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# Heat Flux Gradient Limits for Liquid-Protected Divertors

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# *Problem Definition*

- Work on Liquid Surface Plasma Facing Components and Plasma Surface Interactions has been performed by the ALPS and APEX Programs
- Operating Temperature Windows have been established for different liquids based on allowable limits for Plasma impurities and Power Cycle efficiency requirements
- This work is aimed at establishing limits for the maximum allowable heat flux gradients (i.e. temperature gradients) to prevent film rupture due to thermocapillary effects

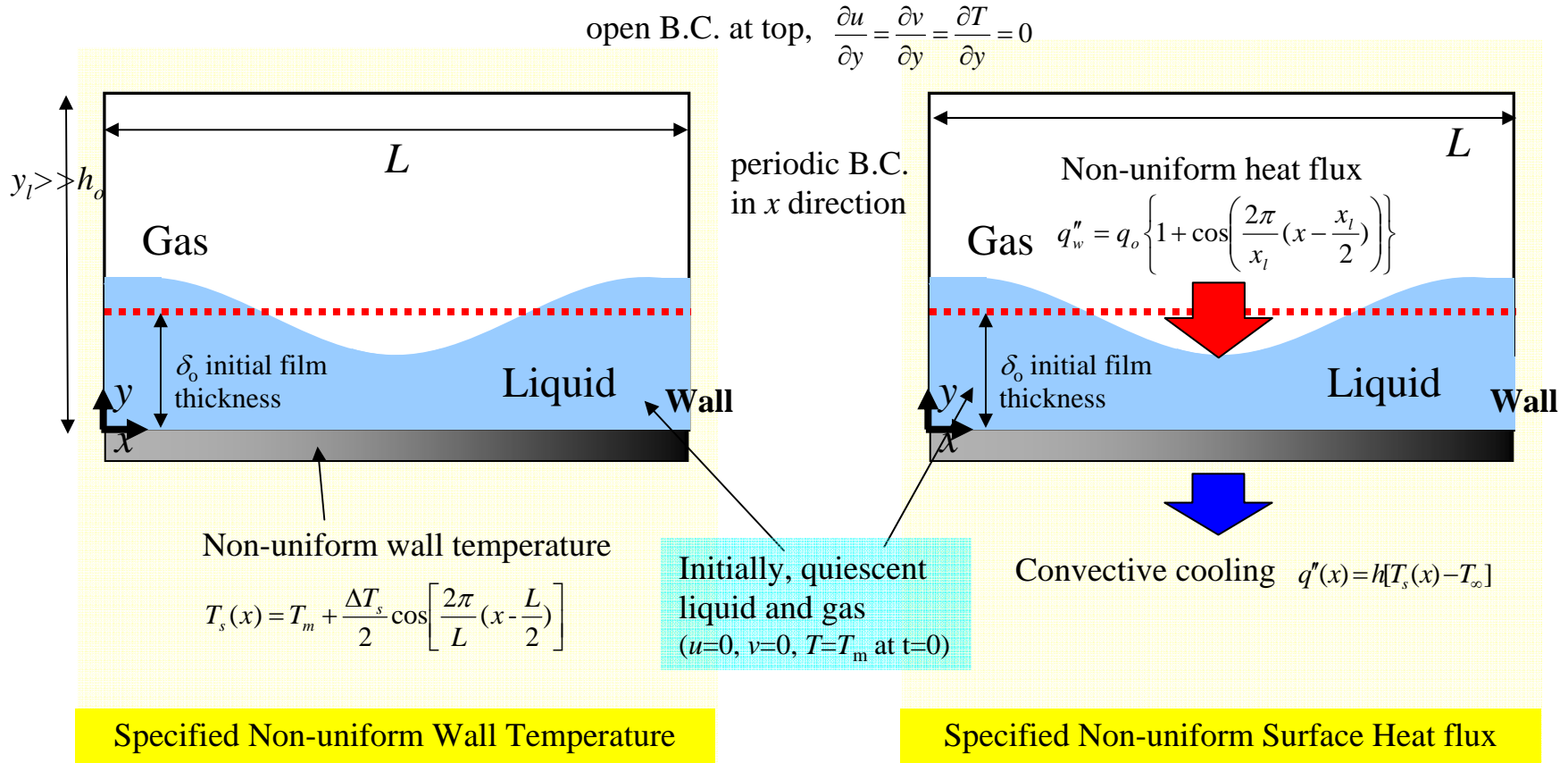


# *Problem Definition*

- Maximum Temperature gradients to prevent film rupture were previously established -- Analysis was extended to determine the maximum allowable heat flux gradients (based on comment made by Don Steiner)
- Spatial Variations in the wall and Liquid Surface Temperatures are expected due to variations in the wall loading
- Thermocapillary forces created by such temperature gradients can lead to film rupture and dry spot formation in regions of elevated local temperatures
- Initial Attention focused on Plasma Facing Components protected by a “non-flowing” thin liquid film (e.g. porous wetted wall)



# Problem Definition



- Two Dimensional Cartesian ( $x$ - $y$ ) Model (assume no variations in toroidal direction)
- Two Dimensional Cylindrical ( $r$ - $z$ ) Model has also been developed (local “hot spot” modeling)



# Variables definition

- Non-dimensional variables

$$a = \frac{\delta_o}{L} \quad y' = \frac{y}{\delta_o} \quad x' = \frac{x}{L} = \frac{ax}{\delta_o}$$

