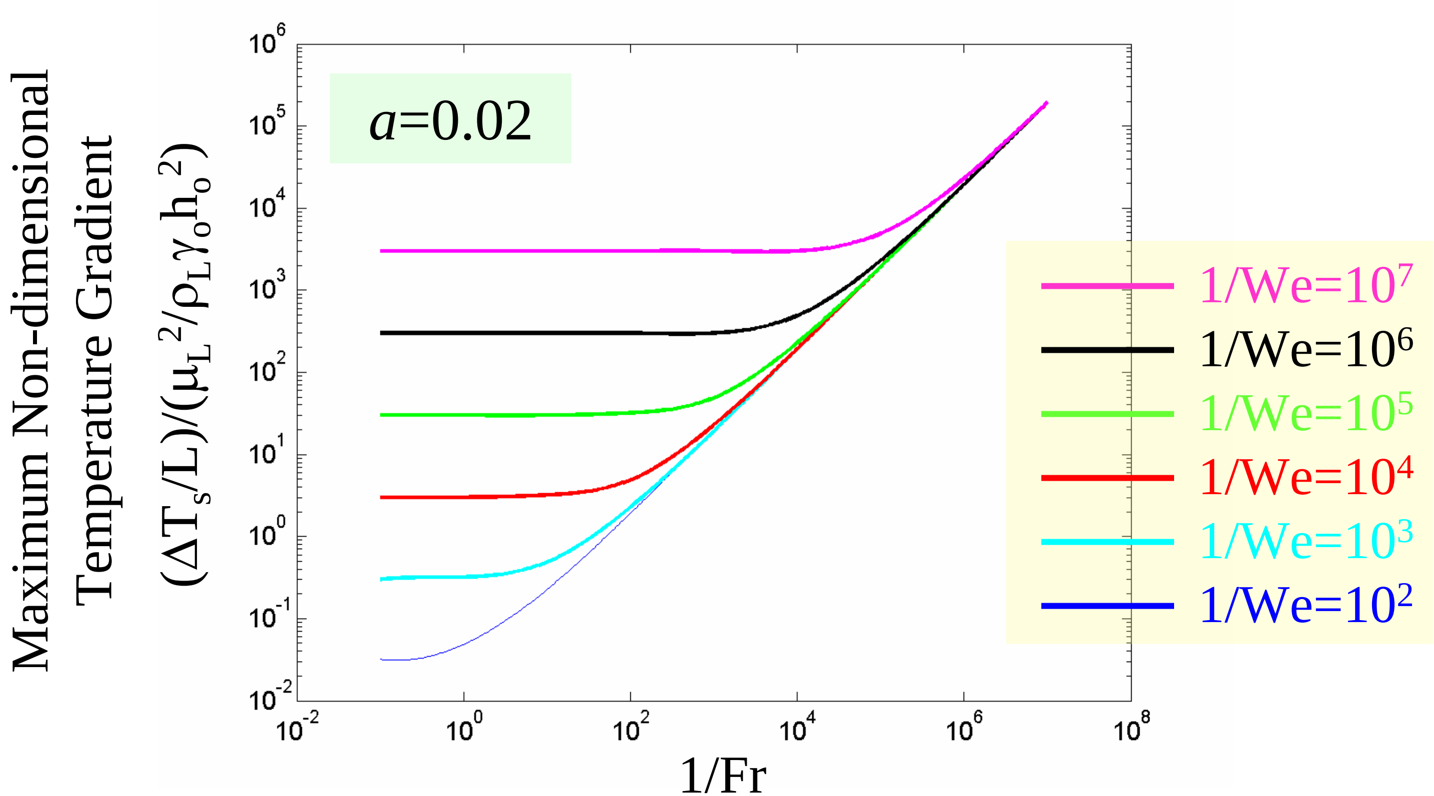
- Long wave theory with surface tension effect (a<<1)

  Governing Equations reduce to: [Bankoff, et al. Phys. Fluids (1990)]

$$\frac{a^2\mathrm{Fr}}{\mathrm{We}}\frac{\partial^3 h}{\partial x^3}h+\frac{\partial h}{\partial x}h+\frac{3}{2}(\mathrm{M}/\mathrm{Pr})\cdot\mathrm{Fr}\frac{\partial T_s}{\partial x}=0$$

- Generalized Charts have been generated for the Maximum non-dimensional temperature gradient (aM/Pr) as a function of the Weber and Froude numbers

