SPECIAL FEATURES OF FIRST-WALL HEAT TRANSFER IN LIQUID-METAL FUSION REACTOR BLANKETS

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The presence of strong magnetic fields and of volumetric heat generation in the fusion reactor environment result in an unusual heat transfer situation for liquid metals, as compared to nonconducting coolants. The effects of velocity profiles and volumetric heat generation on heat transfer in liquid-metal blankets are examined both analytically and numerically. Analysis shows that while the ultimate value of Nusselt number varies greatly, the spatial dependence within the entry region is relatively insensitive to the shape of the velocity profile and the amount of bulk heating. Hence, closed form solutions for fully developed heat transfer can be used together with a normalized entrance region curve to estimate heat transfer throughout the blanket.

I. INTRODUCTION

Heat transfer and fluid flow are important factors determining both the feasibility and performance of liquid-metal-cooled fusion reactor blankets. Peak blanket temperatures are limited by corrosion effects (mass transfer and material interactions), reduced material properties at high temperatures under irradiation, thermal stress limits, and other structural failure modes. These concerns are particularly critical in the first wall, which separates the blanket from the plasma in an environment that includes high thermal loading, pressure stresses, and plasma radiation. Additional constraints are imposed by the thermodynamic efficiency of the power cycle and by the requirement to avoid coolant freezing. The ability to accurately predict thermal-hydraulic behavior of blanket components in the reactor environment is therefore very important.

The presence of strong magnetic fields and volumetric heat generation in liquid-metal blankets creates an unusual heat transfer situation as compared to classical heat transfer in nonconducting fluids. Some of these special features include the following:

1. The flow of liquid metals under high magnetic fields (>1 T) is expected to be laminar, as turbulence is effectively suppressed by magnetic forces. With essentially no eddy diffusivity present, conduction is the only remaining mechanism for heat transport perpendicular to the flow direction. Fortunately, the conductivity of liquid metals is high enough to allow the possibility of adequate cooling without turbulence. Another important outcome of laminarization is that the governing equations are simplified to such an extent that analytic and simple numerical procedures can be used to calculate the temperature profiles exactly.

2. It is believed that unusual, highly nonuniform velocity profiles may exist in liquid-metal flows under a magnetic field, especially in regions where the flow must turn or traverse channels with varying flow areas or varying magnetic field. In developed flow perpendicular to the magnetic field and for uniform conditions in the channel, magnetohydrodynamic (MHD) velocity profiles tend to be very flat—essentially slab flow as far as heat transfer is concerned. In flow parallel to the magnetic field (which is a desirable condition for minimizing the MHD pressure drop), the body force that tends to flatten velocity profiles is absent, and nonuniform velocity profiles can persist over long distances. Furthermore, in the vicinity of magnetic, structural, and flow perturbations (bends, contractions, field gradients, etc.), the velocity profiles can
exhibit strong nonuniformities, such as high-velocity shear layers and stagnant regions. These unusual velocity profiles could result in temperatures much higher than expected.

3. Even in regions where the velocity profiles are fully developed, the temperature profiles in laminar flow require very long distances to develop. The entire length of the blanket may be in the thermal entrance region. This will result in more efficient heat transfer, which in turn leads to lower structure temperatures than would be predicted for fully developed thermal conditions. Larger temperature gradients will appear over longer portions of the blanket channels, and the heat transfer coefficient will be spatially varying.

4. A substantial amount of heat is generated volumetrically in the liquid-metal coolant due to neutrons. The most obvious effect is that the coolant bulk temperature will be affected. Furthermore, volumetric heat generation can alter the temperature profile and heat transfer coefficient—even if the heat generation is spatially uniform. The presence of internal heat generation generally reduces the heat transfer coefficient and results in higher film temperature drops.

In this paper, closed form solutions for fully developed temperature profiles and Nusselt numbers are derived. Using this analysis, the combined effects of nonuniform velocity profiles and bulk heating are studied independently of the thermal entrance length problem. The Nusselt number with volumetric heat generation is expressed in terms of corrections to the solution in the absence of volumetric heating. Volumetric heating generally reduces the Nusselt number. Furthermore, the structure temperature is observed to be highly sensitive to variations in the coolant velocity profiles, which can occur under the influence of a strong magnetic field. A moderate change in the coolant velocity profile can result in large changes in the wall temperatures.