$$u' = \frac{u}{(\mu_L / \rho_L L)}$$

$$v' = \frac{v}{(a\mu_L / \rho_L L)}$$

$$t' = \frac{t}{(\rho_L L^2 / \mu_L)}$$

$$T' = \frac{T - T_m}{\Delta T_s} \quad V_g = \frac{\mu_L}{\rho_L \delta_o}$$

$$We = \frac{\rho_L V_g^2 \delta_o}{\sigma_o} = \frac{\mu_L^2}{\rho_L \sigma_o \delta_o}$$

$$Fr = \frac{V_g^2}{g \delta_o} = \frac{\mu_L^2}{g \rho_L^2 \delta_o^3}$$

$$Pr = \frac{\mu_L c_L}{k_L}$$

$$M = \frac{\gamma_o \Delta T_s \delta_o}{\mu_L \alpha_L}$$



$$q'' = -k_L \frac{\partial T}{\partial y} \quad q_o \sim k_L \frac{\Delta T}{\delta_o}$$

$$\Delta T \sim \frac{\delta_o q_o}{k_L} \quad T' = \frac{T - T_\infty}{(\delta_o q_o / k_L)}$$

$$aNu = \left( \frac{\delta_o}{L} \right) \left( \frac{hL}{k_L} \right) = \frac{\delta_o h}{k_L}$$

$$Q = 1 + \cos \left[ 2\pi \left( x - \frac{1}{2} \right) \right]$$

Specified Non-uniform Wall Temperature

Specified Non-uniform Surface Heat flux



# Governing Equations

- Conservation of Mass  $\frac{\partial u'}{\partial x'} + \frac{\partial v'}{\partial y'} = 0$

- Momentum

$$\sigma' = 1/\text{We} - (\text{M}/\text{Pr})T'$$

$$a^2 \rho^+ \left[ \frac{\partial u'}{\partial t'} + u' \frac{\partial u'}{\partial x'} + v' \frac{\partial u'}{\partial y'} \right] = -\frac{\partial p'}{\partial x'} + a^2 \frac{\partial}{\partial x'} \left( 2\mu^+ \frac{\partial u'}{\partial x'} \right) + \frac{\partial}{\partial y'} \left( \mu^+ \frac{\partial u'}{\partial y'} \right) + a^2 \frac{\partial}{\partial y'} \left( \mu^+ \frac{\partial v'}{\partial x'} \right) + \int \left( \sigma' \kappa \mathbf{n} + \frac{\partial \sigma'}{\partial s} \mathbf{t} \right) \delta ds \cdot \hat{\mathbf{i}}$$

