THE INFLUENCE OF LEAKAGE CURRENTS ON MHD PRESSURE DROP[a]

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ABSTRACT

The MHD pressure drop in the second wall orifice which connects the first wall cooling channel and the header in one of the BCSS blankets is discussed. Though the second wall is thin, the pressure drop in the orifice can be very large because of leakage current effects. If eddy currents leak and do not have to pass through thin walls, the resistivity of the current path may be low and the current intensity may be high, which induces a high pressure drop. The presence of leakage current greatly affects the MHD pressure drop in fusion reactor blankets.

1. INTRODUCTION

Liquid metal flow in channels under magnetic fields has been studied by many researchers. However, most of them treated MHD flow in a single channel, and neglected currents which flow across a number of channels and through outside structures. In a real fusion reactor blanket, channels are not insulated but contact other channels and structures. Eddy currents induced in a channel flow not only in the channel but may leak to other channels and structures. The existence of leakage current may change the MHD pressure drop and the flow distribution greatly. In this report, the leakage current effects in one of the BCSS liquid metal blanket designs[1,2] are discussed as an example.

2. CALCULATIONAL MODELS

Figure 1 shows the liquid metal blanket. It is composed of toroidal first wall cooling channels and poloidal manifolds. There are orifices in the second wall at toroidal ends of the blanket module, which connect the toroidal channel to the manifolds. The MHD pressure drop in the orifices is discussed in the following.

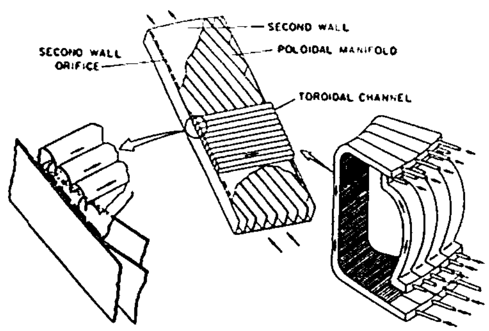

Fig.1. BCSS liquid metal blanket

It is difficult to calculate the velocity profile in the orifices. As the interaction parameter is high in the blanket, the electromagnetic force causes flow redistribution. Yet, the flow may not be fully developed in the orifice, since the second wall is 15 mm thick, which may be thinner than the entry length. Hence, the pressure drop is calculated in two extreme cases where slug flow and fully developed flow are assumed. In the slug flow model, the velocity profile in the channels is assumed to be uniform, and flow redistribution is neglected. The assumption is technically incorrect; however, due to its simplicity, it is a useful tool to introduce this subject. The fully developed flow distribution may appear if the second wall is thick enough. The flow is changing toward this distribution in the orifice, hence it is useful to calculate the pressure drop with this model. The results with two models are compared and the real pressure drop in the orifices is discussed.

In both models, the eddy currents flow only in a plane parallel to the second wall, because there is no electric potential difference in the flow direction. In addition, the following assumptions are made. The coolant is incompressible and the flow is steady. The ribs between the orifices are so thin that the resistivity across them can be neglected. The

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magnetic Reynolds number is small, hence the induced field can be neglected. There is a magnetic field B in the x direction. The magnetic field intensities in the y and z directions are zero. The coolant flows in the z direction, and the bulk velocity is the same in every orifice. In the fully developed flow model, the Hartmann number is assumed to be large so that the boundary layers are thin compared with the ribs.

3. SLUG FLOW MODEL

Figure 2 shows the cross section of the second wall. For simplicity, the ribs between the orifices are neglected. The toroidal width of the blanket module is L and the poloidal width is W. There is an inlet opening between x=0 and x=a, and an outlet opening between x=L-a and x=L. Between x=a and x=L-a is the second wall, the electrical conductivity of which is lower than that of the coolant. But for simplicity, the conductivities are assumed to be the same here. Outside the boundaries of x=0, x=L, and y=±W/2, the conductivity is zero. The coolant velocity is uniformly w_b in the inlet and is $-w_b$ in the outlet.

