2. Introduction to MOS Amplifiers: Transfer Function Biasing & Small-Signal-Model Concepts

Reading: Sedra & Smith Sec. 5.4
(S&S 5th Ed: Sec. 4.4)
NMOS Transfer Function (1)

Transfer Function: Relation between output and input voltages.

To find the transfer function, we start with $v_{GS} = 0$ and increase $v_{GS}$

NMOS $i$-$v$ Characteristics: $i_D = f(v_{GS}, v_{DS})$

KVL: $V_{DD} = R_D i_D + v_{DS}$

* To find the transfer function, we start with $v_{GS} = 0$ and increase $v_{GS}$
NMOS Transfer Function (2)

NMOS $i-v$ Characteristics: $i_D = f(v_{GS}, v_{DS})$

KVL: $V_{DD} = R_D i_D + v_{DS}$

- As we increase $v_{GS}$ passing $V_t$, NMOS will come out of cut-off: $i_D$ increases leading to a decrease in $v_{DS}$ (due to KVL)

- For $v_{GS} < V_t$, NMOS is in cutoff: $i_D = 0$
  
  $$v_{DS} = V_{DD} - R_D i_D = V_{DD}$$
To the right of point A, $v_{GS} > V_t$, and NMOS is ON.

Just to the right of point A:

- $V_{ov} = v_{GS} - V_t$ is small.
- $v_{DS}$ is close to $V_{DD}$ because transfer function cannot have a discontinuity.
- Thus, $v_{DS} > V_{ov} = v_{GS} - V_t$ and NMOS is in saturation.

\[
\begin{align*}
    i_D &= 0.5 \mu_n C_{ox} \frac{W}{L} V_{ov}^2 \\
    v_{DS} &= V_{DD} - \left( 0.5 \mu_n C_{ox} \frac{W}{L} V_{ov}^2 \right) R_D V_{ov}^2
\end{align*}
\]
NMOS Transfer Function (3)

- As $v_{GS}$ increase:
  - $V_{ov} = v_{GS} - V_t$ becomes larger;
  - $v_{DS}$ becomes smaller.
  - At point B, $v_{DS} = V_{ov} = v_{GS} - V_t$

- To the right of point B, $v_{DS} < V_{ov} = v_{GS} - V_t$ and NMOS enters triode.

- Point B is called the "Edge of Saturation"

Exercise: Use NMOS $i-v$ characteristics (and KVL), find $V_{GS|B}$ and $V_{DS|B}$
Graphical analysis of NMOS Transfer Function (1)

- NMOS i-v characteristics $i_D = f(v_{GS}, v_{DS})$ is a surface

* Plot for $V_{t,n} = 1$ V and $\mu_n C_{ox} (W/L) = 2.0$ mA/V²
Graphical analysis of NMOS Transfer Function (1)

Looking at surface with $v_{GS}$ axis pointing out of the paper

Note: surface is truncated (i.e., $v_{GS} < 5$ V)
Graphical analysis of NMOS Transfer Function (3)

- KVL equation is a plane in this space.
- Intersection of KVL plane with the $iv$ characteristic surface is a line.
- NMOS operating point is on this line (depending on the value of $v_{GS}$).

$\forall$ NMOS $i$-$v$ Characterisitics: $i_D = f(v_{GS}, v_{DS})$

KVL: $V_{DD} = R_D i_D + v_{DS}$

If we look from the top ($i_D$ axis out of the paper), we can see the transfer function.
Graphical analysis of NMOS Transfer Function (4)

Looking at surface with $v_{GS}$ axis pointing out of the paper
Graphical analysis of NMOS Transfer Function (5)

NMOS $i$-$v$ Characteristics: $i_D = f(v_{GS}, v_{DS})$

KVL: $V_{DD} = R_D i_D + v_{DS}$

The “Load Line” is the relationship between $i_D$ and $v_{DS}$ imposed by the circuit (outside of NMOS).
Graphical analysis of NMOS Transfer Function (6)

- Every point on the load line corresponds to a specific $v_{GS}$ value.
- As $v_{GS}$ increases, NMOS moves “up” the load line.
Foundation of Transistor Amplifiers

- A voltage amplifier requires \( \frac{v_o}{v_i} = \text{const.} \)
  - Transfer function has to be linear (but NMOS transfer function is NOT).

- Let us consider the response if NMOS remain in saturation at all times:
  - \( v_{GS} \) should be a combination of constant value and a time-varying signal.
The response to a combination of $V_{GS}$ and $v_{gs}$ (signal) can be found from the transfer function. 

Response to the signal appears to be linear!
Although the overall response is non-linear, the transfer function for the signal only is linear!

\[ v_{GS} = V_{GS} + v_{gs} \]
\[ v_{DS} = V_{DS} + v_{ds} \]
\[ i_D = I_{DS} + i_d \]

Non-linear relationship among these parameters

Linear relationship among these parameters

Signal and response

Constant: Bias
An Analogy (1)

- Total Height, $H_b = H_B + \text{response to signal } (h_b)$
- Complicated correlation between total height, $H_b$, and weight of the boat.
- Simple correlation between $h_b$ and added weight
An Analogy (2)

- Bias: $H_B$
- Bias + Signal: $H_b$
- Signal & response to signal: $h_b$
- Non-linear correlations among Bias + Signal: $v_{GS}, v_{DS}, i_D$
- Simple (and linear) correlation between signal and response to the signal: $v_{gs}, v_{ds}, i_d$

Bias: $V_{GS}, V_{DS}, I_D$
Bias + Signal: $v_{GS}, v_{DS}, i_D$
Signal & response: $v_{gs}, v_{ds}, i_d$
**Important Points!**

- **Signal:** We want the response of the circuit to this input.

- **Bias:** State of the system when there is no signal (current and voltages in all elements).
  - Bias is constant in time (may vary extremely slowly compared to signal)
  - Purpose of the bias is to ensure that MOS is in saturation at all times.

- **Response** of the circuit and elements within to the signal is different that the response of the circuit and its elements to Bias (or to Bias + signal):
  - Different transfer function for the circuit
  - Different $iv$ characteristics for the elements, i.e. relationships among $v_{gs}$, $v_{ds}$, $i_d$ is different than relationships among $v_{GS}$, $v_{DS}$, $i_D$. 

Limitations and Constraints

**Floating Boat analogy**
- Boat should float at all times!
  - Sufficient water in the pool
  - Cannot put too much weight (depends on the depth of the water!)

**Transistor**
- MOS should be in saturation at all times!
  - Bias point in Saturation*
    
    \[
    V_{GS} > V_{tn} \\
    V_{DS} > V_{GS} - V_{tn}
    \]
  - Signal amplitude cannot become too large (depends on Bias point!)**
    
    \[
    v_{GS} = V_{GS} + v_{gs} > V_{tn} \\
    v_{DS} = V_{DS} + v_{ds} > V_{GS} + v_{gs} - V_{tn}
    \]

*Equations are for NMOS!
** Obviously, the second set is more restrictive
Issues in developing a MOS amplifier:

1. **How to establish a Bias point** (bias is the state of the system when there is no signal).
   - Stable and robust bias point should be resilient to variations in $\mu_n C_{ox} (W/L), V_t, ...$ due to temperature and/or manufacturing variability.

2. **Find the $i_v$ characteristics of the elements for the signal** (which can be different than their characteristics equation for bias).
   - This will lead to different circuit configurations for bias versus signal

3. **Compute circuit response to the signal**
   - Focus on fundamental MOS amplifier configurations