$$a^4 \rho^+ \left[ \frac{\partial v'}{\partial t'} + u' \frac{\partial v'}{\partial x'} + v' \frac{\partial v'}{\partial y'} \right] = -\frac{\partial p'}{\partial y'} + \frac{\rho^+}{\text{Fr}} + a^4 \frac{\partial}{\partial x'} \left( \mu^+ \frac{\partial v'}{\partial x'} \right) + a^2 \frac{\partial}{\partial x'} \left( \mu^+ \frac{\partial u'}{\partial y'} \right) + a^2 \frac{\partial}{\partial y'} \left( 2\mu^+ \frac{\partial v'}{\partial y'} \right) + a \int \left( \sigma' \kappa \mathbf{n} + \frac{\partial \sigma'}{\partial s} \mathbf{t} \right) \delta ds \cdot \hat{\mathbf{j}}$$

- Energy  $a^2 \rho^+ \left[ \frac{\partial c^+ T'}{\partial t'} + u' \frac{\partial c^+ T'}{\partial x'} + v' \frac{\partial c^+ T'}{\partial y'} \right] = \frac{a^2}{\text{Pr}} \frac{\partial}{\partial x'} \left( k^+ \frac{\partial T'}{\partial x'} \right) + \frac{1}{\text{Pr}} \frac{\partial}{\partial y'} \left( k^+ \frac{\partial T'}{\partial y'} \right)$



# Asymptotic Solution

- Long wave theory with surface tension effect ( $a \ll 1$ ). Governing Equations reduce to :

$$\frac{a^2 Fr}{We} \frac{\partial^3 \delta}{\partial x^3} \delta + \frac{\partial \delta}{\partial x} \delta + \frac{3}{2} (M/Pr) \cdot Fr \frac{\partial T_s}{\partial x} = 0$$

Specified Non-uniform Wall Temperature \*

$$\frac{\sigma_o}{\rho_L g L^2} \frac{\partial^3 \delta}{\partial x^3} \delta + \frac{\partial \delta}{\partial x} \delta + \frac{3}{2} \frac{(Q/L)}{a(\rho_L g k / \gamma_o)} \left[ \frac{\partial \delta Q}{\partial x} + \frac{1}{(aNu)} \frac{\partial Q}{\partial x} \right] = 0$$