An analytic solution is available for the entry length behavior only for a few simple problems, such as slug flow with uniform bulk heating. In general, the combined effects of nonuniform velocity profiles with bulk heating in the entrance region requires a numerical solution. To analyze the thermal entrance region problem, a two-dimensional transient conduction code is used to simulate the three-dimensional steady laminar convection problem. The numerical results indicate that entrance region effects can often be decoupled from the effects of velocity profiles and volumetric heating. This then allows an estimate of heat transfer in the entrance region, knowing only the value of the fully developed Nusselt number, which is generally easier to obtain.

To focus attention on the problems involved, a representative liquid-metal blanket is used—the Blanket Comparison and Selection Study (BCSS) toroidal/poloidal flow tokamak blanket (see Fig. 1). This blanket consists primarily of long “poloidal” channels, slightly off perpendicular to the field, which carry the coolant through the blanket, accepting the majority of the bulk neutron heat deposition. The poloidal channels also act as feed pipes for the “toroidal” channels. The toroidal channels flow past the first wall near the

Fig. 1. Schematic of poloidal/toroidal reference blanket.
plasma, removing both the surface heat flux and some volumetric heat generation. They are directed roughly along the magnetic field lines to minimize the MHD pressure drop and still maintain a high velocity for heat removal. The wall of the toroidal channels facing the surface heat flux is called the "first wall." The wall on the opposite side of the channels is called the "second wall."

II. EFFECTS OF VELOCITY PROFILE AND VOLUMETRIC HEAT GENERATION FOR FULLY DEVELOPED HEAT TRANSFER

II.A. Assumptions and Definitions

Figure 2 shows the model used to analyze the first-wall coolant channel in the reference blanket. The value of the surface heat flux at \( y = a \), the plasma side, is the sum of the first-wall heat flux and the heat generated in the first wall. The heat flux at the second wall \( q_w^- \) accounts for the heat generated in the second wall. The following assumptions are made:

1. The coolant flow is essentially laminarized. Measurements in MHD experiments have determined that a small amount of residual turbulence may exist but at a level that would not significantly impact heat transfer (1 to 2%).\(^3\)

2. The velocity profile is fully developed. In fact, the influence on temperatures due to developing velocity profiles can be very significant. This assumption is made to simplify the analysis.

3. The temperature profile is fully developed, that is (see Nomenclature on p. xxx),

\[
\frac{\partial T}{\partial y} = fn(x) \quad \text{and} \quad \frac{\partial T}{\partial x} = fn(p) .
\]

The thermal entry length problem is treated in more detail in Sec. III.

4. Analysis is performed for two-dimensional flow, that is, flow between two infinite parallel plates.

5. Thermal boundary conditions are simplified to those of constant but different heat flux at each surface.

6. Axial (streamwise) conduction is ignored.

7. The blanket operates under steady-state conditions.

8. Fluid properties are assumed to be independent of temperature.

The energy equation for the flow described above may be written as

\[
\rho c_p u \frac{\partial T}{\partial x} = k \frac{\partial^2 T}{\partial y^2} + \dot{Q}_v(y) ,
\]

with boundary conditions as follows:

\[
k \frac{\partial T}{\partial y} \bigg|_{y=\pm a} = \pm q_w^\pm .
\]

The energy equation and boundary conditions can now be written in nondimensional form as

\[
f(\eta) \frac{\partial T}{\partial \xi} = \frac{\partial^2 T}{\partial \eta^2} + \lambda(\eta)
\]

and

\[
\frac{\partial T}{\partial \eta} \bigg|_{\eta=\pm 1} = \pm 2\gamma^\pm ,
\]

where the nondimensional quantities are given by

\[
\eta = y/a
\]

\[
f = u/u_b , \quad \text{where} \int_{-1}^{1} d\eta f = 2
\]

\[
\xi = \alpha x/u_b a^2
\]

\[
\lambda(\eta) = \dot{Q}_v(\eta)a^2/k
\]

\[
\gamma^\pm = q_w^\pm a/2k
\]

An overall energy balance relates the rise in bulk temperatures to the total energy input in the coolant flow as

\[
\frac{\partial T_b}{\partial x} = \frac{q_w^+ + q_w^- + \int_{-a}^{a} dy \dot{Q}_v}{\rho c_p u_b 2a} ,
\]

where

\[
T_b = \frac{1}{2} \int_{-1}^{1} d\eta f T .
\]

