7. Differential Amplifiers

Sedra & Smith Sec. 2.1.3 and Sec. 8 (MOS Portion)

(S&S 5th Ed: Sec. 2.1.3 and Sec. 7 MOS Portion & ignore frequency-response)
Common-Mode and Differential-Mode Signals & Gain
Consider a linear circuit with TWO inputs

\[ \begin{align*}
  v_2 & \quad v_o \\
  v_1 & \quad \text{By superposition:}
\end{align*} \]

Define:

\[ v_d = v_2 - v_1 \]
\[ v_c = \frac{v_1 + v_2}{2} \]

Difference (or differential) Mode

Common Mode

Substituting for \( v_1 = v_c - \frac{v_d}{2} \) and \( v_2 = v_c + \frac{v_d}{2} \) in the expression for \( v_o \):

\[ v_o = A_1 \cdot \left( v_c - \frac{v_d}{2} \right) + A_2 \cdot \left( v_c + \frac{v_d}{2} \right) = (A_1 + A_2) \cdot v_c + \left( \frac{A_2 - A_1}{2} \right) \cdot v_d \]

\[ v_o = A_c \cdot v_c + A_d \cdot v_d \]
**Differential and common-mode signal/gain is an alternative way of finding the system response**

\[
\begin{align*}
    v_o &= A_1 \cdot v_1 + A_2 \cdot v_2 \\
    v_c &= \frac{v_1 + v_2}{2} \\
    v_d &= v_2 - v_1 \\
    A_c &= A_1 + A_2 \\
    A_d &= \frac{A_2 - A_1}{2} \\
    v_1 &= v_c - \frac{v_d}{2} \\
    v_2 &= v_c + \frac{v_d}{2} \\
    A_1 &= \frac{A_c}{2} - A_d \\
    A_2 &= \frac{A_c}{2} + A_d
\end{align*}
\]

**Differential Gain:** \( A_d \)

**Common Mode Gain:** \( A_c \)

**Common Mode Rejection Ratio (CMRR)**: \(|A_d| / |A_c|\)

* CMRR is usually given in dB: \( \text{CMRR}(\text{dB}) = 20 \log \left( \left| \frac{A_d}{A_c} \right| \right) \)
To find \( v_o \), we can calculate/measure either \( A_1 \) \( A_2 \) pair or \( A_c \) \( A_d \) pair

### Superposition (finding \( A_1 \) and \( A_2 \) ):
1. Set \( v_2 = 0 \), compute \( A_1 \) from
   \[ v_o = A_1 v_1 \]
2. Set \( v_1 = 0 \), compute \( A_2 \) from
   \[ v_o = A_2 v_2 \]
3. For any \( v_1 \) and \( v_2 \):
   \[ v_o = A_1 v_1 + A_2 v_2 \]

### Difference Method (finding \( A_d \) and \( A_c \) ):
1. Set \( v_c = 0 \) (or set \( v_1 = -0.5 v_d \) & \( v_2 = +0.5 v_d \))
   compute \( A_d \) from \( v_o = A_d v_d \)
2. Set \( v_d = 0 \) (or set \( v_1 = +v_c \) & \( v_2 = +v_c \))
   compute \( A_c \) from \( v_o = A_c v_c \)
3. For any \( v_1 \) and \( v_2 \):
   \[ v_o = A_d v_d + A_c v_c \]
   \[ v_d = v_2 - v_1 \quad v_c = 0.5(v_1 + v_2) \]

- Both methods give the same answer for \( v_o \) (or \( A_v \)).
- The choice of the method is driven by application:
  - Easier solution
  - More relevant parameters
Caution

- In Chapter 2.1.3, Sedra & Smith defines $v_d = v_2 - v_1$
  
  $$v_1 = v_c - \frac{v_d}{2} \quad v_2 = v_c + \frac{v_d}{2}$$

- But in Chapter 8, Sedra & Smith uses $v_d = v_1 - v_2$
  
  $$v_1 = v_c + \frac{v_d}{2} \quad v_2 = v_c - \frac{v_d}{2}$$

  While keeping $v_o = v_{o2} - v_{o1}$ as before (this is inconsistent)

- Here we use $v_d = v_2 - v_1$ and $v_o = v_{o2} - v_{o1}$ throughout
  
  $$v_1 = v_c - \frac{v_d}{2} \quad v_2 = v_c + \frac{v_d}{2}$$

- Therefore, $A_d$ (lecture slides) = $-A_d$ (Sedra & Smith) for difference Amplifiers.

- Use Lecture Slides Notation!
Differential Amplifiers: Fundamental Properties
For now, we keep track of “two” output, \(v_{o1}\) and \(v_{o2}\), because there are several ways to configure “one” output from this circuit.