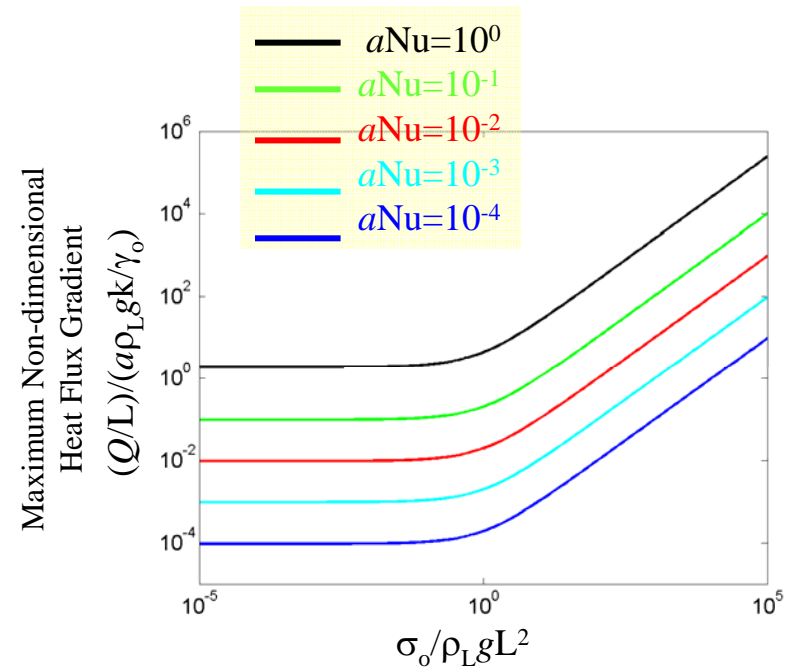
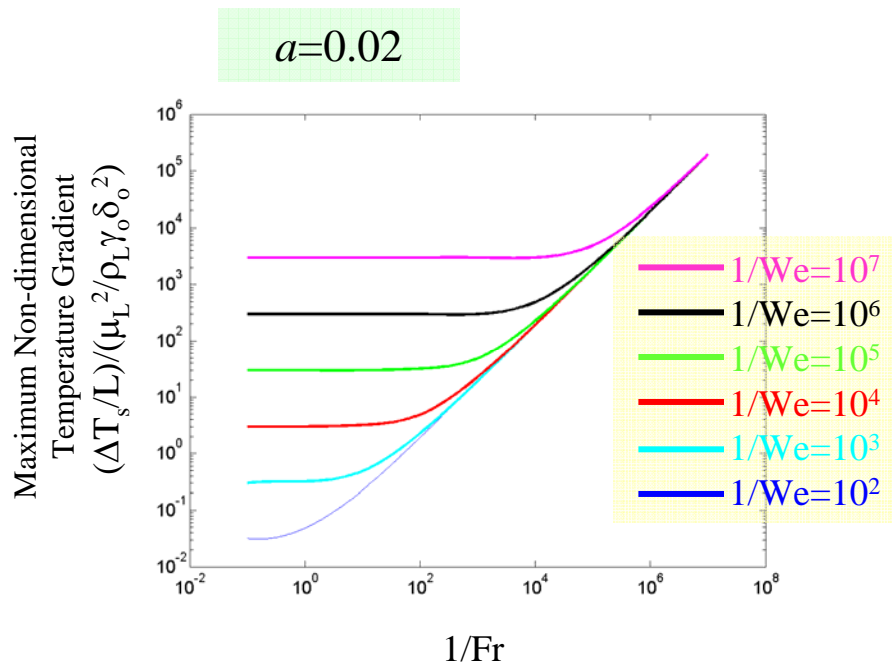
Specified Non-uniform Surface Heat flux

\* [Bankoff, et al. Phys. Fluids (1990)]

- Asymptotic solution used to analyze cases for Lithium, Lithium-lead, Flibe, Tin, and Gallium with different mean temperature and film thickness
- Asymptotic solution produces conservative (i.e. low) temperature gradient limits
- Limits for “High Aspect Ratio” cases analyzed by numerically solving the full set of conservation equations using Level Contour Reconstruction Method



# Asymptotic Solution



## Specified Non-uniform Wall Temperature

- Similar Plots have been obtained for other aspect ratios
- In the limit of zero aspect ratio  $(M/Pr)_{crit} = \frac{\pi^2}{12} \frac{1}{Fr}$

## Specified Non-uniform Surface Heat flux





# Results -- Asymptotic Solution

## – Specified Non-uniform Wall Temp ( $\delta_o=1\text{mm}$ )

- Property ranges

Parameter	Lithium		Lithium-Lead		Flibe		Tin		Gallium	
	573K	773K	573K	773K	573K	773K	1073K	1473K	873K	1273K
<b>Pr</b>	0.042	0.026	0.031	0.013	14	2.4	0.0047	0.0035	0.0058	0.0029
<b>1/Fr</b>	$1.2 \times 10^4$	$2.2 \times 10^5$	$1.9 \times 10^5$	$6.3 \times 10^5$	$1.2 \times 10^3$	$3.8 \times 10^4$	$5.3 \times 10^5$	$6.7 \times 10^5$	$5.7 \times 10^5$	$8.2 \times 10^5$
<b>1/We</b>	$7.8 \times 10^5$	$1.3 \times 10^6$	$9.4 \times 10^5$	$3.0 \times 10^6$	$1.3 \times 10^4$	$4.0 \times 10^5$	$4.2 \times 10^6$	$4.8 \times 10^6$	$6.9 \times 10^6$	$1.0 \times 10^7$
$\frac{\mu_i^2}{\rho_i \gamma_i \delta_o^2}$ [K/m]	2.8	1.5	4.4	1.3	140	4.3	0.72	0.55	1.6	1.1