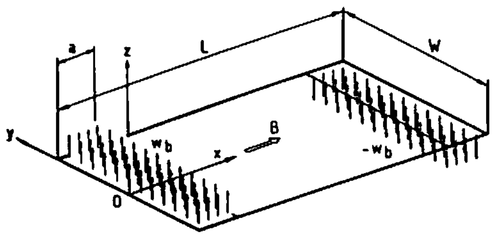

Fig.2. Slug flow model

As the current exists only in the x-y plane, the current distribution can be calculated numerically using a two-dimensional network model. The result depends on the aspect ratios W/a and L/a. Figure 3 shows the current distribution in the case of W/a=10 and L/a=10. The current direction is mainly the y direction, the direction of the electromotive force, in the orifices. The current turns the direction near the edge of the blanket, hence the current intensity is not uniform. The y-directional (poloidal) component of the current and the magnetic field cause an electromagnetic force in the z direction. The y-directional component can be written as follows:

$$i_y = C\sigma_f B w_b \tag{1}$$

where σ_f is the conductivity of the coolant and C is a function of the location. Figure 4 shows the distribution of the value of C. It takes the value zero on the boundaries, y=±W/2, because the current cannot flow in the y direction there. It is large near the edge, x=a, since there is a short circuit. The MHD pressure gradient in the coolant can be written as follows:

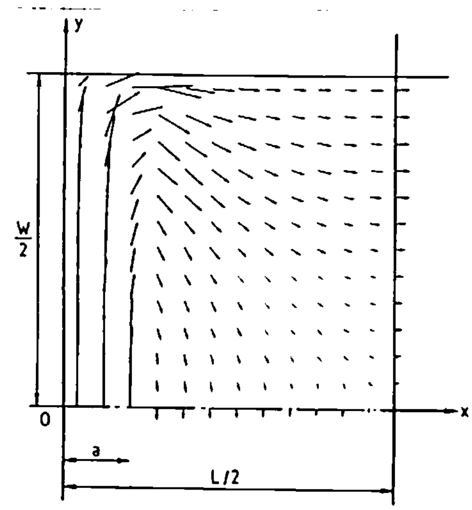

Fig.3. Eddy current distribution (W/a = 10, L/a = 10)

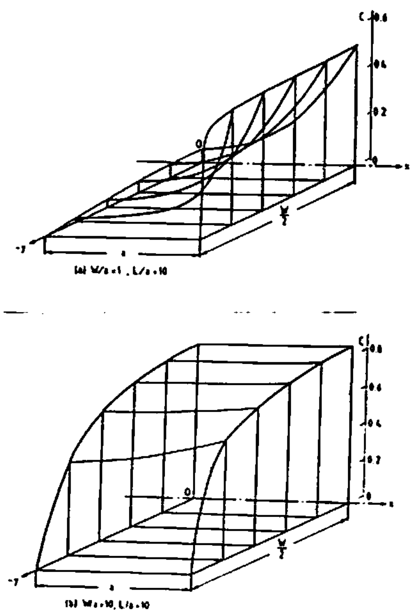

Fig.4. y-directional current intensity

$$-\frac{\partial p}{\partial z} = C\sigma_f B^2 v_b \qquad (2)$$

The nonuniformity of the value of C implies nonuniformity of the pressure gradient. If inter-channel flow redistribution could occur, the coolant would flow in orifices near the poloidal ends of the blanket module. But the bulk velocity should be the same in every orifice in order to cool the first wall uniformly. The value of C is almost uniform in the x direction around y=0 in the case of W/a=10 and L/a=10, which means that flow redistribution in the x direction is not likely to occur. The assumption of slug flow seems to be proper at least in the central orifices, hence we may use eq.2 and the value of C obtained by the model to calculate the average pressure drop in the orifice. The difference between Fig.4 (a) and (b) means that the aspect ratios are important. Figure 5 shows the average value of C in the middle orifice, which changes with the aspect ratios. It increases with increasing value of W/a, and reaches unity.

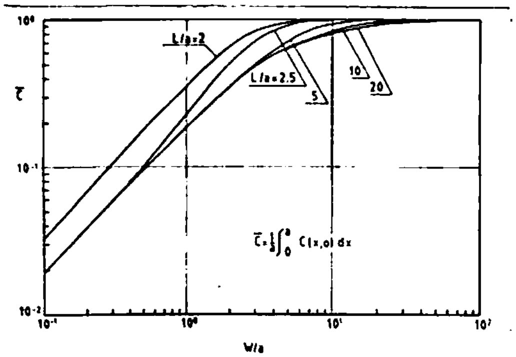

Fig.5. Average pressure gradient coefficient in middle orifice

4. FULLY DEVELOPED FLOW MODEL

Figure 6 shows the geometry and current paths used in the fully developed flow model. The number of orifices is k_{max} and the poloidal width is b. The wall on the right side of the orifice is far thicker than the ribs between the orifices. On the left side is assumed to be a nonconducting wall for simplicity. The conductivity of the ribs and the thick wall is σ_w, while that of the coolant is σ_f. The bulk velocity is v_b in every orifice.

