Diodes

Junction diodes

We will study the diode as the first example of a non-linear element. It is a 2-terminal device, but the current-voltage relationship is such that it depends on the amplitude of the voltage. Remember, for linear elements, this relationship (i.e. the system response) does not depend on the amplitude of the input signal. On a current-voltage graph as above, a linear element would be represented by a straight line through the origin (e.g. for a resistor, capacitor or inductor).

We can model the behavior of a diode at low frequencies as:

For both forward and reverse bias:

\[
i_D = I_S \cdot \left( \frac{v_D}{e^{nV_T}} - 1 \right) \quad v_D = nV_T \cdot \ln\left( \frac{i_D}{I_S} + 1 \right)
\]

\[
\begin{cases}
  I_S & \text{saturation current} \\
  V_T & \text{thermal voltage } = \frac{kT}{q} \approx 25 \text{ mV} \text{ (at room temperature)} \\
  n & \text{typically } n \sim 1 \text{ for ICs, and } n \sim 2 \text{ for discretes}
\end{cases}
\]

Approximations:

\[
\begin{align*}
  v_D & >> V_T & i_D &= I_S \cdot \frac{v_D}{e^{nV_T}} \\
  v_D & << -V_T & i_D &= -I_S
\end{align*}
\]
Large signal approximations

Although the model discussed earlier captures most of the (low frequency) behavior of the diode, it is a non-linear equation. As such, the resulting circuit equations cannot be solved analytically. As the diode iv characteristics cannot be approximated with ONE linear equation (i.e., a line), a “piecewise linear” model is usually used: different portions of the iv characteristics are approximated with straight lines. An example is the “constant voltage” model:

\[
\begin{align*}
(1) \quad v_D &\leq V_{D0} \Rightarrow i_D = 0 \quad \text{(reverse bias)} \\
(2) \quad i_D &\geq 0 \Rightarrow v_D = V_{D0} \quad \text{(forward bias)}
\end{align*}
\]

A typical value for Si junction diode \( V_{D0} = 0.7 \text{ V} \).

To solve circuits that contain diodes, we can use these large signal approximations (btw, this nomenclature will become clear later when we introduce small signal models) and the following method: select a hypothesis and verify at the end.

1. hypothesis reverse bias: \( i_D = 0 \)
   check \( v_D \leq V_{D0} \)
2. hypothesis forward bias: \( v_D = V_{D0} \)
   check \( i_D \geq 0 \)

Examples

1. \( V_{D0} = 1 \text{ V} \)
   \( n = 1 \)

   \[ \text{Diagram} \]

2. \( V_1(V) = 7 \cdot \sin(\omega t) \)
   \( V_2(V) = 5 \)
   \( V_{D0} = 1 \text{ V} \)
   \( n = 1 \)

   \[ \text{Diagram} \]
**Small signal model: basic principle**

**Motivating example**

Consider the circuit below with a diode with $V_{D0} = 0.4$ V. Let’s calculate the response of the circuit ($V_{out}$) to the following input signals (all in V):

1. $V_{in} = 1 \cos(\omega t)$
2. $V_{in} = 1 + 10^{-3} \cos(\omega t)$
3. $V_{in} = 2 + 2 \times 10^{-3} \cos(\omega t)$

Let’s first analyse the circuit using the “constant voltage” diode model.

**Case 1.** $V_{in} = 1 \cos(\omega t)$

If $V_{in} < V_{D0} = 0.4$ V, diode is OFF, $i_D = 0$ and $V_{diode} = V_{in} = 1 \cos(\omega t)$

If $V_{in} > V_{D0} = 0.4$ V, diode is ON, $i_D > 0$ and $V_{diode} = 0.4$ V

As can be seen, the circuit is non-linear. While the input signal is sinusoidal, the output is a sinusoidal signal with values above 0.4 V clipped.
**Case 2 & 3.**

\[ V_{in} = 1 + 10^{-3} \cos(\omega t) \text{ and } V_{in} = 2 + 2 \times 10^{-3} \cos(\omega t) \]

Since \( V_{in} > V_{Do} = 0.4 \text{ V} \), diode is ON, \( i_d > 0 \) and \( V_{diode} = 0.4 \text{ V} \)

Again, we see the non-linear behavior of the circuit as when the input signal is doubled, the output signal has remained the same.