\*  $1/We \propto \delta_o$ ,  $1/Fr \propto \delta_o^3$

Coolant	Mean Temperature [K]	Max. Temp. Gradient : $(\Delta T_s/L)_{\max}$ [K/cm]	
		Asymptotic Solution	Numerical Solution
Lithium	573	13	30
Lithium-Lead	673	173	570
Flibe	673	38	76
Tin	1273	80	113
Ga	1073	211	600



# Results -- Asymptotic Solution

## – Specified Non-uniform surface heat flux

- $\delta_o = 10 \text{ mm}$ ,  $a = 0.02$

Coolant	Mean Temperature [K]	$\sigma_o / \rho_L g L^2$	Max. Heat Flux. Gradient : $(Q''/L)_{\max}$ [(MW/m <sup>2</sup> )/cm]		
			$aNu = 1.0$	$aNu = 0.1$	$aNu = 0.01$
Lithium	573	$2.54 \times 10^{-4}$	0.61	$3.1 \times 10^{-2}$	$3.0 \times 10^{-3}$
Lithium-Lead	673	$2.04 \times 10^{-5}$	4.9	0.24	$2.4 \times 10^{-2}$
Flibe	673	$4.35 \times 10^{-5}$	$6.4 \times 10^{-2}$	$3.2 \times 10^{-3}$	$3.2 \times 10^{-4}$
Tin	1273	$3.04 \times 10^{-5}$	6.8	0.34	$3.3 \times 10^{-2}$
Ga	1073	$4.81 \times 10^{-5}$	19	0.97	$9.5 \times 10^{-2}$

- $\delta_o = 1 \text{ mm}$ ,  $aNu = 1.0$

Coolant	Mean Temperature [K]	Max. Heat Flux. Gradient : $(Q''/L)_{\max}$ [(MW/m <sup>2</sup> )/cm]				
		$a = 0.05$	$a = 0.02$	$a = 0.01$	$a = 0.005$	$a = 0.002$
Lithium	573	$1.9 \times 10^0$	$6.3 \times 10^{-1}$	$3.1 \times 10^{-1}$	$1.5 \times 10^{-1}$	$6.1 \times 10^{-2}$
Lithium-Lead	673	$1.2 \times 10^1$	$4.9 \times 10^0$	$2.4 \times 10^0$	$1.2 \times 10^0$	$4.9 \times 10^{-1}$
Flibe	673	$1.7 \times 10^{-1}$	$6.5 \times 10^{-2}$	$3.2 \times 10^{-2}$	$1.6 \times 10^{-2}$	$6.4 \times 10^{-3}$
Tin	1273	$1.7 \times 10^1$	$6.8 \times 10^0$	$3.4 \times 10^0$	$1.7 \times 10^0$	$6.8 \times 10^{-1}$
Ga	1073	$5.0 \times 10^1$	$1.9 \times 10^1$	$9.7 \times 10^0$	$4.8 \times 10^0$	$1.9 \times 10^0$



# *Conclusions*

- Limiting values for the heat flux gradients (i.e. temperature gradients) to prevent film rupture can be determined
- Generalized charts have been developed to determine the heat flux gradient limits for different fluids, operating temperatures (i.e. properties), and film thickness values
- For thin liquid films, limits may be more restrictive than surface temperature limits based on Plasma impurities limit
- Experimental Validation of Theoretical Model has been initiated



