Time Dependent Radiation Transport in Hohlraums Using Integral Transport Methods

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ABSTRACT

Integral Transport methods were used to calculate the x-ray intensity in a NOVA hohlraum. Two cases were examined: the nominal case in which all five lasers strike either side of the hohlraum and the abnormal case for which five lasers strike the right side but only four strike the left. As expected, the uniform case produces a fairly uniform illumination over the capsule, whereas the abnormal case is highly non-uniform.

I. INTRODUCTION

From the initial illumination of the target capsule to its subsequent burn and disassembly, radiation transport plays an important role throughout the compression, ignition and burn phases of Inertial Fusion Energy (IFE) capsules[1]. Time-dependent diffusion based methods have frequently been used to simulate radiation transport within these target capsules[2,3,4]. It is well known that these methods have difficulty describing the radiation field in optically thin media because of the infinite propagation speed of the radiation in the diffusion approximation.[1] Various methods have been devised to remedy these problems; foremost, flux-limited diffusion methods that seek to limit the propagation speed of the radiation[5,6]. Though improvements have been made, the optically thin regime still poses some problems for diffusion-based methods.

Recently, the time-dependent Integral Transport (IT) method has been shown to achieve highly accurate results for neutral particle transport in optically thick and thin regions with finite propagation speeds.[7,8,9,10] Central to the IT method are the time-dependent single-collision kernels. These kernels contain Heaviside or delta functions that provide causality information for the neutral particle's transport. A neutral particle at a position *r'* at a time *t'* traveling with a speed *v* cannot affect the intensity at a position *r* at a time *t* until enough time has passed for its translation. This is expressed by the condition $(t - t') = |r - r'| / v$. The time-dependent IT method can readily be adapted to x-ray transport.

The transport equation for x-rays in a given frequency group in a heterogeneous medium with an arbitrary isotropic source is:

$$\left(\frac{1}{c}\frac{\partial}{\partial t} + \hat{\Omega}\cdot\vec{\nabla} + \Sigma\right) I_\nu\left(\vec{r}, \hat{\Omega}, t\right) = \frac{Q(\vec{r}, t)}{4\pi}. \tag{1}$$

An integral equation for the mean intensity is obtained by transforming Eqn. 1 using either the method of characteristics or Laplace transform techniques:[7]

$$J_\nu\left(\vec{r}, t\right) = \int_{V'} dt' \int_0^t K\left(\vec{r}, \vec{r}\,'; t, t'\right) Q_\nu\left(\vec{r}\,', t'\right) dr', \tag{2}$$

where: $K_\nu(r,r;t,t')$ is the time-dependent kernel and $Q_\nu(r',t')$ is the time-dependent source:

$$Q_\nu\left(\vec{r}', t'\right) = \Sigma_{s,\nu}\left(\vec{r}', t'\right) J_\nu\left(\vec{r}, t\right) + S\left(\vec{r}', t'\right). \tag{3}$$

The integration is carried out over the volume of interest, *V'*, from *t'*=0 until some later time *t*. Extremely accurate solutions can be obtained provided that a sufficiently accurate quadrature set is used.

The functional form of the homogeneous kernels in the three commonly used geometries has already been derived.[7] The kernels for the two geometries that will be discussed in this paper are:
One-Dimensional Cartesian;

$$K_{1D}\left(x, x'; t, t'\right) = \frac{\exp\left(-\Sigma c\left[t - t'\right]\right)}{2\left(t - t'\right)} H\left(t - t' - \frac{\left|x - x'\right|}{c}\right) \tag{4}$$

Three-Dimensional Cartesian;

$$K_{3D}\left(\vec{r}, \vec{r}\,'; t, t'\right) = \frac{\exp\left(-\Sigma c\left[t - t'\right]\right)}{4\pi c\left(t - t'\right)\left|\vec{r} - \vec{r}\,'\right|} \delta\left(t - t' - \frac{\left|\vec{r} - \vec{r}\,'\right|}{c}\right). \tag{5}$$

The present purpose of this work is to use integral transport methods to simulate x-ray transport within a NOVA hohlraum. To ensure accurate results the IT method was benchmarked against a quasi-analytical method and compared to time-dependent diffusion in an optically thick medium.

