# THE EFFECT OF HARTMANN AND SIDE LAYERS ON HEAT TRANSFER IN MAGNETOHYDRODYNAMIC FLOW

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## ABSTRACT

Analyses were performed of the effect of Hartmann layers and side layers on heat transfer in laminar MHD flow in ducts and the dependence on the magnitude of the Hartmann number. Analytical and numerical results are presented for both fully developed and thermally developing cases. The presence of side layers in a rectangular duct usually increases the heat transfer coefficient on the side layer walls and decreases the heat transfer coefficient on the other two walls. For ducts with uniform thickness and conductivity on all walls, the studies show that a duct with higher conductance ratio gives higher average Nusselt number on the side wall. However, this behavior depends on the combination of Hartmann number and the conductance ratio. The heat generation inside the duct enhances the heat transfer coefficient.

## INTRODUCTION

A magnetic field is known to modify the velocity profile and suppress turbulence in an electrically conducting fluid. Hartmann boundary layers exist in channels along the walls which intersect magnetic field lines. These are usually thin, with steep velocity gradients, and carry a small fraction of the total flow. Another type of hydrodynamic boundary layer, called a side layer, is found when a wall is parallel to the magnetic field lines[1]. In a side layer, there can be a significant amount of fluid flow in a very thin layer. These unusual velocity profiles affect the temperature profile and heat transfer, as compared to the velocity profile for nonconducting fluids. In addition, at very high Hartmann number (Ha) and Interaction parameter (N), these layers become very thin and it may be possible to ignore the detailed velocity profile or to account for the influence of boundary layers on heat transfer in a simplified manner. For "one-dimensional" heat transfer in a side layer, the energy equation was solved for a constant wall heat flux boundary condition. For the thermally developing case, the temperature profile was expanded in a series of Chebyshev polynomials and the coefficients in the Chebyshev expansion were obtained using the spectral-tau method. The thermally developing case with a two-dimensional velocity profile in a square duct was solved numerically using a three-dimensional finite difference computer program.

## FLOW IN A RECTANGULAR DUCT

In the case of MHD flow in a rectangular duct, hydrodynamic boundary layers exist not only at the walls perpendicular to the field, but also at walls parallel to it. The boundary layers on walls parallel to the flow are called side layers[1,2]. The flow characteristics in this type of geometry can be specified for different combinations of boundary conditions. For flow in a rectangular duct with perfectly conductingwalls perpendicular to the field and thin walls of arbitrary conductivity parallel to the field, the exact fully developed velocity profile was obtained by J. C. R. Hunt[1]. The velocity distribution obtained using Hunt's solution (with a conductance ratio of 0.07 for the wall parallel to the field) is plotted in Fig.1 for Hartmann numbers equal to 1000. The most remarkable feature is that the velocity in the boundary layer is much higher than that in the flow core. In addition, a region with negative velocities exists in the vicinity of the interface between the boundary layer and the flow core.

## HEAT TRANSFER OF ONE-DIMENSIONAL SIDE LAYER

Unlike the Hartmann layer, the effect of a side layer on heat transfer has not been addressed in the past few years. A typical example of a side layer can be observed in MHD flows in a rectangular duct at walls parallel to the field (see Fig. 1). To study the effect of side layers on heat transfer in a simplified way, the velocity profile mentioned above is evaluated at the centerpoint of the wall parallel to the magnetic field (that is, in the middle of the side layer). Thus, this velocity profile is a function only of distance away from the wall.

The origin of the coordinate system is taken as the point where the thermal entrance region initiates. The energy equation with neglect of Joule heating is written as:

$$u \frac{\partial T}{\partial x} = k \frac{\partial^2 T}{\partial y^2} \tag{1}$$

where u is the velocity profile given by Hunt[1] as an infinite series solution evaluated at the center of the side layer. In order to determine the temperature in the thermal entrance region, it is convenient to define a temperature $T^+$ as follows:

$$T = T_\infty + T^+ \tag{2}$$

where the fluid temperature in the fully developed region is designated by $T_\infty$. For uniform wall heat flux, the temperature field $T_\infty$ satisfies the following equation:

$$u \frac{\partial T_\infty}{\partial x} = k \frac{\partial^2 T_\infty}{\partial y^2} \tag{3}$$

with boundary conditions as follows:

$$y = 0; \ \frac{\partial T_\infty}{\partial y} = 0$$

$$y = a; \ \frac{\partial T_\infty}{\partial y} = -\frac{q}{\lambda}$$

An overall energy balance relates the rise in bulk temperature to the total energy input in the coolant flow as:

$$\frac{\partial T_\infty}{\partial x} = \frac{\partial T_m}{\partial x} = -\frac{q}{a} \frac{1}{\rho c_p u_m} \tag{4}$$

