Analyzing Amplifier Circuits

Now, we will look at how we can build basic amplifier circuits using MOSFETs. When considering an amplifier circuit, it is important to realize how its behavior depends on the source and load resistances. All of what follows here is for small signals (since only for small signals are these amplifiers linear)!!!!

We use the following notations:

- $R_{out}$: The output resistance of the amplifier, with source $R_{sig}$ attached. This is the resistance seen by the load when looking into the output of the amplifier.
- $R_{in}$: The input resistance of the amplifier, with load $R_L$ attached. This is the resistance seen by the source when looking into the input of the amplifier.
- $v_{out}$: The output voltage of the amplifier, with load $R_L$ attached.
- $A_v = \frac{v_{out}}{v_{in}}$: The gain from the input to the output, with load $R_L$ attached.

[P.S., we use $R_{sig}$ here for the source resistance, since $R_S$ is the typical symbol used for the degeneration resistor attached to the S terminal of the MOSFET].
Thevenin Equivalent Model (for small signals)

To study how the amplifier interacts with other elements, it is easier to replace it by its Thevenin equivalent. The parameters of this model are defined as follows:

- \( R_{in} \): the input resistance of the amplifier with the load attached.
- \( R_o = R_{out}|_{R_L=0} \): the output resistance of the amplifier proper (i.e. with an ideal source)
- \( A_{vo} = A_v|_{R_L=\infty} \): the gain of the amplifier proper (i.e. without load attached)

The Thevenin model presented above is generic and valid for any type of amplifier. A special case is a unilateral amplifier. This is defined as an amplifier where the source does not affect the output resistance, and the load does not affect the input resistance. Therefore, for a unilateral amplifier: \( R_o = R_{out} \). Fortunately, it turns out that all the amplifiers we will encounter in this course are unilateral.

With this Thevenin equivalent model, the behavior of the amplifier when it is connected to other circuit elements can be calculated directly. Again, we’d like to emphasize that this analysis is for small signals only.
Note: For a voltage amplifier, we prefer
- High gain:
- Large input resistance: Ideally $R_{in} = \infty \Rightarrow v_{in} = v_S$
- Small output resistance: Ideally $R_{out} = 0 \Rightarrow v_{out} = A_{vo} \cdot v_{in}$

Similarly, we can analyze the effect (on the small signals) of cascading amplifiers, by replacing them by their Thevenin equivalent model.

For MOSFETs, all four basic amplifier configurations will prove to be unilateral, i.e. $R_o = R_{out}$ and $R_{in}$ does not depend on $R_L$. Since all configurations are unilateral, we can analyze each one independently and analyze the cascaded circuit in any direction.

For BJTs, not all amplifiers are unilateral. In this case, we have to analyze the circuit from right to left, as the input resistance of one stage may depend on the stage attached to it (understand why). This makes cascading stages more tricky for BJTs.
This configuration serves as the gain stage. The disadvantage is high output impedance.

BTW, to be a good voltage amplifier, we want $R_{in}$ large, $R_o$ small and $A_{vo}$ large.

Note: Capacitor $C_S$ is included such that the stage is connected to a current source for biasing (btw, other biasing schemes could be used as well). However, for small signals (at sufficiently high frequencies), this capacitor behaves as a short. As such, for small signals, S is grounded.
Common-Source Configuration with Source Resistance

\[ V_{dd} \]
\[ R_{in} \]
\[ C_{c1} \]
\[ v_{in} \]
\[ v_{gs} \]
\[ v_{ss} \]
\[ g_m \cdot v_{gs} \]
\[ R_{sig} \]
\[ R_{in} \]
\[ R_{G} \]
\[ C_{G} \]
\[ R_{D} \]
\[ R_{L} \]
\[ V_{out} \]

\[ R_{in} = R_{G} \]

\[ R_o = R_D \]

\[ A_v = \frac{g_m \cdot R_D}{1 + g_m \cdot R_S} \]

Feedback lowers the gain of the stage, compared to a pure common-source configuration. However, the benefit is an increased linearity and better high frequency behavior.
This amplifier provides gain and is useful when a specific (low) $R_{in}$ is required. This is, e.g., the case when the impedance needs to be matched, as with transmission lines (e.g. to 50 $\Omega$). Another application of the CG configuration is that it acts as a current buffer (current gain close to unity, small $R_{in}$, large $R_{out}$).
Source Follower (Common-Drain Configuration)