Asymptotic Solution



- Similar Plots have been obtained for other aspect ratios
- In the limit of zero aspect ratio $(M/Pr)_{crit} = \frac{\pi^2}{12}\frac{1}{Fr}$

Results

- Asymptotic solution used to analyze cases for Lithium, Lithium-lead, Flibe, Tin, and Gallium with different mean temperature and film thickness
- Asymptotic solution produces conservative (i.e. low) temperature gradient limits
- Limits for "High Aspect Ratio" cases analyzed by numerically solving the full set of conservation equations using Level Contour Reconstruction Method

*Property Ranges (h_o=1mm)**

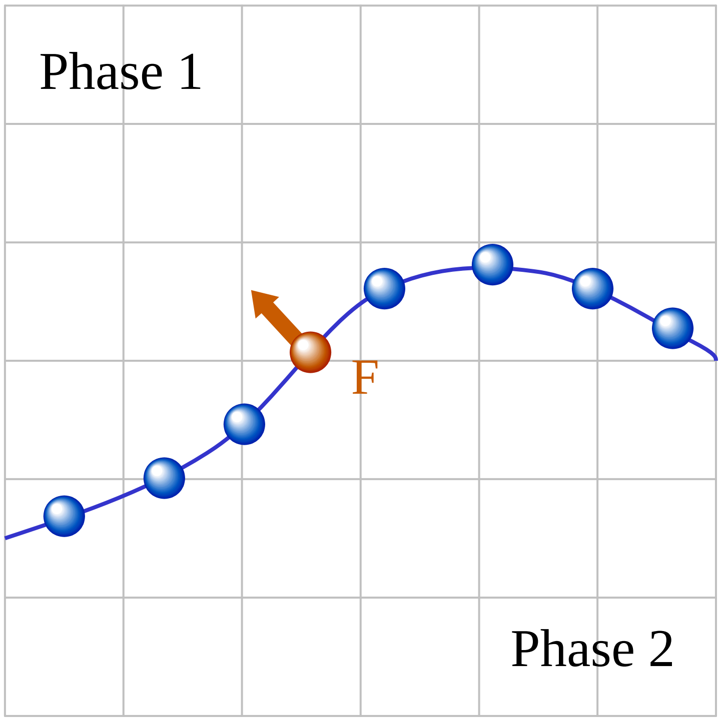
Parameter	Lithium		Lithium-Lead		Flibe		Tin		Gallium	
	573K	773K	573K	773K	573K	773K	1073K	1473K	873K	1273K
Pr	0.042	0.026	0.031	0.013	14	2.4	0.0047	0.0035	0.0058	0.0029
1/Fr	1.2×10^{4}	2.2×10^{5}	1.9×10^{5}	6.3×10^{5}	1.2×10^{3}	3.8×10^{4}	5.3×10^{5}	6.7×10^{5}	5.7×10^{5}	8.2×10^{5}
1/We	7.8×10^{5}	1.3×10^{6}	9.4×10^{5}	3.0×10^{6}	1.3×10^{4}	4.0×10^{5}	4.2×10^{6}	4.8×10^{6}	6.9×10^{6}	1.0×10^{7}
$\frac{\mu_L^2}{\rho_L \gamma_o h_o^2}$ **[K/m]**	2.8	1.5	4.4	1.3	140	4.3	0.72	0.55	1.6	1.1

* $1/We \propto h_o$, $1/Fr \propto h_o^3$

Results -- Asymptotic Solution (h_o=1mm)

Coolant	Mean Temperature [K]	Max. Temp. Gradient $(\Delta T_s/L)_{max}$ [K/cm]
Lithium	573	13
Lithium-Lead	673	173
Flibe	673	38
Tin	1273	80
Ga	1073	211

Numerical Method



- Evolution of the free surface is modeled using the Level Contour Reconstruction Method

- Two Grid Structures
  - Volume - entire computational domain (both phases) discretized by a standard, uniform, stationary, finite difference grid.
  - Phase Interface - discretized by Lagrangian points or elements whose motions are explicitly tracked.