We can simulate cases 2 and 3 with Pspice (PSpice model is very close to the exponential relationship of page 2-1). To make one simulation, we assume \( V_{in} = 1 + 10^{-3} \cos(\omega t) \) for \( t < 1 \text{ s} \) and \( V_{in} = 2 + 2 \times 10^{-3} \cos(\omega t) \) for \( t > 1 \text{ s} \). The simulation results is shown below. We note that PSpice appears to confirm our analysis that and \( V_{diode} \sim 0.4 \text{ V} \).

![Simulation Results](image)

Note that although \( V_{in} = 1 + 10^{-3} \cos(\omega t) \) for \( t < 1 \), for example, the sinusoidal wave does not show on the plot above because \( 10^{-3} \cos(\omega t) \) is much smaller than 1. Let’s now “zoom” on the \( V_{diode} \) plot. We see that the output signal composed of two parts, a “DC” part and a sin wave (similar to the input signal).

![Zoomed Plot](image)
In fact, we find that for \( V_{in} = 1 + 10^{-3}\cos(\omega t) \), \( V_{diode} = 0.386245 + 0.04 \times 10^{-3}\cos(\omega t) \). Interestingly, we simulate the circuit with \( V_{in} = 1\) V, we get \( V_{diode} = 0.386245\) V.

It appears that when the input is made of two components: 1 and \( 10^{-3}\cos(\omega t) \), the output is also made of two parts: 0.386245 and \( 0.04 \times 10^{-3}\cos(\omega t) \) with \( V_{diode} = 0.386245\) V correspondingly to the response of the circuit to \( V_{in} = 1\) V and \( 0.04 \times 10^{-3}\cos(\omega t) \) appear to be the response of the circuit to the smaller input signal of \( 10^{-3}\cos(\omega t) \). This sine-in sine-out behavior reminds us of linear circuits. This is surprising since we know the diode is a non-linear element! The reason for this behavior is that we are looking at small changes at the input, resulting in small changes at the output. **Apparently, for these small changes, our non-linear circuit behaves as a linear one!**

This important behavior can be understood as follows: if we are interested in small changes only, we can get some very useful information by approximating the non-linear curve by its tangent line.

E.g., if I would give you \( r_D \) and \( (V_2 - V_1) \), you can find \( (I_2 - I_1) \), even if you don’t know the exact value of \( V_1 \) or \( I_1 \).

This is called **small signal behavior**. It is one of the most important concepts in circuit design, i.e., **For small signals, non-linear circuits behave as linear ones**. To capture this important behavior, we need to use specific models that target small signals.

**Generic derivation**

To introduce the concept of small signal models, consider a generic system with input-output relationship \( f() \). This system can be linear or non-linear. Note that \( f() \) is not the impulse response.

Assume the input is composed of a large signal component and a small signal component. What ‘large’ and ‘small’ exactly mean, will be formalized a bit later. For now, you can think if it as, e.g., a voltage signal of \( 2\) V + 1 mV.

\[
x_A = X_A + x_a
\]

\[
x_A = \text{large signal}
\]

\[
x_a = \text{small signal}
\]

By definition, the output signal is \( y_A = f(x_A) \). We define \( Y_A = f(X_A) \) to be the response of the circuit when no small signal is present. We now define \( y_a \) such that \( y_A = f(x_A) = Y_A + y_a \). Thus \( y_a \) is the “additional” response of the circuit due to our small signal input.
Taylor series

\[ y_A = f(x_A) \]
\[ = f(X_A) + f^{(1)}(X_A) \cdot (x_A - X_A) + \frac{f^{(2)}(X_A)}{2!} \cdot (x_A - X_A)^2 + ... \]
\[ = f(X_A) + f^{(1)}(X_A) \cdot x_a + \frac{f^{(2)}(X_A)}{2!} \cdot x_a^2 + ... \]
\[ \approx f(X_A) + f^{(1)}(X_A) \cdot x_a \]

Therefore:

\[ y_A = f(x_A) = f(X_A + x_a) = f(X_A) + f^{(1)}(X_A) \cdot x_a = Y_A + y_a \]

\[ \begin{cases} 
Y_A = f(X_A) & \text{large signal behavior (can be non-linear)!} \\
y_a = g(x_a) = f^{(1)}(X_A) \cdot x_a & \text{small signal behavior (linear!)}
\end{cases} \]

So the output contains a large and small signal component as well. Each one of these components can be calculated separately, although not necessarily through the same input-output relationship. Note that this is different from superposition (which can only be used for linear systems).