The Navier-Stokes equation is written as follows:

$$\rho(\underline{v}\cdot\nabla)\underline{v} = -\nabla p + \underline{i}\times\underline{B} + \mu\nabla^2\underline{v} \qquad (3)$$

where ρ is the density and μ is the viscosity. The magnetic field and the current intensity are denoted by $\underline{B}(B,0,0)$ and $\underline{i}(i_x,i_y,0)$. As the flow is fully developed, the velocity is expressed by $\underline{v}(0,0,v)$. We can neglect the first term in fully developed flow. The last term is also negligible, except in boundary layers or free shear layers. Therefore, in the core region,

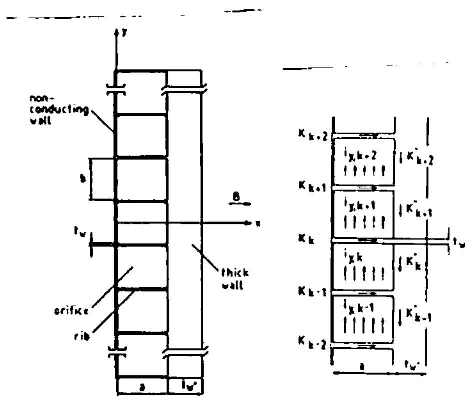

Fig.6. Fully developed flow model and current paths

$$\nabla p = \underline{i}\times\underline{B} \qquad (4)$$

Taking the curl of eq.4, we obtain:

$$\frac{\partial i_y}{\partial x} = \frac{\partial i_y}{\partial y} = 0 \qquad (5)$$

The above equations mean that i_y is constant in each channel except at boundary layers where eq.4 breaks down. The conservation of current can be written as follows:

$$\frac{\partial i_x}{\partial x} + \frac{\partial i_y}{\partial y} = 0 \qquad (6)$$

Substituting eq.5 into eq.6, we obtain:

$$\frac{\partial i_x}{\partial x} = 0 \qquad (7)$$

The x-directional (toroidal) component of the current i_x is zero on the surface of the non-conducting wall at x=0, therefore i_x is identically zero:

$$i_x \equiv 0 \qquad (8)$$

which means that the eddy current flows only in the poloidal direction in the core region of the orifice as shown in Fig.6. The current in the k-th rib $K_k(x)$ satisfies the following equation:

$$K_k(x) = (i_{y,k}-i_{y,k+1})x \qquad (9)$$

where $i_{y,k}$ is the y-directional current density in the k-th orifice. The electric potential difference between the k-th and (k-1)th ribs at x=a is given as follows:

$$\Delta\phi'_{e,k} = \frac{b}{\sigma_w t_w'} K_k' = \frac{ab}{\sigma_w t_w'} i_{y,k} \qquad (10)$$

where K_k' is the current in the thick wall between the k-th and (k-1)th ribs. As the magnetic field is parallel to the x direction, there can be no electromotive force in this direction. The electric potential in the coolant must be constant in the x direction. Therefore, the electric potential changes linearly along the y direction in the core region as it does in the thick wall. The following uniform z-directional velocity is necessary to induce the above potential.

$$v_{core,k} = \left(\frac{1}{\sigma_f} + \frac{a}{\sigma_w t_w'}\right) \frac{1}{B} i_{y,k} \qquad (11)$$

The electric potential in the k-th rib is higher than that at x=a by the following value:

$$\Delta\phi_{e,k} = \frac{i_{y,k} - i_{y,k+1}}{2\sigma_w t_w} (a^2 - x^2) \qquad (12)$$

The difference in the electric potential between the rib and the coolant in the core region near the rib must be compensated by the electromotive force in the boundary layer. Therefore:

$$\int_0^{\delta} v_{BL,k} B \, dy' = -\Delta\phi_{e,k-1} \qquad (13)$$

$$\int_{b-\delta}^{b} v_{BL,k} B \, dy' = \Delta\phi_{e,k} \qquad (14)$$

where y' is the distance from the surface of the (k-1)th rib and δ is the thickness of the boundary layer. From eqs.12~14, the total flow quantity in the layer in the k-th orifice is given as follows:

$$Q_{BL,k} = \frac{a^3}{3\sigma_w t_w B}(2i_{y,k} - i_{y,k+1} - i_{y,k-1}) \qquad (15)$$

The velocity in the core region of the k-th orifice is uniform and is expressed as follows:

$$v_{core,k} = v_b - \frac{1}{ab} Q_{BL,k} \qquad (16)$$

Substituting eqs.11 and 15 into eq.16, we obtain:

$$\frac{1+\phi_w'}{\phi_w'} i_{y,k} = \sigma_f B v_b - \frac{1}{3\phi_w} \frac{a}{b} (2i_{y,k} - i_{y,k+1} - i_{y,k-1}) \qquad (17)$$

where $\phi_w = \sigma_w t_w / \sigma_f a$ and $\phi_w' = \sigma_w t_w' / \sigma_f a$. The current intensity is zero on the outside of the blanket, hence:

$$i_{y,0} = i_{y,k_{max}+1} = 0 \qquad (18)$$

Solving eqs.17 and 18, we can obtain the current intensity in each channel.