- Identical transistors.
- Circuit elements are symmetric about the mid-plane.
- Identical bias voltages at Q1 & Q2 gates (\(V_{G1} = V_{G2}\)).
- Signal voltages & currents are different because \(v_1 \neq v_2\).
Differential Amplifier – Bias

Since \( V_{G1} = V_{G2} = V_G \) and \( V_{S1} = V_{S2} = V_S \)

\[
\begin{align*}
V_{GS1} & = V_{GS2} = V_{GS} \\
V_{OV1} & = V_{OV2} = V_{OV} \\
I_{D1} & = I_{D2} = I_D \\
V_{DS1} & = V_{DS2} = V_{DS}
\end{align*}
\]

Also:

\[
\begin{align*}
g_{m1} & = g_{m2} = g_m \\
r_{o1} & = r_{o2} = r_o
\end{align*}
\]

This is correct even if channel-length modulation is included because

\[
I_{D1}R_D + V_{DS1} = I_{D2}R_D + V_{DS2}
\]
Differential Amplifier – Gain

- Signal voltages & currents are different because $v_1 \neq v_2$
- We cannot use fundamental amplifier configuration for arbitrary values of $v_1$ and $v_2$.
- We have to replace each NMOS with its small-signal model.
Differential Amplifier – Gain

\[ v_{gs1} = v_1 - v_3 \]
\[ v_{gs2} = v_2 - v_3 \]

Node Voltage Method:

**Node \( v_{o1} \):**
\[ \frac{v_{o1}}{R_D} + \frac{v_{o1} - v_3}{r_o} + g_m (v_1 - v_3) = 0 \]

**Node \( v_{o2} \):**
\[ \frac{v_{o2}}{R_D} + \frac{v_{o2} - v_3}{r_o} + g_m (v_2 - v_3) = 0 \]

**Node \( v_3 \):**
\[ \frac{v_3}{R_{SS}} + \frac{v_3 - v_{o2}}{r_o} + \frac{v_3 - v_{o1}}{r_o} - g_m (v_1 - v_3) - g_m (v_2 - v_3) = 0 \]

Above three equations should be solved to find \( v_{o1}, v_{o2} \) and \( v_3 \) (lengthy calculations)

➢ Because the circuit is symmetric, differential/common-mode method is the preferred method to solve this circuit (and we can use fundamental configuration formulas).
Because of summery of the circuit and input signals*:

\[ v_{o1} = v_{o2} \quad \text{and} \quad i_{d1} = i_{d2} = i_d \]

We can solve for \( v_{o1} \) by node voltage method but there is a simpler and more elegant way.

* If you do not see this, set \( v_1 = v_2 = v_c \) in node equations of the previous slide, subtract the first two equations to get \( v_{o1} = v_{o2} \). Ohm’s law on \( R_D \) then gives \( i_{d1} = i_{d2} = i_d \).
Because of the symmetry, the common-mode circuit breaks into two identical “half-circuits”.

\[ V_{ss} \text{ is grounded for signal} \]
Differential Amplifier – Common Mode (3)

The common-mode circuit breaks into two identical half-circuits.

CS Amplifiers with Rs

\[
\begin{align*}
\frac{v_{o1}}{v_c} &= \frac{v_{o2}}{v_c} = -\frac{g_m R_D}{1 + 2g_m R_{SS} + R_D / r_o}
\end{align*}
\]
Differential Amplifier – Differential Mode (1)

**Differential Mode:** Set $v_c = 0$ (or set $v_1 = -v_d/2$ and $v_2 = +v_d/2$)

\[ v_{gs1} = -0.5v_d - v_3 \]
\[ v_{gs2} = +0.5v_d - v_3 \]

**Node Voltage Method:**

**Node $v_{o1}$:**
\[ \frac{v_{o1}}{R_D} + \frac{v_{o1} - v_3}{r_o} + g_m (-0.5v_d - v_3) = 0 \]

**Node $v_{o2}$:**
\[ \frac{v_{o2}}{R_D} + \frac{v_{o2} - v_3}{r_o} + g_m (+0.5v_d - v_3) = 0 \]

**Node $v_3$:**
\[ \frac{v_3}{R_{SS}} + \frac{v_3 - v_{o2}}{r_o} + \frac{v_3 - v_{o1}}{r_o} - g_m (-0.5v_d - v_3) - g_m (+0.5v_d - v_3) = 0 \]

**Node $v_{o1} + Node v_{o2}$:**
\[ \left( \frac{1}{R_D} + \frac{1}{r_o} \right) (v_{o1} + v_{o2}) - \left( \frac{2}{r_o} + 2g_m \right) v_3 = 0 \]