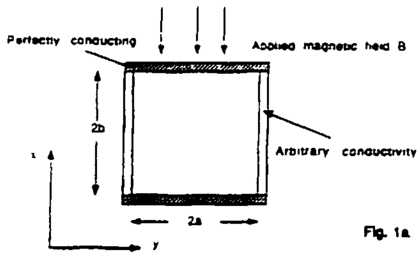


Fig. 1a

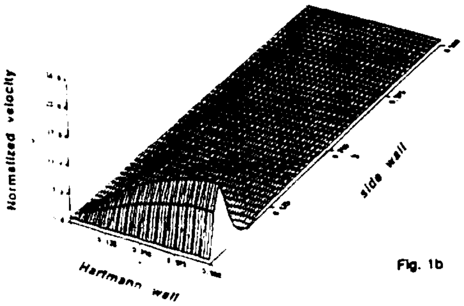


Fig. 1b

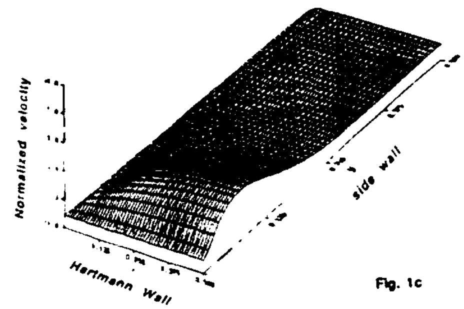


Fig. 1c

Fig. 1 Quarter core velocity profiles for different Hartmann number
Fig. 1a Geometry of Hunt's solution. Fig. 1b Hartmann number = 1000. Fig. 1c Hartmann number = 50

Then, $T_\infty$ yields

$$T_\infty = -\frac{q}{a}\frac{x}{\rho c_p u_m} + G(y) \tag{5}$$

where the function G(y) can be obtained by using direct integration. By substituting this equation into equation (3) the following relation is derived:

$$-\frac{u}{u_m}\frac{q}{a\rho c_p} = k\frac{d^2G}{dy^2} \tag{6}$$

Using equation (1) the following relation is obtainable:

$$u\frac{\partial T^+}{\partial x} = k\frac{\partial^2 T^+}{\partial y^2} \tag{7}$$

with boundary conditions as follows:

$$x = 0; \quad T^+ = -T_\infty;$$

$$y = 0; \quad \frac{\partial T^+}{\partial y} = 0;$$

$$y = a; \quad \frac{\partial T^+}{\partial y} = 0$$

The solution of equation (7) can be expressed as follows:

$$T^+ = \frac{(\mathrm{Re\,Pr})^2 \upsilon k}{a^2 c_p}\sum_{n=1}^{\infty} C_n Y_n \exp\left(-\beta_n u_m \xi\right)$$

$$\xi = \frac{1}{\mathrm{Re\,Pr}}\frac{x}{a} \tag{8}$$

where the Reynolds number Re and Prandtl number Pr are defined as usual, and $\beta_n$ and $Y_n$ are eigenvalues and eigenfunctions of the Sturm-Liouville type differential equation,

$$\frac{d^2Y_n}{d\eta^2} + \beta_n u Y_n = 0 \tag{9}$$

with boundary conditions as follows:

$$\eta = 0; \quad \frac{dY_n}{d\eta} = 0,$$

$$\eta = 1; \quad \frac{dY_n}{d\eta} = 0$$

To solve the above equation, the solution $Y_n$ is expanded in a series of Chebyshev polynomials:

$$Y_n = \sum_{l=0}^{\infty} a_l^n T_l^n \tag{10}$$

The coefficients in the Chebyshev expansion ($a_l$) were obtained using the spectral-tau method. To judge the accuracy of this method, the Hartmann velocity profile was introduced into equation (9) and the results of calculated eigenvalues are compared with Siegel's[3] values for Hartmann number equal to 4 (Table 1).

