6. Cascode Amplifiers and Cascode Current Mirrors

Sedra & Smith Sec. 7 (MOS portion)
(S&S 5th Ed: Sec. 6 MOS portion & ignore frequency response)
Cascode amplifier is a popular building block of ICs

Cascode Configuration

- **Signal circuit:** Current source becomes an open circuit

Cascode amplifier is a two-stage, CS-CG configuration

Signal circuit: Current source becomes an open circuit

![Signal circuit diagram](Image)

**Signal circuit**

- Current source becomes an open circuit

**Cascode Configuration**

- **CG stage**
  - Input: $v_i$, Output: $v_1$, Bias: $I_0$
  - Circuit diagram:
    - Input: $v_i$, Output: $v_1$
    - Bias: $I_0$
- **CS stage**
  - Input: $v_i$, Output: $v_1$, Bias: $I_0$
  - Circuit diagram:
    - Input: $v_i$, Output: $v_1$
    - Bias: $I_0$

**Cascode amplifier is a two-stage, CS-CG configuration**

F. Najmabadi, ECE102, Fall 2012 (2/17)
Small Signal Model of a Cascode Amplifier

- Lengthy analysis to find $A_v$ (and a complicated equation). Simpler to compute open-loop gain ($A_{vo}$) and $R_o$.
- Text book introduces $G_m$ method to find $A_{vo}$ (See S&S Sec. 1)
- Here will find $A_{vo}$ directly from the small signal model.
- However, the solution of and insight into Cascode amplifiers are best obtained using fundamental MOS configurations!

Note that $A_{vo}$ and $R_o$ calculated here are meant to find $A_v$ and guide the choice of the active load. $A_{vo}$ and $R_o$ should be re-calculated for a practical circuit (see slides 14 & 15)
Open-Loop gain of a Cascode amplifier (using small signal model)

\[ A_{vo} = \frac{v_o}{v_i} = -g_{m1}r_{o1} \times (1 + g_{m2}r_{o2}) \approx -g_{m1}r_{o1}g_{m2}r_{o2} \]

Node Voltage Method:

Node \( v_o \):
\[ \frac{v_o - v_1}{r_{o2}} - g_{m2}v_1 = 0 \quad \Rightarrow \quad v_o = (1 + g_{m2}r_{o2})v_1 \]

Node \( v_1 \):
\[ \frac{v_1}{r_{o1}} + g_{m1}v_i + 0 = 0 \quad \Rightarrow \quad v_1 = -g_{m1}r_{o1}v_i \]
Output Resistance of a Cascode amplifier (using small signal model)

Set \( v_i = 0 \), attach a voltage source \( v_x \), compute \( i_x, R_o = v_x / i_x \)

![Cascode Amplifier Diagram]

\[ v_i = v_{gs1} = 0 \rightarrow g_{m1} v_{gs1} \text{ current source becomes open circuit} \]

KVL: \( v_{gs2} = -i_x r_{o1} \)

KCL: \( i_2 = i_x - g_{m2} v_{gs2} = i_x + i_x g_{m2} r_{o1} = i_x (1 + g_{m2} r_{o1}) \)

KVL: \( v_x = i_2 r_{o2} + i_x r_{o1} = i_x (1 + g_{m2} r_{o1}) r_{o2} + i_x r_{o1} \)

\[ v_x = i_x [(1 + g_{m2} r_{o1}) r_{o2} + r_{o1}] \]

\[ R_o = \frac{v_x}{i_x} = r_{o1} + r_{o2} + g_{m2} r_{o1} r_{o2} \]

Note: \( A_v = A_{vo} \times \frac{R'_L + R_o}{R'_L} \)
Gain of a Cascode Amplifier (using MOS Fundamental Configurations)

Cascode (signal circuit)

CG stage

CS stage

**Note:** Open Loop Gain: \((R'_L \to \infty)\)

\[ R_{L1} = R_{i2} = \frac{r_{o2} + R'_L}{1 + g_{m2}r_{o2}} = \infty \quad \Rightarrow \quad A_{vo} = -g_{m1}r_{o1}g_{m2}r_{o2} \]
Output Resistance of a Cascode amplifier
(from Elementary R forms)

\[ R_o = r_o (1 + g_m R) + R \]

\[ R_o = r_{o2} (1 + g_{m2} r_{o1}) + r_{o1} \]

\[ R_o = r_{o1} + r_{o2} + g_{m2} r_{o1} r_{o2} \]

\[ R_o \approx r_{o2} (1 + g_{m2} r_{o1}) + r_{o1} \]

\[ R_o \approx g_{m2} r_{o2} r_{o1} = r_{o1} (1 + g_{m2} r_{o2}) \]

\[ R_o \approx g_{m2} r_{o1} r_{o2} \]
Cascode Amplifier needs a large load

\[ A_{v2} = g_m r_o \parallel R'_L \]

\[ R_{L1} = R_{i2} = \frac{r_{o2} + R'_L}{1 + g_m r_{o2}} \]

\[ A_{v1} = -g_m r_o \parallel R_{i2} \]

\[ R_o \approx g_m r_o r_{o2} \]

For simplicity assume \( r_{o1} = r_{o2} = r_o \) and \( g_m = g_m = g_m \)

<table>
<thead>
<tr>
<th>( R'_L )</th>
<th>( A_{v2} ) (CG)</th>
<th>( R_{i2} = R_{L1} )</th>
<th>( A_{v1} ) (CS)</th>
<th>( A_v = A_{v1} A_{v2} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \infty )</td>
<td>( g_m r_o )</td>
<td>( \infty )</td>
<td>( -g_m r_o )</td>
<td>( -(g_m r_o)^2 )</td>
</tr>
<tr>
<td>( (g_m r_o) r_o = R_o )</td>
<td>( g_m r_o )</td>
<td>( r_o )</td>
<td>( -0.5 g_m r_o )</td>
<td>( -0.5 (g_m r_o)^2 )</td>
</tr>
<tr>
<td>( r_o )</td>
<td>( 0.5 g_m r_o )</td>
<td>( 2/g_m )</td>
<td>( -2 )</td>
<td>( -g_m r_o )</td>
</tr>
</tbody>
</table>

Max. Gain

Practical Gain

Same gain as a single CS Amp.

