Cascode Amplifiers and Cascode Current Mirrors
Cascode amplifier is a popular building block of ICs

Cascode Configuration

CG stage

CS stage

Cascode amplifier is a two-stage, CS-CG configuration
Small Signal Model of a Cascode Amplifier

Cascode Configuration

Small Signal Model

* Text book solves the Cascode amplifier using the small signal model. Text book introduces $G_m$ method to simplify the analysis. We will find $A_{vo}$ from small signal model. However, the solution and insight into Cascode amplifiers can be easily obtained using fundamental MOS configurations!
Open-Loop gain of a Cascode amplifier (using NMOS small signal model)

By KCL around Q2

Node Voltage Method:

Node $v_o$: \[ \frac{v_o - v_1}{r_{o2}} - g_{m2} \cdot v_1 = 0 \quad \Rightarrow \quad v_o = (1 + g_{m2} \cdot r_{o2}) \cdot v_1 \]

Node $v_1$: \[ \frac{v_1}{r_{o1}} + g_{m1} \cdot v_i + 0 = 0 \quad \Rightarrow \quad v_1 = -g_{m1} \cdot r_{o1} \cdot v_i \]

\[ A_{vo} = \frac{v_o}{v_i} = -g_{m1} \cdot r_{o1} \cdot (1 + g_{m2} \cdot r_{o2}) \approx -g_{m1} \cdot r_{o1} \cdot g_{m2} \cdot r_{o2} \]

Exercise: Compute $R_o$ from the small signal circuit 
(Attach a voltage source $v_x$ to the output and compute $i_x$, see S&S pp 509-510)
Distribution of the gain in a Cascode Amp.

Open Loop Gain: \((R_L \rightarrow \infty)\)

\[ A_{v2} = g_{m2}r_{o2}, \quad R_{L1} = R_{i2} = \frac{r_{o2} + R_L}{1 + g_{m2}r_{o2}} = \infty \]

\[ A_{v1} = -g_{m1}r_{o1}, \quad A_{vo} = -g_{m1}r_{o1}g_{m2}r_{o2} \]
Output Resistance of a Cascode amplifier

\[ R_o = r_o (1 + g_m R) + R \]

\[ R_o = r_{o2} (1 + g_{m2} \cdot r_{o1}) + r_{o1} \]

\[ R_o = r_{o1} + r_{o2} + g_{m2} r_{o1} r_{o2} \]
Cascode Amplifier needs a large load

For simplicity assume $r_{o1} = r_{o2} = r_o$ and $g_{m1} = g_{m2} = g_m$

$A_{v2} = g_{m2}(r_{o2} \parallel R_L)$

$R_{L1} = R_{i2} = \frac{r_{o2} + R_L}{1 + g_{m2}r_{o2}}$

$A_{v1} = -g_{m1}(r_{o1} \parallel R_{i2})$

$R_o \approx g_{m2} r_{o1} r_{o2}$

<table>
<thead>
<tr>
<th>$R_L$</th>
<th>$A_{v2}$ (CG)</th>
<th>$R_{i2} = R_{L1}$</th>
<th>$A_{v1}$ (CS)</th>
<th>$A_v = A_{v1} A_{v2}$</th>
<th>Max. Gain</th>
<th>Practical Gain</th>
<th>Same gain as a single CS Amp.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\infty$</td>
<td>$g_m r_o$</td>
<td>$\infty$</td>
<td>$-g_m r_o$</td>
<td>$-(g_m r_o)^2$</td>
<td>$0.5 (g_m r_o)^2$</td>
<td>$0.5 (g_m r_o)^2$</td>
<td>$-0.5 (g_m r_o)^2$</td>
</tr>
<tr>
<td>$(g_m r_o) r_o = R_o$</td>
<td>$g_m r_o$</td>
<td>$r_o$</td>
<td>$-0.5 g_m r_o$</td>
<td>$-0.5 (g_m r_o)^2$</td>
<td>$2/g_m$</td>
<td>$-2$</td>
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<td>$-g_m r_o$</td>
</tr>
</tbody>
</table>

For comparison, a two-stage CS-amplifier (CS-CS) has a gain of $0.5 (g_m r_o)^2$ for $R_L = r_o$ and a gain of $(g_m r_o)^2$ for $R_L = g_m r_o^2$.