The MHD pressure gradient in an orifice is proportional to the current intensity, and can be described as follows:

$$-\frac{dp}{dz} = C\sigma_f B^2 v_b \qquad (19)$$

where C is a function of k_{max}, etc. Figure 7 shows an example of the distribution of C. If there is a single orifice, i.e., $k_{max}=1$, C is calculated by eq.20.

$$C = \frac{1}{1 + \frac{2}{3}\frac{1}{\phi_w}\frac{a}{b} + \frac{1}{\phi_w'}} \qquad (20)$$

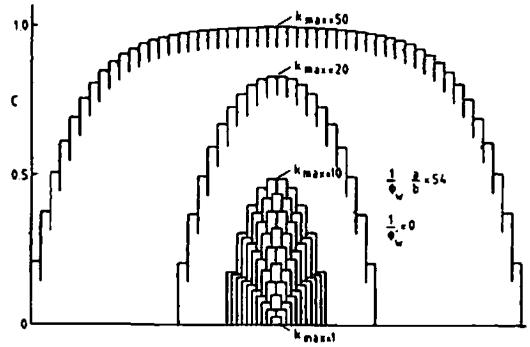

Fig.7. Pressure gradient coefficient

The value of C increases with increasing k_{max}. If there are more than 50 orifices, it reaches unity in the central orifices. It never exceeds unity even if there are more orifices. It is small in orifices near the poloidal ends of the blanket compared with that in central orifices. When $(1/\phi_w)(a/b)$ is larger, the rate of increase of C is low, but C inevitably reaches unity with a sufficiently large number of orifices. When $1/\phi_w'$ is not zero, C does not reach unity but reaches the following value:

$$C = \frac{\phi_w'}{1 + \phi_w'} \qquad (21)$$

The coolant velocity is very high near the rib parallel to the magnetic field. The direction of the boundary layer jet is not always the same as the bulk flow. Figure 8 shows the directions of the jets in the case of $k_{max}=5$. The jets flow in opposite directions with nearly equal velocities on both sides of the rib, which compensate the potential in the rib. The jet velocity decreases with increasing number of orifices, and finally the jets disappear in central orifices.

Slug flow appears there, which was assumed in the previous section.

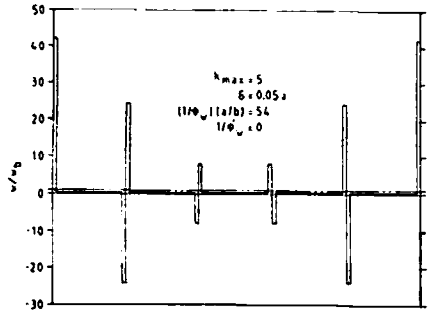

Fig.8. Velocity distribution in poloidal direction

5. EVALUATION OF THE MHD PRESSURE DROP

The MHD pressure drop in the second wall orifice of one of the BCSS liquid metal design is calculated using the values in Table 1. The MHD pressure gradient can be expressed in the same formula, eqs.2 and 19, in both cases with the slug flow and fully developed flow models.

The slug flow model showed that the value of C is almost unity in central orifices when W/a>>1. The electrical conductivity of the wall was assumed to be the same as that of the coolant in the calculation, though the latter is twice as large. But the difference of conductivity can be shown not to affect the result as long as W/a is very large.

The fully developed flow model also showed that the value of C is almost unity in central orifices when there are many orifices and δ_w' can be regarded as zero. If the toroidal length of the second wall is used as the thick wall thickness t_w', the value of C is calculated by eq.21 to be 0.97, which is approximately unity.

Table 1. Parameters of BCSS blanket

Toroidal width of orifice*	a	45 mm
Poloidal width of orifice	b	25 mm
Rib thickness	t_w	3 mm
Toroidal width of blanket module	L	3 m
Poloidal width of blanket module	W	9.3 m
Second wall width**	t_w'	2910 mm
Second wall thickness		15 mm
Number of orifices	k_{max}	332
Magnetic field strength	B	7.5 T
Bulk velocity in orifice	v_b	1.56 m/s
Conductivity of coolant	σ_f	3.0 MS
Conductivity of wall	σ_w	1.5 MS

* As this value is not given in the BCSS design, the above value is assumed.