Numerical Method

- A single field formulation
- Constant but unequal material properties
- Surface tension included as local delta function sources
  - Variable surface tension :

$$\sigma = \sigma_o + \gamma_o (T - T_m)$$

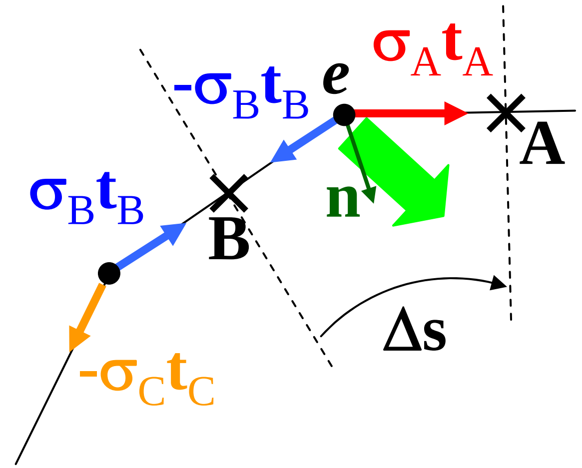
  - Force on a line element

$$\delta F_e = \int_{\Delta s} \left[\sigma \kappa \mathbf{n} + \frac{\partial \sigma}{\partial s} \mathbf{t} \right] \mathrm{d}s = \int_B^A \frac{\partial (\sigma \mathbf{t})}{\partial s} \mathrm{d}s = (\sigma_A \mathbf{t}_A - \sigma_B \mathbf{t}_B)$$

$$\therefore \sigma \kappa \mathbf{n} + \frac{\partial \sigma}{\partial s} \mathbf{t} = \sigma \frac{\partial \mathbf{t}}{\partial s} + \frac{\partial \sigma}{\partial s} \mathbf{t} = \frac{\partial (\sigma \mathbf{t})}{\partial s}$$

normal surface tension force

thermocapillary force



n : unit vector in normal direction

t : unit vector in tangential direction

κ : curvature

Variables definition

- Material property field

$$\frac{\rho_G}{\rho_L} = \rho^+$$

$$\frac{k_G}{k_L} = k^+$$

$$\frac{c_G}{c_L} = c^+$$

$$\frac{\mu_G}{\mu_L} = \mu^+$$

⇒

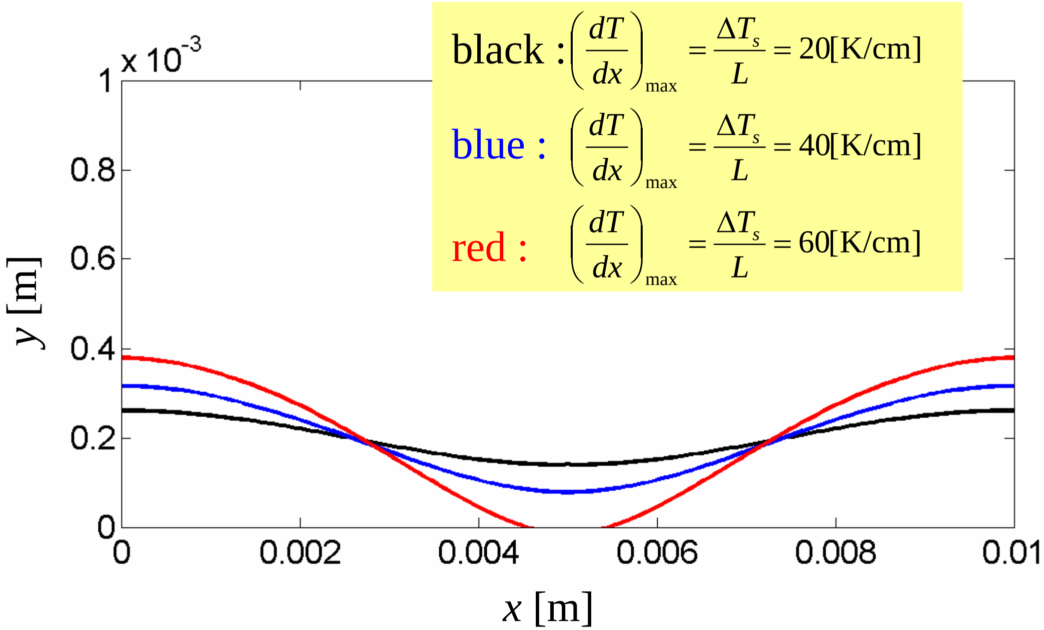
$$\rho = 1 + (\rho^+ - 1) I(\mathbf{x}, t)$$