II. NUMERICAL METHOD

The Heaviside or delta function, within each of the time-dependent kernels, provides causality information for the emitted photons. A finite amount of time must pass before the radiation can affect the mean intensity at a location other than where it was born or scattered. Figure 1 depicts how this causality information is practically used in one-dimensional Cartesian coordinates. Points (x',t'), shown as shaded squares, are within the *cone of communication* which extends backwards in time from the point of interest (x,t) and as such are included in the numerical evaluation of the integrals. Points outside the *cone* have not had enough time to affect the intensity and therefore are excluded.

The IT method is easily applied to problems in either finite or infinite geometries. In an infinite medium, the integration in the spatial domain extends over the region defined between the left and right moving wave fronts. However, for finite geometries, like that presented in Figure 1, the boundaries of the geometry are placed as the limits of integration.

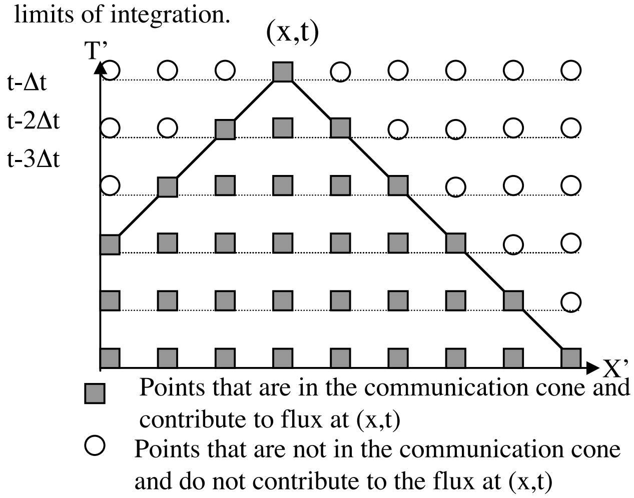

Figure 1. Numerical procedure for 1D-slab.

III. BENCHMARKS

The integral transport method is compared to benchmark results generated by Ganapol.[11] In his paper, Ganapol presents an idealized problem of a unit delta function source in time and space positioned at the center of an infinite medium. Photons released from the source have a velocity of 1 [cm/s]. Therefore, a position '*x*' cm from the source will only have an intensity if '*x*' seconds have passed for photons to travel from the source to that position. The medium is purely scattering with a macroscopic interaction cross section of 1.0 [1/cm]. The material properties are found in Table 1.

Table 1. Material Properties for Benchmarking

Properties	*Value*
Particle Velocity	1.0 [cm/s]
Scattering Cross Section	1.0 [1/cm]
Total Cross Section	1.0 [1/cm]
Source Strength	1.0 [#/cm^3*s]

The source function for the benchmark problem presented in Ref. [11] is:

$$Q_\nu(x',t') = \Sigma_{s,\nu}(x',t')J_\nu(x',t) + S_0\delta(x')\delta(t') \quad (6)$$

Table 2 displays the comparison of the numerical results computed using the IT method with spatial steps of 0.01 cm. The significant figures for which the two results differ are underlined.

Table 2. Comparison with Analytical Benchmark

	x			
t	1.00	2.00	3.00	4.00
1.00	1.8384E-1	-	-	-
3.00	2.3942E-1	9.3835E-2	8.2978E-3	-
5.00	1.9957E-1	1.2105E-1	4.9595E-2	1.1823E-2
7.00	1.7347E-1	1.2293E-1	6.8028E-2	2.8447E-2
9.00	1.5528E-1	1.1935E-1	7.6384E-2	4.0186E-2

As evident from the table, the IT results are in excellent agreement to the benchmark values presented by Ganapol. For every case except one, the IT method and the analytical method agree to 5 significant figures. For information on the numerical method used in the IT method refer to Ref. [10].