# Extra Slides



# Results -- Asymptotic Solution ( $\delta_o=10$ mm, $a=0.02$ )

Coolant	Mean Temperature [K]	$\sigma_o/\rho_L g L^2$	Max. Heat Flux. Gradient $(Q''/L)_{\max}$ [(MW/m <sup>2</sup> )/cm]		
			$aNu=1.0$	$aNu=0.1$	$aNu=0.1$
<b>Lithium</b>	573	$2.54 \times 10^{-4}$	0.61	$3.1 \times 10^{-2}$	$3.0 \times 10^{-3}$
<b>Lithium-Lead</b>	673	$2.04 \times 10^{-5}$	4.9	0.24	$2.4 \times 10^{-2}$
<b>Flibe</b>	673	$4.35 \times 10^{-5}$	$6.4 \times 10^{-2}$	$3.2 \times 10^{-3}$	$3.2 \times 10^{-4}$
<b>Tin</b>	1273	$3.04 \times 10^{-5}$	6.8	0.34	$3.3 \times 10^{-2}$
<b>Ga</b>	1073	$4.81 \times 10^{-5}$	19	0.97	$9.5 \times 10^{-2}$



# Results -- Asymptotic Solution ( $\delta_o=1$ mm, $a=0.02$ )

Coolant	Mean Temperature [K]	$\sigma_o/\rho_L g L^2$	Max. Heat Flux. Gradient $(Q''/L)_{\max}$ [(MW/m <sup>2</sup> )/cm]		
			$aNu=1.0$	$aNu=0.1$	$aNu=0.1$
<b>Lithium</b>	573	$2.54 \times 10^{-2}$	0.63	$3.2 \times 10^{-2}$	$3.1 \times 10^{-3}$
<b>Lithium-Lead</b>	673	$2.04 \times 10^{-3}$	4.9	0.24	$2.4 \times 10^{-2}$
<b>Flibe</b>	673	$4.35 \times 10^{-3}$	$6.5 \times 10^{-2}$	$3.3 \times 10^{-3}$	$3.2 \times 10^{-4}$
<b>Tin</b>	1273	$3.04 \times 10^{-3}$	6.8	0.34	$3.3 \times 10^{-2}$
<b>Ga</b>	1073	$4.81 \times 10^{-3}$	19	0.98	$9.6 \times 10^{-2}$



# Results -- Asymptotic Solution ( $\delta_o=0.1$ mm, $a=0.02$ )

Coolant	Mean Temperature [K]	$\sigma_o/\rho_L g L^2$	Max. Heat Flux. Gradient $(Q''/L)_{\max}$ [(MW/m <sup>2</sup> )/cm]		
			$aNu=1.0$	$aNu=0.1$	$aNu=0.1$
<b>Lithium</b>	573	$2.54 \times 10^0$	2.6	0.11	$1.1 \times 10^{-2}$
<b>Lithium-Lead</b>	673	$2.04 \times 10^{-1}$	6.2	0.3	$2.9 \times 10^{-2}$
<b>Flibe</b>	673	$4.35 \times 10^{-1}$	0.1	$4.8 \times 10^{-3}$	$4.7 \times 10^{-4}$
<b>Tin</b>	1273	$3.04 \times 10^{-1}$	9.5	0.46	$4.4 \times 10^{-2}$
<b>Ga</b>	1073	$4.81 \times 10^{-1}$	31	1.5	0.14



# Results -- Asymptotic Solution ( $\delta_o=1$ mm, $aNu=1.0$ )

Coolant	Mean Temperature [K]	Max. Heat Flux. Gradient $(Q''/L)_{\max}$ [(MW/m <sup>2</sup> )/cm]				
		$a=0.05$	$a=0.02$	$a=0.01$	$a=0.005$	$a=0.002$
Lithium	573	$1.9 \times 10^0$	$6.3 \times 10^{-1}$	$3.1 \times 10^{-1}$	$1.5 \times 10^{-1}$	$6.1 \times 10^{-2}$
Lithium-Lead	673	$1.2 \times 10^1$	$4.9 \times 10^0$	$2.4 \times 10^0$	$1.2 \times 10^0$	$4.9 \times 10^{-1}$
Flibe	673	$1.7 \times 10^{-1}$	$6.5 \times 10^{-2}$	$3.2 \times 10^{-2}$	$1.6 \times 10^{-2}$	$6.4 \times 10^{-3}$
Tin	1273	$1.7 \times 10^1$	$6.8 \times 10^0$	$3.4 \times 10^0$	$1.7 \times 10^0$	$6.8 \times 10^{-1}$
Ga	1073	$5.0 \times 10^1$	$1.9 \times 10^1$	$9.7 \times 10^0$	$4.8 \times 10^0$	$1.9 \times 10^0$