Condition for a signal to be considered small:

\[ \left| f^{(1)}(X_A) \cdot x_a \right| >> \left| \frac{f^{(2)}(X_A)}{2!} \cdot x_a^2 \right| \quad \Rightarrow \quad |x_a| << 2 \cdot \frac{f^{(1)}(X_A)}{f^{(2)}(X_A)} \]

Physical interpretation:

\[ y_A = Y_A + y_a = f(X_A) + f^{(1)}(X_A) \cdot x_a \]
**Diode small signal model**

Next, we apply the large-small signal theory to diodes, as an example of a non-linear element.

\[
i_D = I_S \cdot \left( \frac{v_D}{nV_T} - 1 \right) \quad \text{or} \quad f(x) = I_S \cdot \left( \frac{x}{e^{nV_T}} - 1 \right)
\]

Large signals:

\[
I_D = f(V_D) = I_S \cdot \left( \frac{v_D}{nV_T} - 1 \right)
\]

or use the large signal approximation with two line segments (this is what is used most frequently).

Small signals:

\[
i_D = f^{(1)}(V_D) \cdot v_d = \left[ I_S \cdot \frac{e^{v_D}}{nV_T} \right]_{x=V_D} \cdot v_d = \left[ I_S \cdot \frac{v_D}{nV_T} \right] \cdot v_d = \left[ \frac{I_D + I_S}{nV_T} \right] \cdot v_d
\]

Small signal model:

\[
i_d = \frac{v_d}{r_D} \quad \quad r_D = \frac{nV_T}{I_D + I_S} \quad \text{(forward or reverse bias)}
\]

\[
r_D \approx \frac{nV_T}{I_D} \quad \text{(in forward bias only!)}
\]

So, the small signal equivalent model for a diode is a resistor \( r_D \), the value of which depends on the bias condition. In other words, for small signals, the diode behaves as if it is a resistor, but the resistor value depends on the large signal bias of the diode.
What constitutes a small signal?

\[ |v_d| \ll 2 \cdot \frac{f^{(1)}(V_D)}{f^{(2)}(V_D)} = 2 \cdot \frac{I_S \cdot e^{aV_T}}{nV_T} \cdot \left( nV_T \right)^x = 2nV_T \]

So roughly anything around 10 mV or smaller behaves as a small signal for diodes. In terms of currents, anything in the range of \( \mu A \) would be small.

**How to use small signal model:**

It is essential to remember that small signal model is applicable ONLY to the small signal input. To solve a circuit, we have divide the problem into:

1) **Response to Large Signals:** Set, the small signal source is set to zero and use large signal model to find the Large signal response of the circuit.

2) **Response to Small Signals:** Set, the large signal source is set to zero and use small signal model to find the small signal response of the circuit.

3) The Total response of the circuit is the sum of two.

Note that to set a voltage source to zero, it becomes a short to ground (i.e. we can replace the DC voltage source with a ground when considering small signals). To set a current source to zero, we replace it with an open circuit.