Table 1: Eigenvalues of Equation (9)
(Hartmann Number = 4)

| | Present Method | Seigel's Solution |
|---|---|---|
| n | $\beta_n$ | $\beta_n$ |
| 1 | 14.78 | 14.7866 |
| 2 | 56.52 | 56.5509 |
| 3 | 124.855 | 124.9158 |
| 4 | 219.742 | 219.8498 |
| 5 | 341.17 | 341.3385 |

The coefficients $C_n$ in equation (8) can be obtained using the initial condition for $T^+$ at x =0.

$$C_n = -\frac{\int_0^1 u\frac{G(\eta) - C_0}{(\mathrm{RePr})^2\upsilon k/(a^2C_p)}Y_n\, d\eta}{\int_0^1 u\, Y_n^2\, d\eta}$$

$$C_0 = -\frac{\int_0^1 u\frac{G(\eta) - C_0}{(\mathrm{RePr})^2\upsilon k/(a^2c_p)}d\eta}{\int_0^1 u\, d\eta} \tag{11}$$

The above integrations are done using Gauss-Legendre quadrature with 96 roots. The local nusselt number Nu is defined as :

$$Nu = \frac{h(2a)}{\lambda} = \frac{2a}{\lambda}\frac{q}{T_m - T_w} = -2a\frac{(\partial T/\partial y)_{y=a}}{T_m - T_w} \tag{12}$$

In the case of constant wall heat flux, $T_m$-$T_w$ is given by the following equation:

$$T_m - T_w = -G(1) - \frac{qa}{\lambda}\sum_{n=1}^{\infty} C_n Y_n(1) \exp(-u_m \beta_n \xi) \tag{13}$$

Consequently, the Nusselt number becomes:

$$Nu = \frac{2qa/\lambda}{G(1) + \frac{qa}{\lambda}\sum_{n=1}^{\infty} C_n Y_n(1) \exp(-u_m \beta_n \xi)} \tag{14}$$

Figure 2 shows the relation between local Nusselt number and nondimensional axial location for different Hartmann numbers. The results indicate that the Nusselt number for side layers increases indefinitely as the Hartmann number increases. This behavior is expected, since as the Hartmann number increases the side layer becomes thinner and the flow rate in the side layer increases.

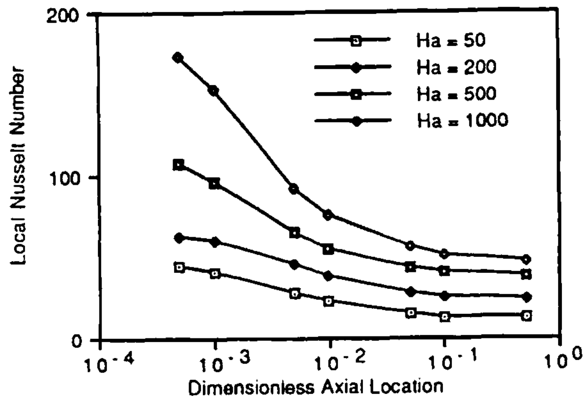


Fig. 2 Local Nusselt number vs. dimensionless axial locations for one-dimensional side layer

HEAT TRANSFER OF MHD FLOW IN A RECTANGULAR DUCT

It is known that the presence of side layers increases the heat transfer on those walls, but there are many details which are unknown. For instance, it is not clear whether the increased heat transfer on the side layer walls occurs at the expense of decreased heat transfer on the other two walls. The magnitude of the increase in heat transfer as a function of wall conductance ratio has not yet been quantified. To better address the heat transfer issues, a fully three-dimensional finite difference heat transfer code was developed. This code solves the energy equation, given any arbitrary velocity profile, using the hybrid differencing algorithm. The code was first used to analyze the problem of thermally developing flow in a square duct, with uniform heat flux boundary conditions on all four walls. The velocity profile was the exact solution for flow in a square duct (due to J. C. R. Hunt). A nonuniform grid was used, with a fine mesh in the side layer (e.g. for Hartmann number equal to 1000, 15 mesh points are used within one side layer).