# Results -- Asymptotic Solution ( $\delta_o=1$ mm, $aNu=0.1$ )

Coolant	Mean Temperature [K]	Max. Heat Flux. Gradient $(Q''/L)_{\max}$ [(MW/m <sup>2</sup> )/cm]				
		$a=0.05$	$a=0.02$	$a=0.01$	$a=0.005$	$a=0.002$
Lithium	573	$9.2 \times 10^{-2}$	$3.2 \times 10^{-2}$	$1.5 \times 10^{-2}$	$7.7 \times 10^{-3}$	$3.0 \times 10^{-3}$
Lithium-Lead	673	$6.2 \times 10^{-1}$	$2.4 \times 10^{-1}$	$1.2 \times 10^{-1}$	$6.1 \times 10^{-2}$	$2.4 \times 10^{-2}$
Flibe	673	$8.4 \times 10^{-3}$	$3.3 \times 10^{-3}$	$1.6 \times 10^{-3}$	$8.1 \times 10^{-4}$	$3.2 \times 10^{-4}$
Tin	1273	$8.7 \times 10^{-1}$	$3.4 \times 10^{-1}$	$1.7 \times 10^{-1}$	$8.5 \times 10^{-2}$	$3.4 \times 10^{-2}$
Ga	1073	$2.5 \times 10^0$	$9.8 \times 10^{-1}$	$4.9 \times 10^{-1}$	$2.4 \times 10^{-1}$	$9.7 \times 10^{-2}$



# Results -- Asymptotic Solution ( $\delta_o=1$ mm, $aNu=0.01$ )

Coolant	Mean Temperature [K]	Max. Heat Flux. Gradient $(Q''/L)_{\max}$ [(MW/m <sup>2</sup> )/cm]				
		$a=0.05$	$a=0.02$	$a=0.01$	$a=0.005$	$a=0.002$
Lithium	573	$8.9 \times 10^{-3}$	$3.1 \times 10^{-3}$	$1.5 \times 10^{-3}$	$7.5 \times 10^{-4}$	$3.0 \times 10^{-4}$
Lithium-Lead	673	$6.1 \times 10^{-2}$	$2.4 \times 10^{-2}$	$1.2 \times 10^{-2}$	$6.0 \times 10^{-3}$	$2.4 \times 10^{-3}$
Flibe	673	$8.2 \times 10^{-4}$	$3.2 \times 10^{-4}$	$1.6 \times 10^{-4}$	$8.0 \times 10^{-5}$	$3.2 \times 10^{-5}$
Tin	1273	$8.5 \times 10^{-2}$	$3.3 \times 10^{-2}$	$1.7 \times 10^{-2}$	$8.3 \times 10^{-3}$	$3.3 \times 10^{-3}$
Ga	1073	$2.5 \times 10^{-1}$	$9.6 \times 10^{-2}$	$4.8 \times 10^{-2}$	$2.4 \times 10^{-2}$	$9.5 \times 10^{-3}$