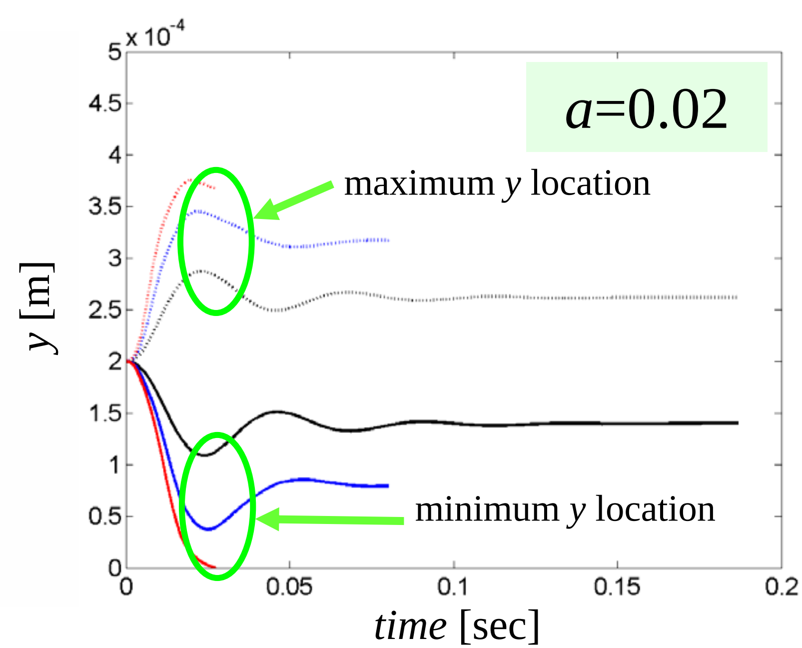
$$k = 1 + (k^+ - 1) I(\mathbf{x}, t)$$

$$c = 1 + (c^+ - 1) I(\mathbf{x}, t)$$

$$\mu = 1 + (\mu^+ - 1) I(\mathbf{x}, t)$$

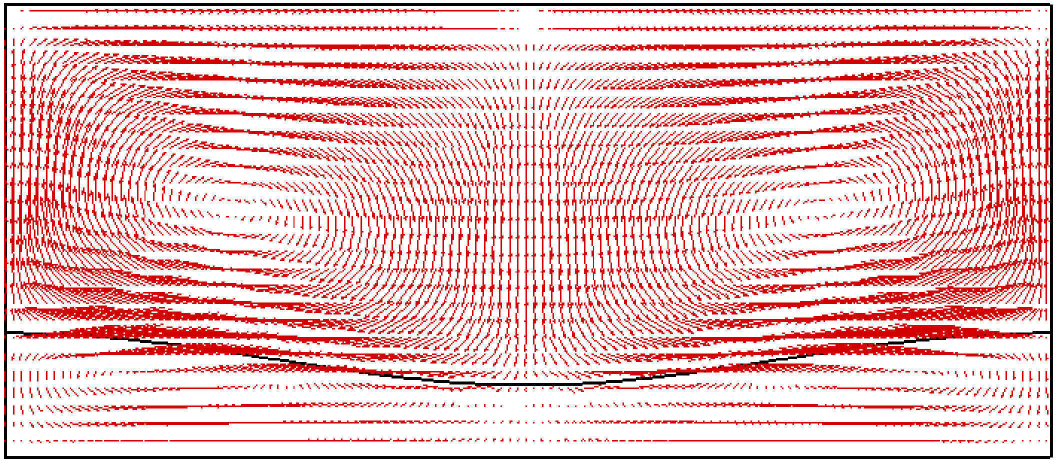
Dimensional Simulation (Lithium)