In terms of dimensionless variables and using the condition of fully developed temperature profile, Eq. (6) reduces to
\[
\frac{\partial T_b}{\partial \xi} = \frac{\partial T}{\partial \xi} = \gamma^+ + \gamma^- + \lambda_a ,
\]
where
\[
\lambda_a = -\frac{1}{2} \int_{-1}^{1} d\eta \lambda(\eta) .
\]

Two Nusselt numbers are defined, based on the heat flux at the first and second walls:
\[
 Nu^\pm = \frac{q_w^\pm 4a}{k(T_w^\pm - T_b)} ,
\]
or in terms of dimensionless parameters:
\[
 Nu^\pm = \frac{8\gamma^\pm}{T_w^\pm - T_b} .
\]

II.C. Derivation in the Case with Bulk Heating

For the case with heat generation, Eqs. (4) and (7) are combined to obtain
\[
\frac{\partial^2 T^0}{\partial \eta^2} = (\gamma^+ + \gamma^- + \lambda_a) f - \lambda .
\]
A solution of the following form is adopted:
\[
 T(\xi, \eta) = T^0(\xi, \eta) + \theta(\eta) + \lambda_a \xi .
\]
In Eq. (16), \( \lambda_a \xi \) accounts for the temperature rise due to bulk heating and satisfies Eq. (7). The function \( \theta(\eta) \) accounts for the effect of \( Q_0 \) on the temperature profile. Equation (16) may be replaced in Eq. (15), while using Eq. (10) for \( T^0 \) to obtain
\[
\frac{d^2 \theta}{d\eta^2} = \lambda_a f - \lambda ,
\]
with the boundary conditions:
\[
\frac{\partial T}{\partial \eta} = \frac{\partial T^0}{\partial \eta} \quad \text{at} \quad \eta = -1 \quad \text{and} \quad 1 .
\]
This leads to the relation:
\[
\frac{\partial \theta}{\partial \eta} = 0 \quad \text{at} \quad \eta = -1 \quad \text{and} \quad 1 .
\]
Equation (18) represents the fact that the surface heat flux at the first and second walls are set by boundary conditions [Eq. (3)] and do not depend on the presence of heat generation. Equation (17) may be integrated twice from \(-1\) to \(\eta\) while using Eq. (19) to obtain \( \theta \) as
\[
\theta - \theta_w = \int_{-1}^{\eta} \int_{-1}^{\eta} (\lambda_a f - \lambda) \, d\eta .
\]
The temperature profile, Eq. (16), may now be written as:
\[
T = T^0 + \lambda_a \xi + \theta_w + \int_{-1}^{\eta} \int_{-1}^{\eta} d\eta (\lambda_a f - \lambda) .
\]
By computing the bulk temperatures from Eq. (21) and using the Nusselt number definition [Eq. (9)], we obtain
\[
1/Nu_0^\pm = 1/Nu_0^\pm + (I_2 + I_3^\pm) \left( \lambda_a / 8 \gamma^\pm \right) ,
\]
where \( I_2^\pm \) and \( I_3^\pm \) are defined by
\[
 I_2 = \frac{1}{2} \int_{-1}^{1} \int_{-1}^{\eta} \int_{-1}^{\eta} d\eta (\lambda / \lambda_a - f) .
\]
and
\[
 I_3^\pm = \int_{-1}^{1} \int_{-1}^{\eta} d\eta (f - \lambda / \lambda_a) .
\]
The general relation for Nusselt number is then
\[
1/Nu^\pm = \frac{1}{4} - I_1 / 8 - I_1 (q_w^+ / q_w^- - 1) / 16
\]
and
\[
(Nu_0^+ = 8.235 \quad \text{if} \quad q_w^- = 0 ,
\]
which compares favorably with the results of Kays.\(^5\)