IV. TRANSPORT VS. DIFFUSION

The IT and time-dependent diffusion methods are compared for transport in a homogeneous one-dimensional Cartesian medium. For all cases the material is a slab with a width of 20 mean free lengths and has the material properties found in Table 1.

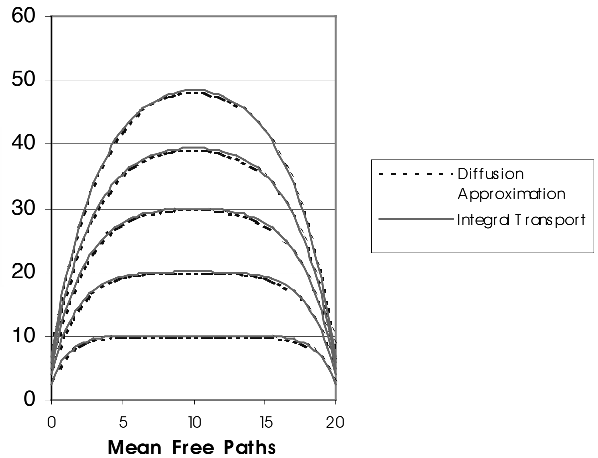

Figure 2. IT-Diffusion comparison for uniform source. The curves are every 10 mean free times, starting with t=10.

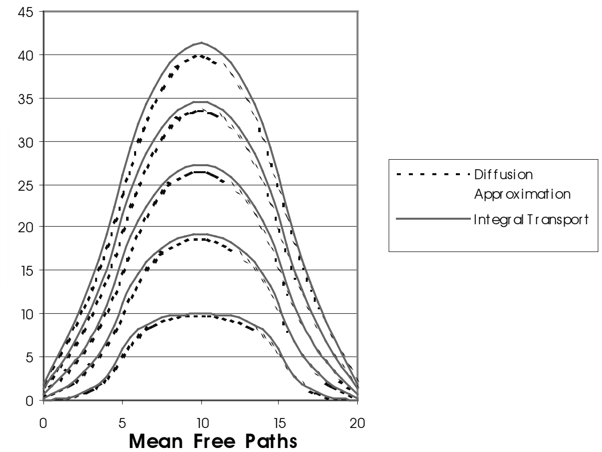

Figure 3. IT-Diffusion comparison for localized source. The curves are every 10 mean free times, starting with t=10.

The first case examined is a one-dimensional slab with a uniform source distributed throughout its interior. Both methods were run for 50 mean free times. Results are shown in Figure 2 for every 10 mean free times.

As shown in Figure 2, the time-dependent diffusion and integral transport calculations are in excellent agreement in the center of the slab. However, at the last time step presented, there is a 25% error between the diffusion-based calculations and the IT method at the vacuum boundaries. Again this is expected, as diffusion theory is in best agreement with transport in a diffusive medium with points several mean free paths away from boundaries.

In the second case, the comparison is for an isotropic, uniformly distributed source localized in the central 10 mean free lengths of the slab. The source strength for this localized source is 1 particle per cm^3s. As before, the time-dependent calculations were run for 50 mean free times. As shown in Figure 3, the agreement between the diffusion and IT methods is poor. After 10 mean free times, the diffusion-based solution differs by a maximum of 1% over the interior of the source region. As time progresses the agreement between the two calculational methods increases. After 50 mean free times, the diffusion results are 8% lower than those predicted by the IT method. As time advances, the difference between the two methods continues to increase. If the material was not as thick or if the source distribution was even more localized, there would be an even greater discrepancy between the diffusion based and IT methods for interior points.