**Node $v_3$:**
\[ -\frac{1}{r_o} (v_{o1} + v_{o2}) + \left( \frac{1}{R_{SS}} + \frac{2}{r_o} - 2g_m \right) v_3 = 0 \]

Only possible solution:
\[ v_{o1} + v_{o2} = 0 \Rightarrow v_{o1} = -v_{o2} \]
\[ v_3 = 0 \]
Because of the symmetry, the differential-mode circuit also breaks into two identical half-circuits.

\[ v_3 = 0 \quad \text{and} \quad v_{o1} = -v_{o2} \implies i_{d1} = -i_{d2} \]

\[ \frac{v_{o1}}{-0.5v_d} = -g_m (r_o || R_D) , \quad \frac{v_{o2}}{+0.5v_d} = -g_m (r_o || R_D) \]
Concept of “Half Circuit”

For a symmetric circuit, differential- and common-mode analysis can be performed using “half-circuits.”
Common Mode Half-circuit

1. Currents about symmetry line are equal.
2. Voltages about the symmetry line are equal (e.g., $v_{o1} = v_{o2}$)
3. No current crosses the symmetry line.
Differential-Mode “Half Circuit”

1. Currents about the symmetry line are equal in value and opposite in sign.
2. Voltages about the symmetry line are equal in value and opposite in sign.
3. Voltage at the summery line is zero.
Step 1:
Divide **ALL elements** that cross the symmetry line (e.g., $R_L$) and/or are located on the symmetry line (current source) such that we have a symmetric circuit (only wires should cross the symmetry line, nothing should be located on the symmetry line!)
Step 2: Common Mode Half-circuit
1. Currents about symmetry line are equal (e.g., $i_{d1} = i_{d2}$).
2. Voltages about the symmetry line are equal (e.g., $v_{o1} = v_{o2}$).
3. No current crosses the symmetry line.

$\mathbf{v_{o1,c} = v_{o2,c}}$
Constructing “Half Circuit”— Differential Mode

**Step 3: Differential Mode Half-Circuit**

1. Currents about symmetry line are equal but opposite sign (e.g., $i_{d1} = -i_{d2}$)
2. Voltages about the symmetry line are equal but opposite sign (e.g., $v_{o1} = -v_{o2}$)
3. Voltage on the symmetry line is zero.

\[ v_{o1,d} = -v_{o2,d} \]
“Half-Circuit” works only if the circuit is symmetric!

- Half circuits for common-mode and differential mode are different.
- Bias circuit is similar to Half circuit for common mode.
- Not all difference amplifiers are symmetric. Look at the load carefully!

- We can still use half circuit concept if the deviation from perfect symmetry is small (i.e., if one transistor has $R_D$ and the other $R_D + \Delta R_D$ with $\Delta R_D <\ll R_D$).
  - However, we need to solve BOTH half-circuits (see slide 30)
Why are Differential Amplifiers popular?

- They are much less sensitive to noise (CMRR >>1).
- Biasing: Relatively easy direct coupling of stages:
  - Biasing resistor ($R_{SS}$) does not affect the differential gain (and does not need a by-pass capacitor).
  - No need for precise biasing of the gate in ICs
  - DC amplifiers (no coupling/bypass capacitors).
- ...
Why is a large CMRR useful?

- A major goal in circuit design is to minimize the noise level (or improve signal-to-noise ratio). Noise comes from many sources (thermal, EM, ...)

- A regular amplifier “amplifies” both signal and noise.

- However, if the signal is applied between two inputs and we use a difference amplifier with a large CMRR, the signal is amplified a lot more than the noise which improves the signal to noise ratio.*

\[
\begin{align*}
v_1 &= v_{\text{sig}} + v_{\text{noise}} \\
v_o &= A \cdot v_1 = A \cdot v_{\text{sig}} + A \cdot v_{\text{noise}} \\
v_1 &= -0.5v_{\text{sig}} + v_{\text{noise}} \quad \& \quad v_2 = +0.5v_{\text{sig}} + v_{\text{noise}} \\
v_d &= v_2 - v_1 = v_{\text{sig}} \quad \& \quad v_c = v_{\text{noise}} \\
v_o &= A_d \cdot v_d + A_c \cdot v_c = A_d \cdot v_{\text{sig}} + \frac{A_d}{CMRR} \cdot v_{\text{noise}}
\end{align*}
\]

* Assuming that noise levels are similar to both inputs.
Comparing a differential amplifier
two identical CS amplifiers (perfectly matched)
Comparison of a differential amplifier with two identical CS amplifiers – Differential Mode

Differential amplifier

\[ v_{o1,d} = -g_m \left( r_o \parallel R_D \right) (-0.5v_d) \]

\[ v_{o2,d} = -g_m \left( r_o \parallel R_D \right) (+0.5v_d) \]

\[ v_{od} = v_{o2,d} - v_{o1,d} = -g_m \left( r_o \parallel R_D \right) v_d \]

\[ A_d = v_{od} / v_d = -g_m \left( r_o \parallel R_D \right) \]