II.B. Derivation in the Case of No Bulk Heating

The case with no bulk heating (\( \lambda = 0 \)) is denoted by superscript 0. Equations (4) and (7) are combined and reduced to obtain
\[
\frac{\partial^2 T^0}{\partial \eta^2} = (\gamma^+ + \gamma^-) f(\eta) .
\]
This equation may be integrated using the boundary conditions and known velocity profile \( f(\eta) \) to obtain the temperature distribution. Nusselt numbers at the first and second walls may be calculated according to Eq. (9) (the subscript 0 is used on the Nusselt number to denote the case with no heat generation):
\[
1/Nu_0^\pm = \frac{1}{4} - I_1 / 8 - I_1 (q_w^+ / q_w^- - 1) / 16 ,
\]
where \( I_1 \) is defined as
\[
 I_1 = \int_{-1}^{1} \int_{-1}^{\eta} \int_{-1}^{\eta} d\eta f(\eta) .
\]
Equation (11) can be compared with results for limiting cases available in the literature. For slug flow (\( f = 1 \)), in which \( I_1 = \frac{1}{4} \), Eq. (11) yields
\[
 Nu_0^+ = 12 \quad \text{if} \quad q_w^+ = q_w^- \]
and
\[
 Nu_0^- = 6 \quad \text{if} \quad q_w^- = 0 ,
\]
which is consistent with the results of Hartnett and Irvine.\(^4\) For the case of parabolic velocity profile, where \( I_1 = 1.0286 \), Eq. (11) yields
\[
 Nu_0^+ = 8.235 \quad \text{if} \quad q_w^+ = q_w^- \]
and
\[
 Nu_0^- = 5.385 \quad \text{if} \quad q_w^- = 0 ,
\]
which compares favorably with the results of Kays.\(^5\)
1.0286, \( I_2 = 0.0857\lambda_w \), and \( I_3^x = 0 \), Eq. (22) reduces to

\[
1/\text{Nu} = 1/\text{Nu}_0 + (3/140)(\dot{Q}_w a / q_w),
\]

which is identical to the results obtained by Sparrow et al.\(^6\)

**II.D. Results: Fully Developed Nusselt Numbers**

To present the results, the first-wall channels of the reference blanket were analyzed. Table I lists the numerical values of the relevant parameters for this blanket.

Due to lack of detailed knowledge of velocity profiles in the toroidal first-wall channels and due to predictions that the velocity profiles might be nonuniform and asymmetric, several different velocity profiles are examined, including slug, couette, parabola, power, and Gaussian relations. The general formula for the power relation velocity profile is

\[
\frac{u}{u_b} = f = \frac{n+1}{n} \left[ 1 - \left( \frac{y}{a} \right)^n \right].
\]

The parameter \( n \) is introduced, where \( n = 2 \) is the parabola and \( n \to \infty \) is the slug velocity profile. Gaussian velocity profiles, which are shown in Fig. 3, have the general formula:

\[
f = \frac{s(y/a + 1)}{1 - \exp(-s^2) - s^2 \exp(-s^2)}.
\]

It should be stressed here that Eq. (28) is not the result of any analytical consideration. It simply is an algebraic fit (with certain conditions on the values at the boundaries and on the integral of \( u \)) to help investigate the effects of unusual velocity profiles on the heat transfer.

Using slug, couette, and parabolic velocity profiles, the fully developed temperature profiles are presented in Fig. 4, with and without spatially constant

**TABLE I**

**Specifications of the First-Wall Channels for the Reference Blanket**

<table>
<thead>
<tr>
<th>Specification</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>First-wall thickness (mm)</td>
<td>7.5</td>
</tr>
<tr>
<td>Second-wall thickness (mm)</td>
<td>15</td>
</tr>
<tr>
<td>Coolant channel width, ( 2a ) (mm)</td>
<td>45</td>
</tr>
<tr>
<td>Heat generation in vanadium ( \dot{Q}_{w,v} ) (MW/m(^2))</td>
<td>25</td>
</tr>
<tr>
<td>Heat generation in lithium ( \dot{Q}_{w,Li} ) (MW/m(^2))</td>
<td>25</td>
</tr>
<tr>
<td>Surface heat flux ( q_s ) (MW/m(^2))</td>
<td>0.5</td>
</tr>
<tr>
<td>Dimensionless surface heat flux ( \gamma^- )</td>
<td>45.76</td>
</tr>
<tr>
<td>Dimensionless surface heat flux ( \gamma^+ )</td>
<td>274.5</td>
</tr>
<tr>
<td>Dimensionless volumetric heat generation ( \lambda )</td>
<td>274.5</td>
</tr>
</tbody>
</table>

bulk heating. The peak wall temperature is obviously very sensitive to the velocity profile and, to a lesser but still significant degree, to the presence of bulk heating.