This configuration acts as a voltage buffer. It provides no gain, but has low output impedance. It is typically the last stage in a multi-stage amplifier.
Examples

Example: Source follower

\[
A_v = \frac{g_m \cdot r_o}{1 + g_m \cdot r_o}
\]

\[
R_o = \frac{1}{g_m} \parallel r_o
\]

\[
R_{in} = R_G
\]

\[
\frac{v_{out}}{v_{in}} = A_v = A_v \cdot \frac{R_L}{R_L + R_o} = \frac{g_m \cdot r_o}{1 + g_m \cdot r_o} \cdot \frac{R_L}{R_L + 1/g_m \parallel r_o} = \frac{g_m \cdot (r_o \parallel R_L)}{1 + g_m \cdot (r_o \parallel R_L)}
\]

\[
\frac{v_{out}}{v_S} = \frac{R_{in}}{R_{in} + R_{sig}} \cdot A_v
\]

Example: Cascaded amplifier
By grouping the different factors in this expression, we can find a physical interpretation for the cascading. This physical interpretation can be used to guide simulation or analysis of the different stages separately, before combining them into a cascaded amplifier.

\[
\frac{v_{out}}{v_S} = \frac{v_1}{v_S} \cdot \frac{v_2}{v_2} = \left[ \frac{R_{in1} \cdot A_{vo1} \cdot R_{in2}}{R_{in1} + R_S} \right] \cdot \left[ \frac{R_{in2} \cdot A_{vo2} \cdot R_L}{R_{in2} + R_{o1}} \right] \cdot \left[ A_{vo2} \cdot \frac{R_L}{R_L + R_{o2}} \right]
\]

Gain of stage 1 with actual source and loaded by stage 2

Gain of stage 2 with ideal source and loaded by \( R_L \)
Earlier, we analyzed a current mirror. This analysis was valid for all types of signals, large or small. However, it is instructive to look at how this circuit behaves for small signals in particular. Specifically, we can calculate the small signal resistance seen when looking in the two terminals:

\[
R_1 = \frac{1}{g_{m1}} / / r_{o1} \approx \frac{1}{g_{m1}}
\]

\[
R_2 = r_{o2}
\]

Also, we see that the current mirroring action occurs for small signals as well. If we assume the load \( R_L \) is small:

\[
v_1 = i_{in} \cdot R_1 \approx i_{in} \cdot \frac{g_{m1}}{g_{m1}}
\]

\[
i_{out} = g_{m2} \cdot v_1 \cdot \frac{(r_{o2} / / R_1)}{R_L} \approx g_{m2} \cdot v_1 \approx i_{in} \cdot \frac{g_{m2}}{g_{m1}}
\]

Since \( \frac{g_{m2}}{g_{m1}} = \frac{I_{D2}}{I_{D1}} \), we have the same current mirroring for small signals as we had for large signals.

[Note: if \( R_L \) is not small, we can see that the current \( i = g_{m2} \cdot v_1 \approx i_{in} \cdot \frac{g_{m2}}{g_{m1}} \) is basically mirrored into \( r_{o2} / / R_L \) instead of just \( R_L \).]
A current source is based on this current mirror. Therefore, it essentially behaves as illustrated below. This corresponds to a non-ideal current source, i.e. a current source with a source resistance.

For example, consider the source follower we studied before. The current source is implemented with MOSFETs now.
Elementary Configurations: Small Signal Analysis

To allow fast analysis of circuits, it is good to remember the small signal analysis of a few important elementary configurations.

\[
A_{vo} = \frac{g_m \cdot (r_o \parallel r_2)}{1 + g_m \cdot (r_o \parallel r_2)}
\]
The resistance seen in the directions indicated above can be calculated using regular small signal analysis. However, for quite a few of those, we actually have done this already.

(a) $R_{in}$ of CS configuration if $R_G = \infty$.

(b) $R_o$ of CS configuration if $R_D = \infty$.

(c) $R_{in}$ of CG configuration.

(d) $R_I$ of current mirror (i.e. diode-connected MOS), in series with $R$.

(e) This one is new and needs to be calculated. Note that this is the output resistance of the CS configuration with resistance if $r_o \neq \infty$.

For example, what is the small signal resistance seen?

\[ R = r_{o2} + r_{01} + g_m \cdot r_{o2} \cdot r_{01} \approx g_m \cdot r_{o2} \cdot r_{01} \]