**Example** (same circuit as that of pages 2-3 to 2-5)

\( V_{in} = 1 \, \text{V} + 10^{-3} \cos(\omega t) \, \text{V} \)

**Large signals**

\( V_{in} = 1 \, \text{V} \quad \Rightarrow \quad V_{diode} = V_{D0} = 0.4 \, \text{V} \)

\( V_{out} = V_{in} - V_{diode} = 0.6 \, \text{V} \)

\( I_D = 0.6 \, \text{V} / 1 \, \text{k} \Omega = 0.6 \, \text{mA} \)

\( \Rightarrow \quad r_D = 41.67 \, \Omega \)

**Small signals**

\[ \frac{v_{diode}}{v_{in}} = \frac{r_D}{r_D + 1 \, \text{k} \Omega} = 0.04 \]

Or by voltage divider formula \( v_{diode} = 0.04 \times 10^{-3} \cos (\omega t) \)

Summing the responses to Large and Small signals: \( V_{diode} = 0.4 + 0.04 \times 10^{-3} \cos(\omega t) \, \text{V} \) which is very similar to PSpice result (page 2-5) of \( V_{diode} = 0.386245 + 0.04 \times 10^{-3} \cos (\omega t) \, \text{V} \).
Comparing these results, we note that the first term is approximate (0.4 vs 0.39) since it is based on an approximate large signal model (although this accuracy is quite sufficient for most problems). If we had used a more accurate large signal model (e.g., the exponential one), we could have found this value more exactly as well (e.g., for MOSFETs, the large signal models we will use will be more exact). Considering the small signal components, we notice that our model is able to predict those fairly well.

**Summary:** Using large + small signal analysis allows us to capture the fact that small signal changes manifest themselves in a linear way at the output. The small signal model predicts these small signal relationships. On the other hand, for the large signals, we will need to use the non-linear large signal models. How well we can predict these two components (large or small) will depend on the accuracy of the underlying models.

Strictly speaking, the Large signal should be constant (i.e., a DC signal). This DC value is usually referred to as the bias condition. The signals that carry the interesting information are the small signals; and we are interested in the relative changes around the large-signal bias conditions. These small signals are often AC signals although they can be small DC signals, e.g., see the example below for a voltage-controlled attenuator). For circuits with AC small signals, it is possible to use (sufficiently large) capacitors, called coupling capacitors, to couple the small signals to the circuit at input and output. For the DC bias, these capacitors behave as open circuits. For the AC signals, these capacitors behave as approximately as short circuits (assuming the frequency is high enough and the capacitor is large). Note that the behavior of the capacitors only results from the frequency of the signals (not whether they are large or small).

Lastly, if the small signal is an AC signal, the “large” component of the signal has to be constant only in the context of the period of the AC signal (i.e., the “large” signal can be slowly-varying such that it is almost a constant over several period of the AC signal). Note that it in this case, value of $r_D$ will change slowly (corresponding to the changes in the large signal).

**Example:** In this circuit, $v_s$ is a small signal sinusoidal source, and capacitors are large, find $v_0$

![Circuit Diagram]

**Large Signal Analysis:**
Small signal = 0,
Caps open (large signal is DC)

$I_D = I$
Small Signal Analysis
Large signal = 0 (current source become an open circuit),
Large capacitors: capacitor impedances are small.
Replace the diode with its small signal model,

By voltage divider formula, we get \( v_o = \frac{r_d}{r_d + R_s} = v_s \)

Where \( r_D \approx \frac{nV_T}{I_D} = \frac{nV_T}{I} \)

**Another Example:** Voltage-controlled Attenuator:
In this circuit, \( v_s \) is a small signal sinusoidal source, \( V_c \) is a large DC sources (its value can be changed) and capacitors are large, find \( v_0 \)

Large Signal Analysis:
Small signal = 0,
Caps open (large signal is DC)

\[ I_D = \frac{V_C - V_{D0}}{R_C} \quad r_D \approx \frac{nV_T}{I_D} \]

Small Signal Analysis
Large signal = 0 (\( V_c \) source become a short circuit),
Large capacitors: capacitor impedances are small.
Replace the diode with its small signal model,
Note that \( R_C, r_D, \) and \( R_L \) are in parallel. Defining \( R_p = R_C \parallel r_D \parallel R_L \)

By voltage divider formula, we get \( \frac{v_o}{v_i} = \frac{R_p}{R_p + R} \)

If \( r_D \ll R_L \) and \( r_D \ll R_C \), then \( R_p \approx r_D \) and

\[ \frac{v_o}{v_i} = \frac{r_D}{r_D + R} \]

Note: If \( V_c \) increases, \( I_D \) increases (see large signal analysis) reducing \( r_D \) and decreasing \( v_o \). If \( V_c \) decreases, \( I_D \) decreases (see large signal analysis) increasing \( r_D \) and increasing \( v_o \).