The results of investigations on the effect of velocity profiles and heat generation are condensed in Figs. 5 and 6. In these figures, the abscissa does not represent a quantitative parameter. Rather, it presents the velocity profiles from Eqs. (27) and (28) in a progression of increasing nonuniformity. It starts with
Fig. 5. First-wall Nusselt number and temperature rise under fully developed conditions for different velocity profiles, using parameters listed in Table I.

Fig. 6. Second-wall Nusselt number and temperature rise under fully developed conditions for different velocity profiles, using parameters listed in Table I.
slug flow \((n \to \infty)\). Thereafter, the parameter \(n\) in the power relation velocity profiles decreases down to \(n = 2\) for parabolic profiles. At this point, the abscissa is switched to represent the Gaussian velocity profiles, starting with a skewing factor \(s = 1\). Figure 3 demonstrates the fact that \(s = 1\) is symmetric and very close to a parabolic profile. Thereafter, the skewing factor increases and the profile becomes more asymmetric. Also, note that there are two velocity profiles for each skewing parameter— one with the velocity peaked near the first wall and the other near the second wall (one being the mirror image of the other).

The ordinates in Figs. 5 and 6 are the inverse Nusselt number and the temperature difference between the wall and the bulk coolant for the first and second walls, respectively. While the actual temperature differences for the toroidal channel may be up to two to three times smaller (since it is in the entrance region), the ratios of the Nusselt numbers and temperature differences for the different cases should remain more or less the same as the velocity profile and heat generation are varied.

Figure 5 shows that the first-wall temperature increases only moderately as the velocity profile changes from slug flow all the way to parabolic. The first-wall temperature increases drastically, however, as the velocity near the second wall increases with the skewed profiles. This kind of behavior (higher velocity near the second wall) is predicted to happen in the toroidal first-wall channels. It can be seen that even a moderately skewed profile \((s = 3)\) increases the first-wall temperature by a factor of 2, which can be more than enough to close the blanket design window.

One potential solution that has often been suggested is to somehow direct more of the coolant near the first wall, e.g., by installing guide plates at the entrance of the toroidal channels. This is modeled by the Gaussian profile with higher velocity near the first wall. Figure 5 shows that this quickly reduces the first-wall temperature as expected. Figure 6, however, shows a drastic increase in second-wall temperature, to the extent that the blanket may fail due to large second-wall temperatures. There is, however, a small window of skewed velocity profiles \((s = 1\) or 2 with peak velocity near the first wall) that successfully reduces the first-wall temperature while only moderately increasing the second-wall temperature.

The effect of spatially uniform heat generation on the first- and second-wall heat transfer is also included in Figs. 5 and 6. The heat generation effect is identically zero for slug flow. It starts to grow as the velocity profile approaches a parabola, but it is still only about a few percent. The heat generation effect, however, quickly increases as the velocity profile is skewed. It can be seen from Fig. 5 that this effect can get as high as 30% for a moderate skewing factor of \(s = 4\). This effect can be even higher for the second wall, as can be observed from Fig. 6.

The effect of nonuniform heat generation is shown in Fig. 7. An exponential variation in the \(y\) direction is considered \([\tilde{Q}(\eta) = \exp(c\eta)]\) since it approximates the actual distribution in a neutron environment. Care is taken to preserve the total energy input (constant \(\lambda_0\)) as the profile is varied:

\[
\lambda(\eta) = \frac{2c\lambda_0 \exp(c\eta)}{[\exp(c) - \exp(-c)]}.
\]

In Fig. 7, the inverse Nusselt number and the temperature differences at the first and second walls are plotted while varying the coefficient in the exponential relation. It can be seen that as the profile becomes more nonuniform (increasing \(c\)), \(Nu^-\) decreases, indicating a higher first-wall temperature. This is expected because an increase in \(c\) leads to a higher portion of the heat generated close to the first wall. The second-wall Nusselt number \(Nu^+\) increases as \(c\) is increased, causing a lower second-wall temperature.