** $t_w' = L - 2a$

Both models showed that the pressure gradient coefficient C is almost unity in the central orifices of the BCSS blanket. This fact suggests that the pressure drop with the real velocity distribution may also be the same. The pressure drop in orifices near the poloidal ends of the blanket module is affected by the calculational model, but it is lower than that of the central orifices. The higher pressure drop is more critical, hence we discuss the value in the central orifices in the following.

We assumed that the ribs are thin and the resistivity across them can be neglected, but in the BCSS blanket, the ribs are not so thin. If we take this effect into account, eq.22 should be used instead of eqs.2 and 19.

$$-\frac{dp}{dz} = C\,\frac{b}{b/\sigma_f + t_w/\sigma_w}\,B^2 v_b \qquad (22)$$

Using this equation and the value of C=1, the pressure gradient is calculated to be 212 MPa/m. As the second wall thickness is 15 mm, the MHD pressure drop in the orifice is calculated to be 3.18 MPa. The error in this value is considered to be at most several percent.

On first inspection, the pressure drop in the orifice seemed to be negligible compared with that in the toroidal channel and the poloidal manifold. However, the total pressure drop in the blanket exclusive of the orifices has been calculated to be 3.03 MPa. It is obvious that the pressure drop in the orifice cannot be neglected.

The velocity in the orifice of 1.56 m/s is about 5 times as high as that in the poloidal manifold. This is a part of the reason why the pressure gradient is so high. But the value of C is about 100 times as large as that in the manifold. The major reason of the high pressure gradient in the orifice is the high value of C.

6. DISCUSSIONS

The reason for the high pressure drop in the orifice is that eddy current can flow across the channels. If there was a single channel, or if the channel was insulated from neighboring channels, the pressure drop would not be so high. In liquid metal blanket designs, we usually expect that the MHD pressure gradient can be expressed in the following equation:

$$-\frac{dp}{dz} = \sigma_f B^2 V\,\frac{\phi_w}{1+\phi_w} \qquad (23)$$

where B is the magnetic field strength perpendicular to the flow of velocity V, and ϕ_w is the wall conductance ratio. Since ϕ_w is usually between 0.01~0.1, the MHD pressure gradient is 1~10% of the value in the perfectly conducting wall case. The equation is proper in a single channel. We can use eq.23 in the case with parallel channels, as long as eddy currents do not leak from the channel. But we cannot use it when eddy currents leak to neighboring channels and/or structures which are in contact with the channel.

Parallel channels partitioned by common walls as shown in Fig.9 are often used in fusion blankets. If the common walls are split and the channels are divided into separate channels, each wall carries a current in different direction. But in the parallel channels in contact with each other, the currents are canceled and disappear. The electric potential distribution along the wall also disappears and it becomes uniform. The contact of the channels has the effect of making the wall an ideal conductor.

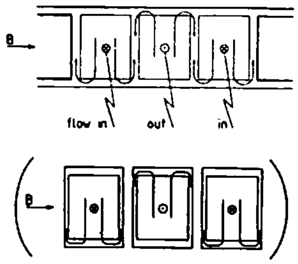

Fig.9. Current paths in parallel channels

Even if currents leak, the MHD pressure drop is low as long as the currents have to flow in thin walls which have high resistance. But the wall perpendicular to the magnetic field is very thick in the case of the second wall orifice. The currents do not have to flow in thin walls, and so the current intensity is high. The MHD pressure drop is 10~100 times higher than that in the case the currents do not leak. Therefore, the pressure drop is high even though the flow length is short. This example is an extreme case. But whenever currents leak, the MHD pressure drop is always affected.

In current designs of liquid metal blankets, leakage current effects are usually neglected. As they are undesirable, a fusion reactor blanket should be designed so that the effects are minimized. But even if it is designed carefully, there may be an unknown current path. Further studies are necessary on the leakage current effects.

7. CONCLUSIONS

The MHD pressure drop in the orifice of BCSS blanket was calculated, and the leakage current effects were shown to be critical.

The MHD pressure drop in the second wall orifice of one of the BCSS designs was calculated with a slug flow model and a fully developed flow model. As there is no insulation between the orifices, eddy currents leak to neighboring orifices. The current return path is the second wall, the cross-sectional area of which is very large. Since the resistivity of the current path is low, the current intensity is high, which induces a very high pressure drop of more than 3 MPa.

When currents leak from the channel, the MHD pressure drop is affected. As it is very difficult to insulate each cooling channel from the remainder of the blanket, special attention should be given to the leakage current effects.

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