An application of this circuit is in a speakerphone. A frequent problem is that some speakers speak quietly (or are far from the microphone) and some speak loudly (or are close). If \( v_i \) is the output of the microphone and \( v_o \) is attached to a high-gain amplifier and phone system, control voltage can compensate for changes in \( v_i \) (\( V_c \) can be, for example, the output of a peak detector circuit with \( v_i \) as the input).
**Small signal capacitance**

Thus far, we have studied diodes for small and large signals, but only in quasi-steady state behavior, i.e. moderate frequencies. To understand their high frequency behavior, we have to look a bit into the device physics.

**Open circuit**

pn-junction: p-type and n-type material

Each type contains majority carriers and minority carriers, as well as ionized dopant atoms. The concentrations are such that the material is neutrally charged.

In n-type silicon:  
\[ n >> p \]  
\[ N_D = n - p \]  
e\(^-\) (electrons) are majority carriers  
dopant ions are positively charged (donors)

In p-type silicon:  
\[ p >> n \]  
\[ N_A = p - n \]  
e\(^+\) (holes) are majority carriers  
dopant ions are negatively charged (acceptors)

At the junction, majority carriers will experience a stark difference in concentration and start diffusing to the other side.

\[ e^- \text{ diffuse from n-type to p-type} \]
\[ e^+ \text{ diffuse from p-type to n-type} \]

\[ \rightarrow \] on the other side of junction, they recombine with majority carriers on that side

\[ \rightarrow \] region around junction is depleted of free charges and only the fixed ions remain

\[ \rightarrow \] depletion region (or space-charge region): negatively charged in p-type and positively charged in n-type

\[ \rightarrow \] electrical field from n-type to p-type

\[ \rightarrow \] internal barrier voltage over the junction: voltage drop from n-type to p-type

\[ \rightarrow \] electrical field opposes the diffusion of majority carriers

\[ \rightarrow \] the net diffusion of carriers is represented as diffusion current \( I_D \). Its value is a function of the electrical field.

In addition, in each material, the minority carriers move around. When they happen to cross into the depletion region, they experience the electrical field that sweeps them to the other side.

\[ \rightarrow \] This gives rise to a current, called the drift current \( I_S \). This current is a function of the number of minority carriers (which depends on temperature). It does not depend on the strength of the electrical field (the field just needs to sweep them across).

In equilibrium and when no external voltage is applied to the diode, these currents perfectly cancel each other and no net current flows:  
\[ I_D = I_S \]  
\[ I = I_D - I_S = 0 \]
Reverse bias

A negative voltage is applied over the diode
- increased voltage drop over the junction compared to just the internal barrier voltage
- depletion region expands, which increases the electrical field
- lowers the diffusion current \( I_D \) to almost zero (the drift current remains the same).
- small negative current flows through the diode in reverse bias: \( I = I_D - I_S \approx -I_S \)

All of this relates to steady state. When the voltage changes in reverse bias, the charge stored in the depletion region has to change (as the depletion region expands or shrinks). This relationship between voltage and charge increase is captured by the notion of capacitance. In general:
\[
Q = C \cdot V
\]

When looking at the depletion region, we see that it indeed resembles a parallel plate capacitor. However, since the width of this region also varies with the voltage, it does not behave really as a parallel plate capacitor you are familiar with; the distance between the plates themselves changes.
- charge \( Q \) – voltage \( V \) relationship is non-linear
- represents a non-constant capacitance, called the junction capacitance (it decreases as the diode voltage becomes more negative)
- captures the following effect: when the reverse bias voltage is changed, extra charge needs to be provided or taken away. The result is an extra transient current needed to (dis)charge the depletion region. This is different from the steady state current \( I = I_D - I_S \).