### III. THERMAL ENTRY LENGTH CONSIDERATIONS

As a result of the laminarization of liquid-metal flow under a magnetic field, eddy diffusivity is absent and the thermal entry length can be very long. The degree to which the temperature profile is developed can be estimated by the Fourier number, which is the ratio of the fluid residence time to the cross-channel conduction time.

\[
Fo = \frac{x/u}{4a^2/\alpha} = \alpha x/4a^2 u,
\]

![Fig. 7. Nusselt numbers and temperatures for an exponentially varying heat generation \([\tilde{Q}(\eta) = \exp(c\eta)]\), holding the total heat input constant. A parabolic velocity profile was used with \(\gamma^+ = 45.76\), \(\gamma^- = 274.5\), \(\lambda_0 = 274.5\).](image-url)
where
\[ x = \text{coordinate along the channel length} \]
\[ a = \text{channel half width} \]
\[ u = \text{fluid velocity} \]
\[ \alpha = \text{thermal diffusivity}. \]

In the reference design, the entry length is of the order of 100 m, whereas the channel length is \(-3 \text{ m}\). Therefore, the temperature profiles are always far from being fully developed.

In the following analysis, the entry length problem is considered only for temperature profiles. The velocity profile is assumed to be fully developed, although this is likely to be incorrect for some blanket geometries in which the hydrodynamic entry length is also long. Under these assumptions, the energy equation governing the coolant temperature is
\[ \rho c_p \frac{\partial T}{\partial x} = \kappa \left( \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right) + Q_0. \quad (31) \]

Analytic treatment of Eq. (31) can be performed in certain special cases, for example, one-dimensional cross-channel conduction under slug flow with no bulk heating or with exponential bulk heating. More general cases of Eq. (31) have been investigated numerically and are presented below.

III.A. Analysis

Numerical techniques for two- or three-dimensional convective flows are not always stable. In addition, there are few numerical packages for convection, as opposed to several well-known all-purpose conduction numerical packages. Therefore, a conduction code was employed to solve the laminarized convective flow problem. This technique may be used in any situation with laminar flow.

One approximation is needed for this technique to be valid: Streamwise conduction must be negligible. That is,
\[ \frac{\partial^2 T}{\partial x^2} \ll \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}. \quad (32) \]

Streamwise conduction has been treated, and its effects on the thermal solution of this problem are found to be negligible.

A normalized velocity distribution \( f \) may be defined as
\[ f(y, z) = u(y, z)/u_b. \quad (33) \]

At this point, a pseudotime can be defined as
\[ t_p = x/u_b. \quad (34) \]

A spatially varying “effective heat capacity” is next defined as
\[ \bar{\rho} c_p(y, z) = \rho c_p f(y, z). \quad (35) \]

After incorporating Eqs. (32) through (35), Eq. (31) may be rewritten as
\[ \bar{\rho} c_p \frac{\partial T}{\partial t_p} = \kappa \left( \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right) + Q_0, \quad (36) \]

which is identical to a two-dimensional transient conduction equation. The steady-state convection problem (including entrance region effects) is thus converted to a transient convection problem by introducing an artificial, spatially dependent heat capacity function.

The transformation of boundary conditions to the new coordinates is straightforward. Even boundary conditions that are varying in the direction of flow (x direction) are treated easily. They simply become time-dependent boundary conditions in the new system.

III.B. Results

As discussed earlier, temperature profiles throughout the entire toroidal first-wall channel are in the entry length region. This results in heat transfer coefficients that vary considerably and are higher than predicted for fully developed flow. Figure 8 shows the entrance region Nusselt numbers for several different velocity profiles. In this figure, the Nusselt numbers are normalized by their fully developed values for two reasons. First, it is possible to estimate how fast the profiles are developing, since all the curves go to unity. Second, being normalized to their fully developed values, any differences in the normalized Nusselt numbers are directly attributed to the nonlinear effects of the entrance region, heat generation, and velocity profiles. It is seen from Fig. 8 that all the Nusselt numbers start very high at the entrance point and drop more or less...
in a similar fashion. It is also observed that the normalized Nusselt number for all velocity profiles except slug flow are very close and that slug flow delivers the longest entry length.