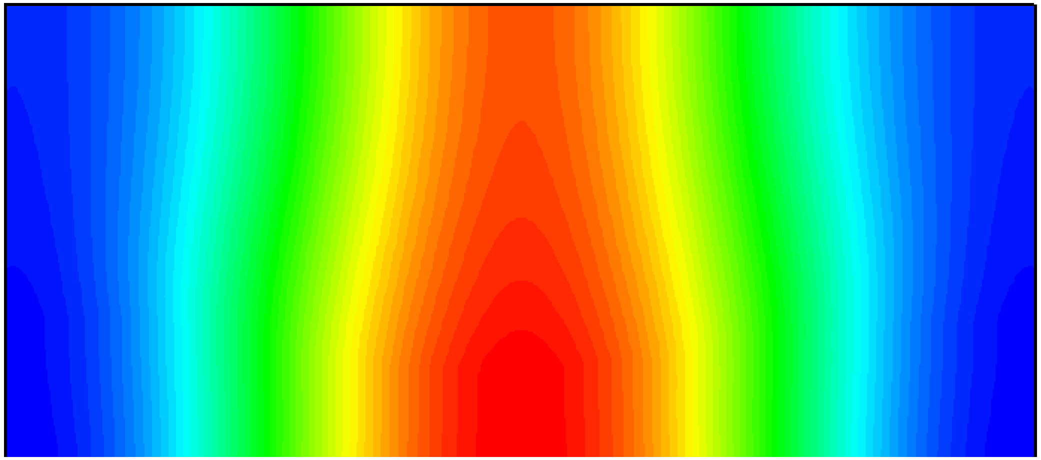


- Two-dimensional simulation with 1[cm]×0.1[cm] box size, 250×50 resolution, and h_o=0.2 mm

Dimensional Simulation (Lithium)