We are often interested in only small changes around an operating point (i.e. what happens when the reverse bias voltage is changed only a little bit). The change in charge and resulting current can then be modeled by a small signal capacitance. It can be interpreted as the tangent line to the non-linear \( Q-V \) curve:
\[
C_j = \frac{dQ}{dV}
\]

From device physics:
\[
C_j = \frac{C_{j0}}{\left(1 - \frac{V_D}{V_0}\right)^m}
\]

\( V_D \) bias voltage over the diode (negative in reverse bias)
\( C_{j0} \) small signal junction capacitance when there is zero voltage over the diode
\( V_0 \) internal junction voltage when there is zero voltage over the diode
\( m \) exponent that depends on how the concentration of dopants changes across the junction. Typical values are from 1/3 to 1/2.

The expressions of \( C_{j0} \) and \( V_0 \) as a function of the physical parameters of the device can be found in the book.
Forward bias

A positive voltage is applied over the diode
→ decreased voltage drop over the junction
→ shrinking of the depletion layer, which lowers the electrical field
→ diffusion current increases, resulting in a net positive current in forward bias: \( I = I_D - I_S \)

As in reserve bias, the depletion region charge has to change, giving rise to a junction capacitance. The same equation could be used, but it turns out that the accuracy is rather poor. Instead, circuit designers often use the following rule of thumb for the small signal junction capacitance in forward bias:

\[
C_j \approx 2 \cdot C_{j0}
\]

In addition to the junction capacitance, there is another charge-related phenomenon in forward bias. The increased diffusion current pushes more majority carriers from one side to the other, where they are now minority carriers. They will then diffuse further and slowly recombine. The resulting concentration of minority carriers is shown in the figure below.

→ these extra carriers again represent a charge, of which the value depends on the voltage (since the diffusion current depends on the voltage).
→ we can capture this charge – voltage relationship as a capacitance: diffusion capacitance

Here as well, we are mainly interested in the small signal behavior around a bias point. The small signal diffusion capacitance can be found as

\[
C_d = \frac{\tau_T \cdot I_D}{V_T}
\]

\( I_D \) bias current through the diode
\( \tau_T \) transit time

Note that there is no diffusion capacitance in reserve bias.
High frequency diode models
The large and small signal models that we discussed earlier in this chapter, do not take into account the capacitive effects, and therefore strictly speaking only apply to low frequency behavior.

When studying large signals at high frequencies, we need to add the non-linear junction and diffusion capacitances to the models we used before (e.g. the exponential I-V relationship).

For small signals, the high frequency equivalent model of the diode is shown below. It consists of the small signal resistance in parallel with the small signal capacitances.

\[
C_j + C_d
\]
\[
r_D = \frac{nV_T}{I_D + I_S}
\]
\[
C_j \approx 2 \cdot C_{j0}
\]
\[
C_d = \frac{\tau_T \cdot I_D}{V_T}
\]

All these small signal values depend on the bias conditions. By changing the bias, we can therefore manipulate the small signal behavior.

Note: Coupling capacitors are much larger then the junction or diffusion capacitances. At moderate frequencies, we typically use the low-frequency model of the diode (i.e. without capacitances), but consider the effect of coupling capacitors. Only at high frequencies do we also need to take into account the capacitive effects of the diodes themselves.

Example: VCO
Many communication circuits contain a voltage controlled oscillator (VCO). They can be based on the resonance of an LC tank. Suppose this is achieved by using a diode in reverse bias, in parallel with an inductor (there are other components as well to sustain the oscillation, but we focus here just on the resonance behavior). For small signals, this combination will behave as an RLC circuit.

\[
\text{resonance frequency is } f_{\text{res}} = \frac{1}{2\pi} \cdot \frac{1}{\sqrt{LC}}
\]

We are given: \(C_{j0} = 530 \text{ fF}, V_0 = 0.73 \text{ V}, m = 0.5, L = 11.9 \text{ nH.}\)

When \(V_D = 0\) (diode bias voltage): \(C_j = 530 \text{ fF and } f_{\text{res}} = 2 \text{ GHz.}\)

When \(V_D = -2\text{V:}\) \(C_j = 274 \text{ fF and } f_{\text{res}} = 2.79 \text{ GHz.}\)

By varying the bias voltage of the diode, we are able to change the oscillating frequency.