It is useful to separate the effects of volumetric heat generation and velocity profiles on the entrance region. Figure 8 indicates that the development of the Nusselt number in the entrance region depends only weakly on the velocity profile. To see the effect of volumetric heat generation, Fig. 9 shows the developing Nusselt numbers for various velocity profiles with and without volumetric heat generation. This figure clearly indicates that the principal effect of volumetric heat generation is on the magnitude of the fully developed Nusselt number and not on the rate at which the Nusselt number develops. Therefore, it is a reasonable approximation to assume

\[
\text{Nu}(x, u, \dot{Q}_v) = \frac{\text{Nu}(x, u, 0)}{\text{Nu}_{id}(u, 0)} \cdot \text{Nu}_{id}(u, \dot{Q}_v).
\]  

(37)

In this equation, the arguments \(x\), \(u\), and \(\dot{Q}_v\) indicate the functional dependence of the Nusselt number on the entrance region, velocity profile, and volumetric heat generation, respectively. In other words, the entrance region Nusselt number may be estimated as the product of the fully developed Nusselt number (obtained from Figs. 5 and 6) and the ratio of entrance region to fully developed Nusselt numbers with no volumetric heat generation.

The exit location of the BCSS reference blanket is shown in Fig. 8. It reaffirms the fact that the entire blanket is in the entrance region and that the average Nusselt number in the toroidal channels can be as high as two to three times the fully developed Nusselt number, depending on the velocity profile.

IV. CONCLUDING REMARKS

A closed form solution for the Nusselt number has been obtained for fully developed laminar flow between parallel plates with arbitrary velocity and volumetric heat generation profiles. Together with numerical solutions in the entrance region, the analysis predicts several aspects of heat transfer in liquid-metal blankets, which include the following:

1. Volumetric heat generation could strongly affect the Nusselt number values. This effect is even more pronounced for nonuniform velocity profiles.

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Fig. 9. Entrance region Nusselt number for various velocity profiles, with (solid line) or without (broken line) internal heat generation, using parameters listed in Table 1.
2. The effect of velocity profiles and volumetric heating on the rate of development of the Nusselt number in the entrance region is moderate at most.

3. Hence, for design purposes an estimate of heat transfer may be obtained using the fully developed Nusselt number together with the normalized entry region curve (Fig. 8).

NOMENCLATURE

\[ a = \text{channel half-width} \]
\[ c_p = \text{specific heat} \]
\[ f = \text{nondimensional velocity, } u/u_b \]
\[ Fo = \alpha L/4a^2u_b, \text{ Fourier number} \]
\[ h = \text{heat transfer coefficient} \]
\[ k = \text{thermal conductivity} \]
\[ n = \text{arbitrary integer parameter} \]
\[ Nu = 4ah/k, \text{ Nusselt number} \]
\[ Pr = c_p\mu/k, \text{ Prandtl number} \]
\[ q_w = \text{wall heat flux} \]
\[ \dot{Q}_v = \text{volumetric heat generation rate} \]
\[ \dot{Q}_{v,a} = \text{average volumetric heat generation rate} \]
\[ s = \text{velocity profile skewing factor} \]
\[ t = \text{time} \]
\[ t_p = x/u_b, \text{ pseudotime} \]
\[ T = \text{temperature} \]
\[ T_b = \text{bulk temperature, } \frac{\int dy Tu}{\int dy u} \]
\[ T_w = \text{wall temperature} \]

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\[ \alpha = k/\rho c_p, \text{ thermal diffusivity} \]
\[ \gamma = q_wa/2k, \text{ nondimensional surface heat flux} \]
\[ \epsilon_h = \text{eddy diffusivity for heat transfer} \]
\[ \epsilon_m = \text{eddy diffusivity for momentum transfer} \]
\[ \xi = ax/u_b a^2, \text{ nondimensional axial coordinate} \]
\[ \eta = y/a, \text{ nondimensional } y \text{ coordinate} \]
\[ \theta = \text{temperature increment due to bulk heating} \]
\[ \lambda = \dot{Q}_v a^2/k, \text{ nondimensional volumetric heating} \]

\[ \lambda_u = \frac{1}{2} \int_{-1}^{1} d\eta \lambda, \text{ average nondimensional volumetric heating} \]
\[ \mu = \text{viscosity} \]
\[ \rho = \text{mass density} \]

Subscripts and Superscripts

+ = first wall
- = second wall
0 = value with no bulk heating

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REFERENCES