For comparison, a two-stage CS-amplifier (CS-CS) has a gain of \( 0.5 (g_m r_o)^2 \) for \( R'_L = r_o \) and a gain of \( (g_m r_o)^2 \) for \( R'_L = g_m r_o^2 \).

- Cascode amplifier needs a large load \( (R'_L = g_m r_o^2) \).
Cascode amplifier needs a large load to get a high gain

Gain did not increase compared to a CS amplifier.

This is still a useful circuit because of its high gain-bandwidth (we see this later).

To get a high gain, \( A_v = -0.5(g_m r_o)^2 \), we need to increase the small-signal resistance of the current mirror to 
\[ \approx (g_m r_o) r_o \]
- Cascode current mirror
Cascode Current mirror

- Identical MOS: Same $\mu C_{ox}$ and $V_t$, & \( \frac{(W/L)_4}{(W/L)_3} = \frac{(W/L)_2}{(W/L)_1} \)
  - $v_{GS1} = v_{GS2}$ & $v_{GS3} = v_{GS4}$

- Usually: (W/L)$_1$ = (W/L)$_3$ and (W/L)$_2$ = (W/L)$_4$
  - $v_{GS1} = v_{GS2} = v_{GS3} = v_{GS4} = v_{GS}$

- Q1 and Q3 are always in saturation
- Q2 and Q4 both have to be in saturation for current mirror to work
  - $V_{DS2} > V_{GS} - V_t$
  - $V_{DS4} > V_{GS} - V_t$

- Straight forward to show \( I_o = \frac{(W/L)_2}{(W/L)_1} I_{ref} \)

Exercise: Show that a single current mirror (no cascoding) works only if $V_{D2} > V_{OV} - V_{SS}$ and a cascode current mirror requires $V_{D4} > 2V_{OV} - V_{SS}$
Small signal resistance of a cascode current mirror is quite large

\[ \bar{R} = r_{o4}(1 + g_{m4}r_{o2}) + r_{o2} \]

Transistor numbering is different in different circuits
Be careful in applying formulas!
It is best to use elementary R forms to find \( \bar{R} \) instead of formula above.
PMOS cascode current mirror is similar to NMOS version

NMOS Cascode current mirror

PMOS Cascode current mirror
Cascode amplifier with a cascode current mirror/active load

Exercise: Draw the circuit for a PMOS cascode with a cascode current mirror (cascode current mirror would be made of NMOS).

\[ I_{D1} = I_{D2} = I_{D3} = I_{D4} \]
Gain of a Cascode amplifier with a cascode current mirror/active load

**Q2 (CG Amp)**

\[ A_{v2} = \frac{v_o}{v_1} \approx g_{m2} \left( r_{o2} \parallel R'_L \right) \approx g_{m2} r_{o2} \]

\[
R_{L1} = R_{i2} = \frac{r_{o2} + r_{o3} (1 + g_{m3} r_{o4}) + r_{o4}}{1 + g_{m2} r_{o2}} \\
\approx \frac{g_{m3} r_{o3} r_{o4}}{g_{m2} r_{o2}}
\]

**Q1 (CS Amp)**

\[ A_{v1} = \frac{v_1}{v_i} = -g_{m1} \left( r_{o1} \parallel \frac{g_{m3} r_{o3} r_{o4}}{g_{m2} r_{o2}} \right) \]

\[ A_{vo} = A_{v1} A_{v2} = -\frac{g_{m1} g_{m2} g_{m3} r_{o1} r_{o2} r_{o3} r_{o4}}{g_{m2} r_{o1} r_{o2} + g_{m3} r_{o3} r_{o4}} \]

**Value for the same \( g_m \) and \( r_o \)**

<table>
<thead>
<tr>
<th>( A_{v2} )</th>
<th>( R_{L1} = R_{i2} \approx r_o )</th>
</tr>
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<tbody>
<tr>
<td>( A_{v1} \approx -0.5 g_m r_o )</td>
<td>( A_{vo} = -0.5 (g_m r_o)^2 )</td>
</tr>
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</table>

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Output Resistance of a Cascode amplifier with a cascode current mirror/active load

\[ R_1 = r_{o3}(1 + g_{m3}r_{o4}) + r_{o4} \]

\[ R_2 = r_{o2}(1 + g_{m2}r_{o1}) + r_{o1} \]

\[ R_o = R_1 \parallel R_2 \]

Value for the same \( g_m \) and \( r_o \)

\[ R_1 \approx g_m r_o^2 \]

\[ R_2 \approx g_m r_o^2 \]

\[ R_o = 0.5 g_m r_o^2 \]
Why Cascode Amplifiers are popular?

Drawbacks:

- While $A_{vo}$ are similar, Cascode has a very $R_o$ (MΩ level).
  - should be followed with a CS or CD stage (infinite load for cascode)
  - BJT cascodes are not useful.
- Low voltage headroom ($V_{DD}$ across 4 MOS)
  - Folded cascodes solve this.

Benefits:

- Much better high-frequency response (high gain-bandwidth).*
- Simpler biasing.

* We will see this later in our discussion of frequency response.
Folded Cascode increases voltage overhead*

* Folded cascode only helps the voltage overhead issue for difference amplifiers (see S&S pages 999-1000)