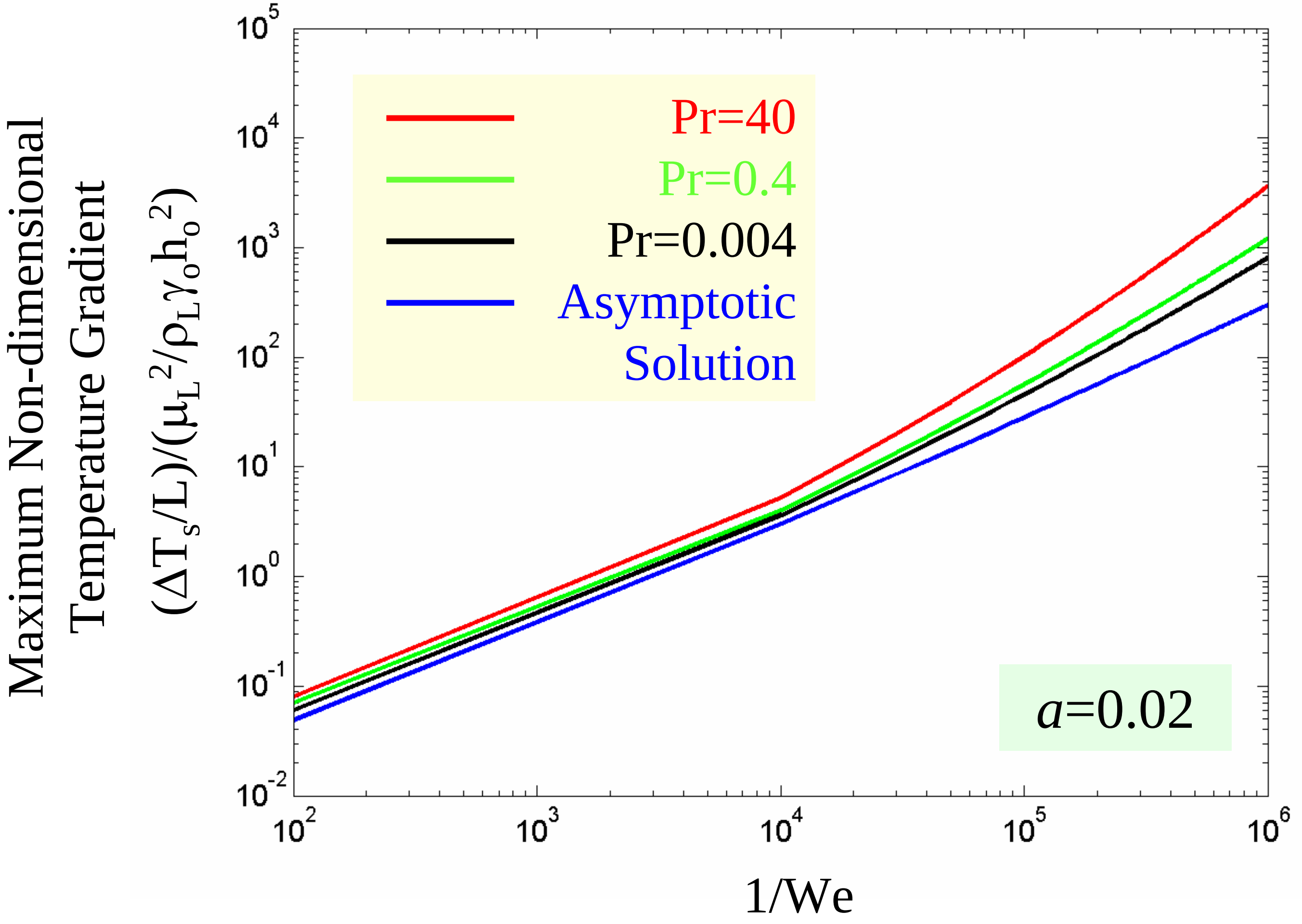
velocity vector plot



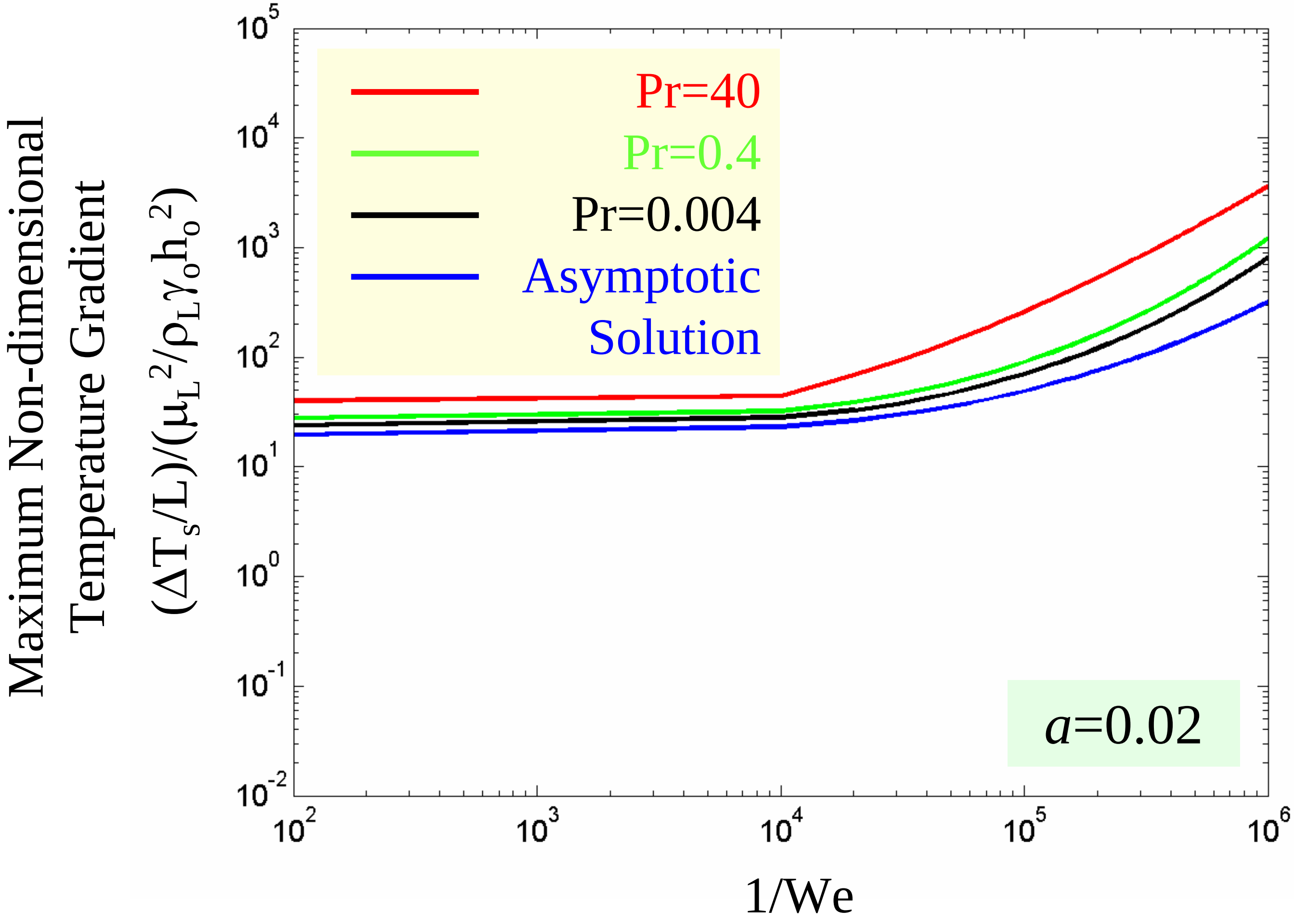
temperature plot

- Two-dimensional simulation with 1.0[cm]×0.1[cm] box size and 250×50 resolution
- h_o=0.2 mm, ΔT_s=20 K