A "side layer" Nusselt number is defined as the average Nusselt number for the wall with the side layer. This is plotted against dimensionless axial location for different Hartmann numbers in Figure 3. The results indicate that the average side layer Nusselt number increases with increasing Hartmann number. This confirms the expectation that side layers are beneficial in increasing heat transfer at the wall parallel to the magnetic field due to the increased flow in the side layer. The wall average Nusselt number for the Hartmann layer vs. dimensionless axial location is plotted in Figure 4 for the same cases. As the Hartmann number increases, the flow in the side layer increases, and the core velocity therefore decreases. This tends to make the Nusselt number decrease on the Hartmann layer walls. However, for Hartmann numbers of 20 and 50, the results are very close together, with the Nusselt number slightly less for Ha = 20. This is probably due to the fact that the Hartmann boundary layer is thicker for lower Hartmann number. Figures 5 and 6 show the temperature surface plots for two different Hartmann numbers at about 50 diameters downstream. The reduction of heat transfer coefficient on the Hartmann wall at high Hartmann number results in a much higher temperature distribution along that wall. The wall temperature distributions along the side layer and Hartmann layer are illustrated in Fig. 7. Note that the horizontal axis represents the distance along either the side layer wall or the Hartmann layer wall, depending on which curve is being considered. The maximum wall temperature occurs on the Hartmann layer wall at the interface of the side layer and core. This is probably due to the negative velocity where the side layer meets the core.

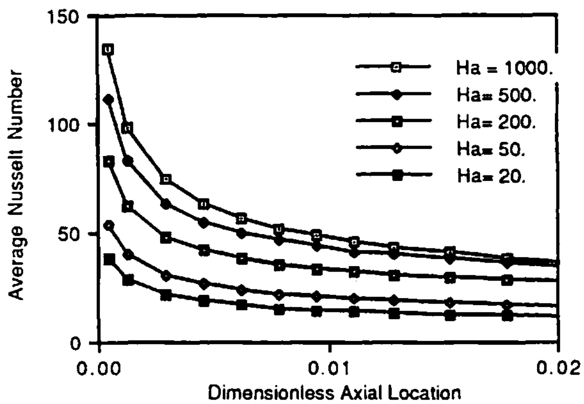


Fig. 3 Side layer Nusselt number vs dimensionless axial location for different Hartmann numbers

To focus attention on the effect of wall conductivity on heat transfer, Walker's[6] analytic solution for MHD flow in a conducting rectangular duct was used. This velocity solution is similar to Hunt's, but allows for arbitrary (but equal) conductivities on both walls. Using this velocity profile, the heat transfer is investigated when the heat flux is applied to only one wall, which has the side layer, to simulate the reactor first wall condition. The results of investigations on the effect of conductance ratios are condensed in Figs. 8 and 9. Fig. 8 shows that the maximum wall temperature occurs at the corner of the duct, and the temperature difference ($T_w$-$T_m$) is reduced when the conductance ratio increases. Fig. 9 indicates that the average side layer wall Nusselt number begins to decrease on the side walls at a wall conductance ratio less than ≈0.05 (this range of conductance ratio is relevant to fusion reactor blanket design) at the same dimensionless axial position. This result is particularly interesting since the MHD pressure drop increases when a highly electrically conducting duct is introduced using liquid metal fluid for a fusion blanket design. To reduce the pressure drop, the concept of using an insulated wall in a liquid metal blanket coolant duct will not give a better heat transfer coefficient than a conducting duct. The design of the liquid metal blanket will therefore require the optimization of minimizing pressure drop and also must providing a reasonable heat transfer coefficient. To explain the reduction of heat transfer on the side wall at lower conductance ratio, the core velocity vs

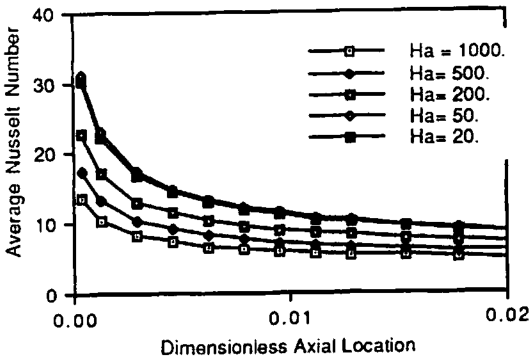


Fig. 4 Average Hartmann layer Nusselt number vs dimensionless axial location for different Hartmann numbers