V. HOHLRAUM RESULTS

X-ray transport within a NOVA hohlraum (Figure 4) will be calculated. Several assumptions are made which will simplify Eqn. 3 into a more readily calculated form. These assumptions are:

1. The capsule is assumed to be totally black. Any x-rays incident on the capsule will be absorbed and not re-emitted.
2. The hohlraum's albedo is independent of position, time and incident intensity. Therefore, the albedo is set to a constant 0.8 throughout the simulation. X-rays are emitted isotropically from the surface of the hohlraum.
3. The geometry of the hohlraum and capsule remains fixed throughout the simulation. No provisions are made for the deformation of the capsule and hohlraum.
4. The filler gas within the hohlraum does not have any opacity. Therefore, x-rays traveling within the hohlraum will not scatter or be absorbed. Thus, only the x-ray mean intensity incident on the surfaces of the capsule and the hohlraum will be calculated.

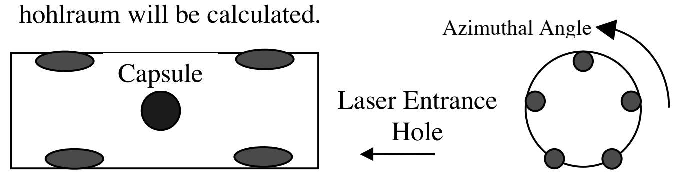

Figure 4. NOVA calculational model.

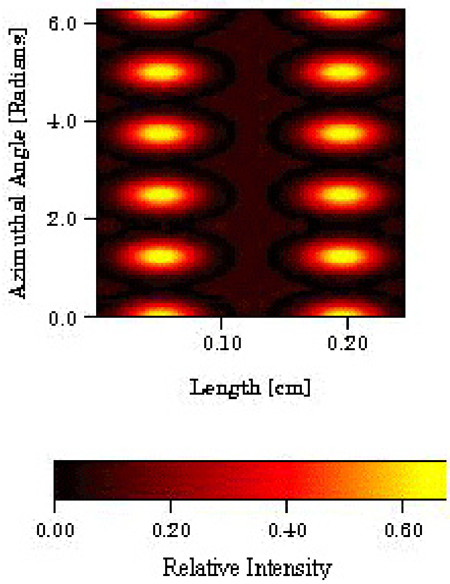

Figure 5. Uniform hohlraum illumination.

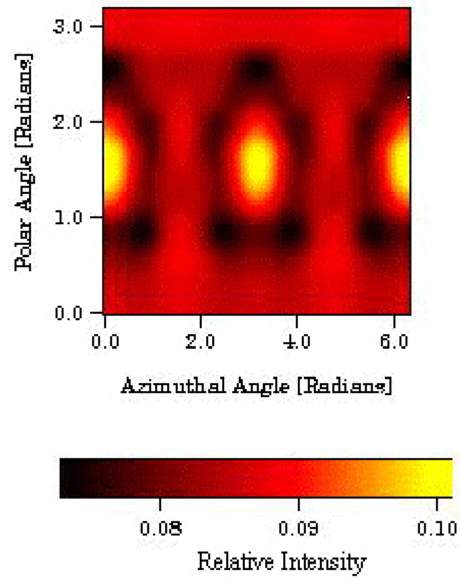

Figure 6. Uniform capsule illumination.

With these assumptions the time-dependent integral transport equation is:

$$J_\nu(\vec{r},t)=\int_0^t dt'\int_S \frac{S(\vec{r}',t')}{4\pi c(t-t')\left|\vec{r}-\vec{r}'\right|}\delta\left(t-t'-\frac{\left|\vec{r}-\vec{r}'^2\right|}{v}\right)dS. \quad (7)$$

The integration is carried out over the surface of the hohlraum, *S*, because that is the only location that possesses any x-ray sources.

Although the simplifying assumptions have reduced the previous full-fledged integral equation into an *ordinary* integral, solving Eqn. 7 numerically still poses some difficulties. A *view-factor* calculation must be performed because the capsule in the center of the hohlraum can block the line of sight between two surface points on the hohlraum. Any x-ray that intersects the capsule in the center of the hohlraum is absorbed, as the capsule is assumed to be completely black.

The spatial distribution of the x-ray sources and their respective intensity profiles are modeled after the NOVA experiment. Figure 4 shows the modeled source input used in the numerical modeling of the NOVA hohlraum and capsule. For each case the calculation was allowed to run for 25 picoseconds, at which time the capsule illumination was plotted.