# Results -- Asymptotic Solution ( $L=5$ cm, $aNu=1.0$ )

Coolant	Mean Temperature [K]	Max. Heat Flux. Gradient $(Q''/L)_{\max}$ [(MW/m <sup>2</sup> )/cm]				
		$a=0.05$	$a=0.02$	$a=0.01$	$a=0.005$	$a=0.002$
Lithium	573	$1.6 \times 10^0$	$6.3 \times 10^{-1}$	$3.2 \times 10^{-1}$	$1.6 \times 10^{-1}$	$6.4 \times 10^{-2}$
Lithium-Lead	673	$1.2 \times 10^1$	$4.9 \times 10^0$	$2.5 \times 10^0$	$1.3 \times 10^0$	$5.2 \times 10^{-1}$
Flibe	673	$1.6 \times 10^{-1}$	$6.5 \times 10^{-2}$	$3.3 \times 10^{-2}$	$1.7 \times 10^{-2}$	$8.5 \times 10^{-3}$
Tin	1273	$1.4 \times 10^1$	$6.8 \times 10^0$	$3.4 \times 10^0$	$1.7 \times 10^0$	$6.8 \times 10^{-1}$
Ga	1073	$4.8 \times 10^1$	$1.9 \times 10^1$	$9.5 \times 10^0$	$4.8 \times 10^0$	$1.9 \times 10^0$



# Results -- Asymptotic Solution ( $L=5$ cm, $aNu=0.1$ )

Coolant	Mean Temperature [K]	Max. Heat Flux. Gradient $(Q''/L)_{\max}$ [(MW/m <sup>2</sup> )/cm]				
		$a=0.05$	$a=0.02$	$a=0.01$	$a=0.005$	$a=0.002$
Lithium	573	$8.0 \times 10^{-2}$	$3.2 \times 10^{-2}$	$1.6 \times 10^{-2}$	$8.0 \times 10^{-3}$	$3.2 \times 10^{-3}$
Lithium-Lead	673	$6.0 \times 10^{-1}$	$2.4 \times 10^{-1}$	$1.2 \times 10^{-1}$	$6.0 \times 10^{-2}$	$2.4 \times 10^{-2}$
Flibe	673	$8.3 \times 10^{-3}$	$3.3 \times 10^{-3}$	$1.7 \times 10^{-3}$	$8.5 \times 10^{-4}$	$3.4 \times 10^{-4}$
Tin	1273	$8.5 \times 10^{-1}$	$3.4 \times 10^{-1}$	$1.7 \times 10^{-1}$	$8.5 \times 10^{-2}$	$3.4 \times 10^{-2}$
Ga	1073	$2.5 \times 10^0$	$9.8 \times 10^{-1}$	$4.9 \times 10^{-1}$	$2.5 \times 10^{-1}$	$1.0 \times 10^{-1}$



# Results -- Asymptotic Solution ( $L=5$ cm, $aNu=0.01$ )

Coolant	Mean Temperature [K]	Max. Heat Flux. Gradient $(Q''/L)_{\max}$ [(MW/m <sup>2</sup> )/cm]				
		$a=0.05$	$a=0.02$	$a=0.01$	$a=0.005$	$a=0.002$
Lithium	573	$7.8 \times 10^{-3}$	$3.1 \times 10^{-3}$	$1.6 \times 10^{-3}$	$8.0 \times 10^{-4}$	$3.2 \times 10^{-4}$
Lithium-Lead	673	$6.0 \times 10^{-2}$	$2.4 \times 10^{-2}$	$1.2 \times 10^{-2}$	$6.0 \times 10^{-3}$	$2.4 \times 10^{-3}$
Flibe	673	$8.0 \times 10^{-4}$	$3.2 \times 10^{-4}$	$1.6 \times 10^{-4}$	$8.0 \times 10^{-5}$	$3.2 \times 10^{-5}$
Tin	1273	$8.3 \times 10^{-2}$	$3.3 \times 10^{-2}$	$1.7 \times 10^{-2}$	$8.5 \times 10^{-3}$	$3.4 \times 10^{-3}$
Ga	1073	$2.4 \times 10^{-1}$	$9.6 \times 10^{-2}$	$4.8 \times 10^{-2}$	$2.4 \times 10^{-2}$	$9.6 \times 10^{-3}$