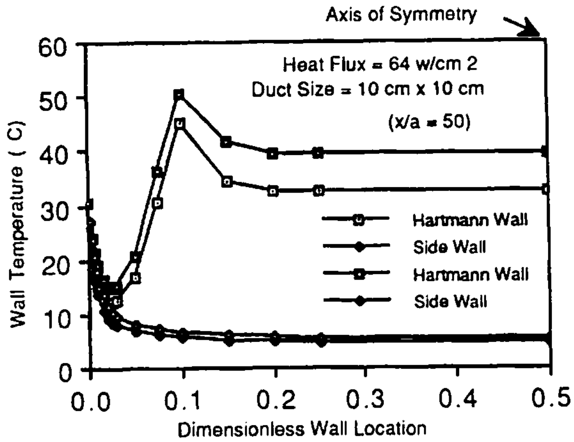


Fig. 7 Wall temperature distribution along Hartmann and side wall

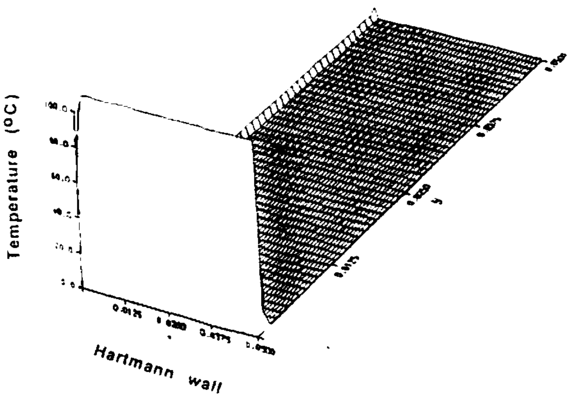


Fig. 5 Quarter core temperature surface plot at 15 diameter downstream using Hunt's solution for Ha = 1000
(four-side heated, duct size = 10 cm x 10 cm, q" = 64W/cm$^2$)

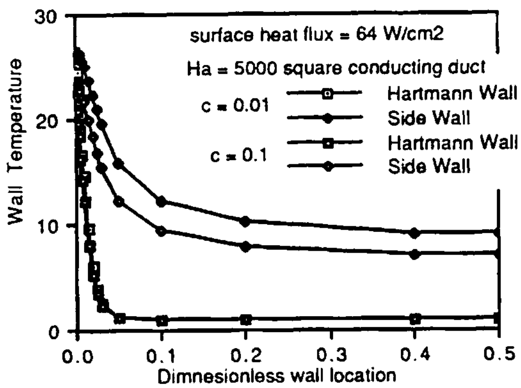


Fig. 8 Wall temperature distribution along Hartmann and side wall (Effect of conductance ratio on heat transfer)

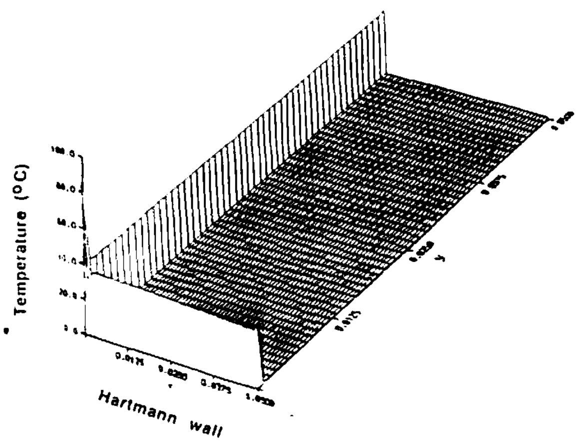


Fig. 6 Quarter core temperature surface plot at 15 diameter downstream using Hunt's solution for Ha = 50
(four-side heated, duct size = 10 cm x 10 cm, q" = 64W/cm$^2$)

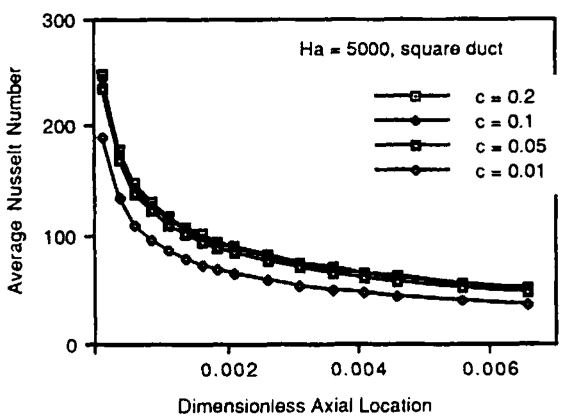


Fig. 9 Nusselt Number Behavior for Different Wall Conductance Ratios

conductance ratio is plotted in Fig. 10. The core velocity increases (side layer flow decreses) at a lower conductance ratio, hence the Nusselt number decreases. The predictions of core velocity based on the core flow approximation are also shown in Fig. 10. The results indicate that the core velocity calculated using the core flow approximation is very different than that calculated by the full solution.