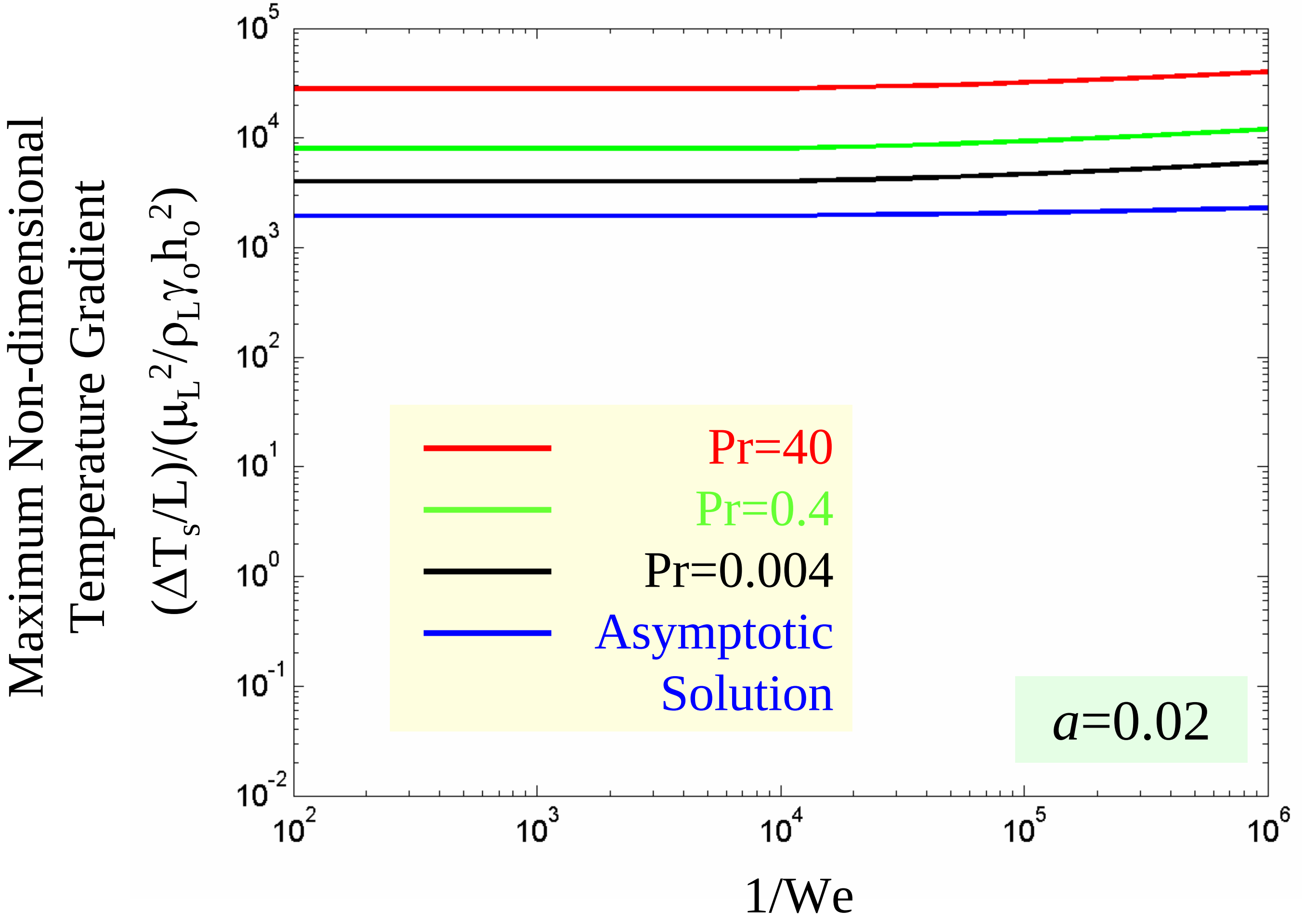
Results **(1/Fr=1)**



Results (1/Fr=10^3)