- Cascode amplifier needs a large load ($R_L = g_m r_o^2$).
- Cascode amplifiers are popular because of their large gain-bandwidth.
Cascode amplifier needs a large load to get a high gain

\[ R_L = r_{o3} \]
\[ A_v \approx -g_m r_o \]

- Gain did not increase compared to a CS amplifier.
- This is still a useful circuit because of its high gain-bandwidth (we see this later).
- To get a high gain, \( A_v = -0.5(g_m r_o)^2 \), we need to increase the small-signal resistance of the current mirror to \( \approx (g_m r_o) r_o \)
  
  - Cascode current mirror
Cascode Current mirror

- Identical MOS: Same $\mu C_{ox}$ and $V_t$, and
  \[
  \frac{(W/L)_4}{(W/L)_3} = \frac{(W/L)_2}{(W/L)_1}
  \]
- Usually: $(W/L)_1 = (W/L)_3$ and $(W/L)_2 = (W/L)_4$
  - $V_{GS1} = V_{GS3} = V_{GS}$

- Q1 and Q3 are always in saturation
- Q2 and Q4 have to be in saturation for current mirror to work
  - $V_{DS2} > V_{GS} - V_t$
  - $V_{DS4} > V_{GS} - V_t$
- Straight forward to show
  \[
  I_o = \frac{(W/L)_2}{(W/L)_1} I_{ref}
  \]

**Exercise:** Show that a single current mirror (no cascoding) works only if $V_{D2} > V_{OV} - V_{SS}$ and a cascode current mirror requires $V_{D4} > 2V_{OV} - V_{SS}$
Small signal resistance of a cascode current mirror is quite large

\[ R_o = r_{o4} (1 + g_{m4} \cdot r_{o2}) + r_{o2} \]
Cascode amplifier with a cascode current mirror/active load

$I_{D1} = I_{D2} = I_{D3} = I_{D4}$
Gain of a Cascode amplifier with a cascode current mirror/active load

- A high gain, $A_v \approx -0.5(g_m r_o)^2$, high gain-bandwidth circuit.
- Draw-back: Low voltage headroom because 4 MOS should be in saturation for a given $V_{DD}$.

Gain expressions:
- $A_{v2} = v_o / v_1 = g_{m2} (r_{o2} \parallel R_L) \approx g_{m2} r_{o2}$
- $R_{L1} = R_{i2} = \frac{r_{o2} + r_{o3} (1 + g_{m3} \cdot r_{o4}) + r_{o4}}{1 + g_{m2} r_{o2}} \approx \frac{g_{m3} r_{o3} r_{o4}}{g_{m2} r_{o2}}$
- $A_{v1} = v_1 / v_i = -g_{m1} (r_{o1} \parallel \frac{g_{m3} r_{o3} r_{o4}}{g_{m2} r_{o2}})$
- $A_v = A_{v1} A_{v2} = -\frac{g_{m1} g_{m2} g_{m3} r_{o1} r_{o2} r_{o3} r_{o4}}{g_{m2} r_{o1} r_{o2} + g_{m3} r_{o3} r_{o4}}$

Value for the same $g_m$ and $r_o$:
- $A_{v2} \approx g_m r_o$
- $R_{L1} = R_{i2} \approx r_o$
- $A_{v1} = -0.5 g_m r_o$
- $A_v = -0.5 (g_m r_o)^2$
Folded Cascode increases voltage overhead

NMOS CS stage
Biased with $I_1 - I_2$

PMOS CG stage
Biased with $I_2$

Bias

Signal