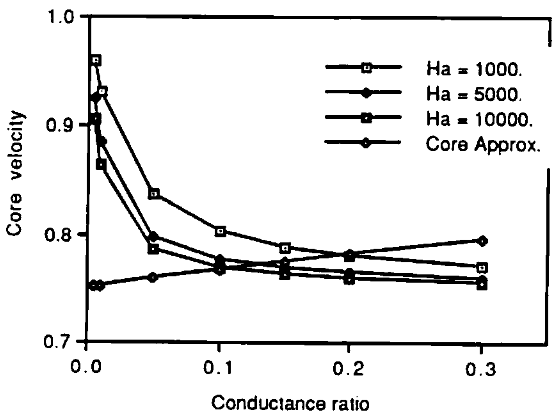


Fig. 10 Core velocity vs. conductance ratio for different Hartmann numbers

In a fusion reactor, the heat generation due to neutron interaction within a blanket flow channel duct is about the same as the heat transported into the duct due to the surface heat flux. The effect of the heat generated volumetrically in the liquid-metal coolant may reduce the heat transfer coefficient when a skewed velocity profile is introduced into a fusion enviorment[4]. To address this problem more realistically, Walker's velocity profile is considered with one side wall heated, in addition to bulk heating. The average Nusselt numbers of the side layer wall with uniform heat flux boundary are plotted against the dimensionless axial locations for the bulk heating cases in Fig. 11 to compare with the results without bulk heating. The heat transfer coefficients increase about 40% at x/d equal to 50 for conductance ratio = 0.1 and about 20% at the same location for 0.01 conductance ratio. These results indicate that the presence of internal heat generation increases the coolant bulk temperature, hence the Nusselt number increases. This result is consistent with the previous study our group has done[4]. In that study they showed that the effect of volumetric heat generation affects heat transfer and the effect is more pronounced for nonuniform velocity profile. The presence of side layer results in the relationship between nonuniform velocity profile and Nusselt number.

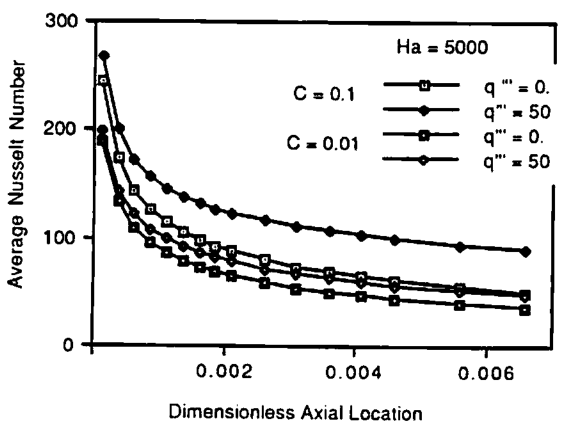


Fig. 11 The effect of wall conductance ratio and bulk heating on heat transfer


SUMMARY

Analytical solutions for the Nusselt number have been obtained for fully developed one-dimensional side flow in thermally developing and developed regions. The results indicate that the existance of side layer increases the heat transfer coefficient indefinitely with the Hartmann number. Two-dimensional fully developed velocity profiles for arectangular duct were used to study the heat transfer in a rectangular duct which includes both side layers and Hartmann layers. The analyses were performed using a three-dimensional finite difference computer program for thermally developing regions. The results confirm the expectation that side layers are beneficial in increasing heat transfer. However, the heat transfer coefficients on the Hartmann layer walls get worse as the side layers get better. The hottest temperature occurs on the Hartmann layer wall at the interface of the side layer and core when all four walls are heated, and at the corner when only one wall is heated. The studies show that the enhancement of heat transfer coefficient depends on wall conductance ratio and the presence of internal heat generation increases the coolant bulk temperature, hence the Nusselt number increases.


NOMENCLATURE

a = one half of channel width
c = conductance ratio
$c_p$ = heat capacity
h = heat transfer coefficient
k = thermal diffusivity
Nu = Nusselt number
q'' = heat flux at wall
q''' = volumetric heat generation rate
Pe = Peclet number
T = temperature
u = velocity of fluid
x = axial coordinate
y = coordinate normal to side layer wall

Greek

$\nu$ = kinematic viscosity
$\lambda$ = thermal conductivity
$\eta$ = dimensionless y coordinate

Subscripts

w = wall
m = mean value


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