Two different scenarios were investigated. The first case simulated the nominal illumination of the NOVA capsule. The capsule (Figure 5) is symmetrically illuminated by five x-ray sources on either side of the hohlraum, where the azimuthal angle for the hohlraum is defined in Figure 4. Figure 6 is a two dimensional representation of the capsule illumination. The two angular ordinates are defined in spherical coordinates from the center of the capsule. As shown in the Figure 6, IT predicts that the capsule illumination varies by no more than 20% for any given point. As expected, the *hottest* portion of the capsule is the area that faces all five sources. The *coolest* portion of the capsule is the region on either side, which only has a partial line of sight to any given source. These regions are evident from the five cool regions, which are in a ring just outside the central hot spot.

The second case simulated a non-uniform illumination of the NOVA capsule. As shown on the left side of Figure 7, the right side of the capsule is illuminated by five sources whereas the left is illuminated only by four.

As shown in Figure 8, the region of the capsule that has the most direct line of sight to the source that is turned off, is the *coolest*. The asymmetry in the illumination affects not only the portions of the capsule nearest to it, but the illumination over the entire capsule. The capsule illumination for this asymmetrical case varies approximately by a factor of two

VI. SUMMARY AND CONCLUSION

It was shown that the Integral Transport method produces very accurate results and can model three-dimensional radiation transport in IFE hohlraums.

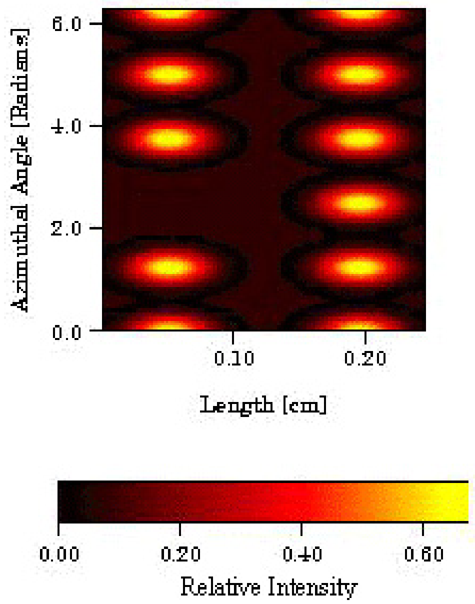

Figure 7. Non-uniform hohlraum illumination.

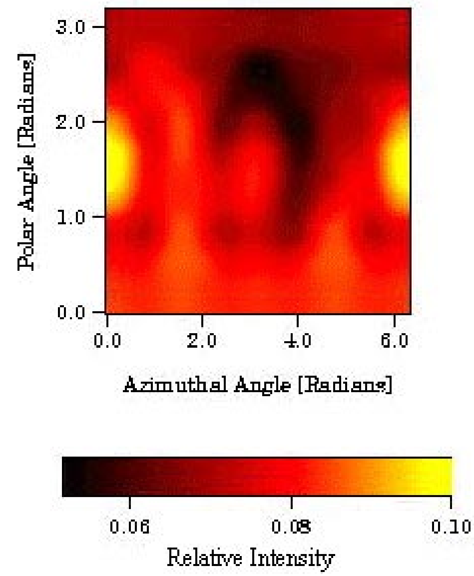

Figure 8. Non-uniform capsule illumination.

Simulations showed that a source distribution modeled after the NOVA experiment yielded a uniform capsule illumination to within 20%. A surprising result was that small asymmetries in the source distribution produced capsule illuminations that differ by a factor of two.

Future work will include the ability to model moving spatial boundaries. This will be done through the use of a time-step method for the specific intensity whereby the temporal integration is updated every time-step without the need to integrate back to the starting time. This would decrease the run-time by reducing the temporal domain of the integration and allowing material boundaries to move after the updated intensity is computed.

ACKNOWLEGEMENT

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