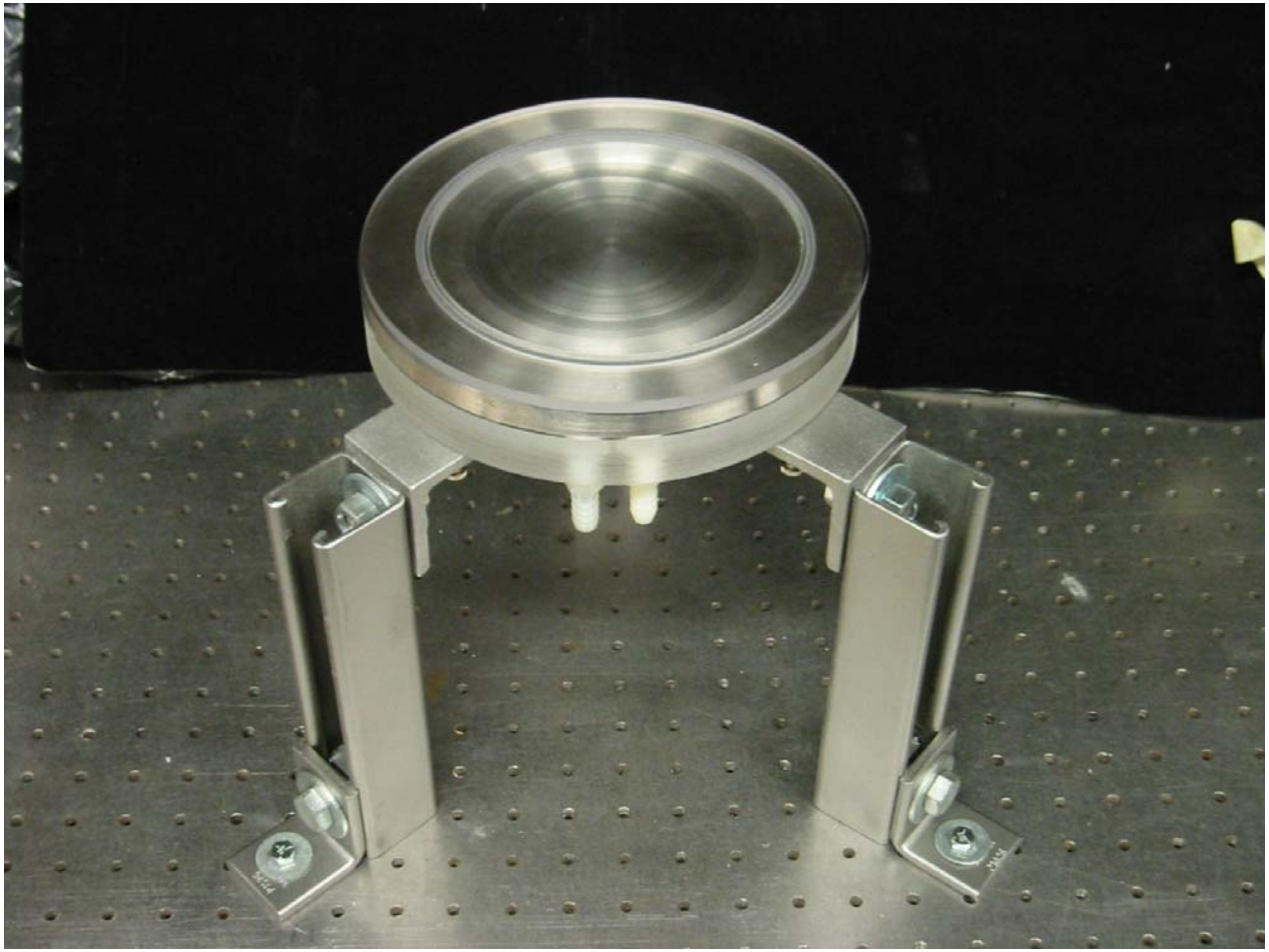
Results (1/Fr=10^5)



Results -- Numerical Solution (h_o=1mm)

Coolant	Mean Temperature [K]	Max. Temp. Gradient $(\Delta T_s/L)_{max}$ [K/cm]	
		Num. Sol.	Asymp. Sol.
Lithium	573	30	13
Lithium-Lead	673	570	173
Flibe	673	76	38
Tin	1273	113	80
Ga	1073	600	211

Experimental Validation



Conclusions

- Limiting values for the temperature gradients (i.e. heat flux gradients) to prevent film rupture can be determined
- Generalized charts have been developed to determine the temperature gradient limits for different fluids, operating temperatures (i.e. properties), and film thickness values
- For thin liquid films, limits may be more restrictive than surface temperature limits based on Plasma impurities limit
- Experimental Validation of Theoretical Model has been initiated
- Preliminary results for Axisymmetric geometry (hot spot model) produce more restrictive limits for temperature gradients