Half-Circuits

Identical

Two CS amplifiers

\[ v_{o1,d}, v_{o2,d}, v_{od}, \text{ and differential gain, } A_d, \text{ are identical.} \]
Comparison of a differential amplifier with two identical CS amplifiers – Common Mode

Differential amplifier

Two CS amplifiers

Half-Circuits

\[ v_{o1,c} = v_{o2,c} = -\frac{g_m R_D}{1 + 2 g_m R_{SS} + R_D / r_o} v_c \]

\[ v_{oc} = v_{o2,c} - v_{o1,c} = 0 \]

\[ A_c = v_{oc} / v_c = 0 \]

\[ v_{o1,c} = v_{o2,c} = -g_m (r_o \parallel R_D) v_c \]

\[ v_{oc} = v_{o2,c} - v_{o1,c} = 0 \]

\[ A_c = v_{oc} / v_c = 0 \]

\[ \blacktriangleright \quad v_{o1,c} \text{ & } v_{o2,c} \text{ are different! But } v_{oc} = 0 \text{ and CMMR} = \infty. \]
Comparison of a differential amplifier with two identical CS amplifiers – Summary

For perfectly matched circuits, there is no difference between a differential amplifier and two identical CS amplifiers.

- But one can never make **perfectly** matched circuits!
Consider a “slight” mis-match in the load resistors

We will ignore $r_o$ in the this analysis (to make equations simpler)
“Slightly” mis-matched loads – Differential Mode

**Differential amplifier**

\[ v_{o1,d} = -g_m(R_D)(-0.5v_d) \]
\[ v_{o2,d} = -g_m(R_D+\Delta R_D)(+0.5v_d) \]
\[ v_{od} = v_{o2,d} - v_{o1,d} = -g_m(R_D+0.5\Delta R_D)v_d \]
\[ A_d = \frac{v_{od}}{v_d} = -g_m(R_D+0.5\Delta R_D) \]

\( v_{o1}, v_{o2}, v_{od}, \) and differential gain, \( A_d \), are identical.

**Two CS amplifiers**

Half-Circuits

F. Najmabadi, ECE102, Fall 2012 (31/33)
“Slightly” mis-matched loads – Common Mode

Differential amplifier

\[
\begin{align*}
\nu_{o1,c} &= -\frac{g_m R_D}{1 + 2g_m R_{SS}} \nu_c, \\
\nu_{o2,c} &= -\frac{g_m (R_D + \Delta R_D)}{1 + 2g_m R_{SS}} \nu_c \\
\nu_{oc} &= \nu_{o2,c} - \nu_{o1,c} = -\frac{g_m \Delta R_D}{1 + 2g_m R_{SS}} \nu_c \\
A_c &= \frac{\nu_{oc}}{\nu_c} = -\frac{g_m \Delta R_D}{1 + 2g_m R_{SS}}
\end{align*}
\]

Two CS amplifiers

\[
\begin{align*}
\nu_{o1,c} &= -g_m R_D \nu_c \\
\nu_{o2,c} &= -g_m (R_D + \Delta R_D) \nu_c \\
\nu_{oc} &= \nu_{o2,c} - \nu_{o1,c} = +g_m \Delta R_D \nu_c \\
A_c &= \frac{\nu_{oc}}{\nu_c} = +g_m \Delta R_D
\end{align*}
\]

\[\Rightarrow\ \nu_{o1} \text{ and } \nu_{o2} \text{ are different. In addition, } \nu_{oc} \neq 0 \text{ and CMMR} \neq \infty.\]
A differential amplifier increases CMRR substantially for a slight mis-match ($\Delta R_D \neq 0$)

**Two CS Amplifiers**

$$A_d = -g_m (R_D + 0.5\Delta R_D)$$

$$A_c = +g_m \Delta R_D$$

$$\text{CMRR} \approx \frac{1}{\Delta R_D / R_D}$$

**Differential Amplifier**

$$A_d = -g_m (R_D + 0.5\Delta R_D)$$

$$A_c = -\frac{g_m \Delta R_D}{1 + 2g_m R_{SS}}$$

$$\text{CMRR} \approx \frac{1 + 2g_m R_{SS}}{\Delta R_D / R_D}$$

- Differential amplifier reduces $A_c$ and increases CMRR substantially (by a factor of: $1 + 2g_m R_{SS}$).
- The common-mode half-circuits for a differential amplifier are CS amplifiers with $R_S$ (thus common mode gain is much smaller than two CS amplifiers).
- We should use a large $R_{SS}$ in a differential amplifier!

* Exercise: Compare a differential amplifier and two CS amplifiers with a mis-match in $g_m$.