Transistor Amplifiers

Lecture notes: Sec. 6

Sedra & Smith (6th Ed): Sec. 5.6, 5.8, 6.6 & 6.8
Sedra & Smith (5th Ed): Sec. 4.6, 4.8, 5.6 & 5.8
How to add signal to the bias

1. Direct Coupling
   - Use bias with 2 voltage supplies
     - For the first stage, bias such that $V_{GS} = 0$
     - For follow-up stages, match bias voltages between stages
   - Difficult biasing problem
   - Used in ICs
   - Amplifies “DC” signals!

2. Capacitive Coupling
   - Use a capacitor to separate bias voltage from the signal.
   - Simplified biasing problem.
   - Used in discrete circuits
   - Only amplifies “AC” signals
Capacitive coupling is based on the fact that capacitors appear as open circuit in bias.

- At a high enough frequency, $Z_c = 1/ (\omega C)$, becomes small (effectively, capacitors become short circuit).
  - **Mid-band** parameters of an Amplifier.*

- At low frequencies, $Z_c$ cannot be ignored. As $Z_c$ depends on frequency, amplifier is NOT linear (for an arbitrary signal) for these low frequencies.
  - Capacitors introduce a lower cut-off frequency for an amplifier (i.e., amplifier should be operated above this frequency).

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In ECE102, you will see that transistor amplifiers also have an “upper” cut-off frequency.
What are amplifier parameters?

Voltage Gain: \( A_v = \frac{v_o}{v_i} \)

Open-loop Gain: \( A_{vo} = \frac{v_o}{v_i\bigg|_{R_L \rightarrow \infty}} \)

Input Resistance: \( R_i = \frac{v_i}{i_i} \)

Output Resistance of Amplifier: \( R_o = \frac{v_o}{i_o\bigg|_{v_i \rightarrow 0}} \)

Output Resistance of the circuit: \( R_{out} = \frac{v_o}{i_o\bigg|_{v_{sig} \rightarrow 0}} \)

Output resistance is the Thevenin resistance between the output terminals!

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Observations on the amplifier parameters

Overall Gain:

\[ A = \frac{v_o}{v_{\text{sig}}} = \frac{v_i}{v_{\text{sig}}} \times \frac{v_o}{v_i} = \frac{R_i}{R_i + R_{\text{sig}}} A_v \]

- Value of \( R_i \) is important.
  - For \( R_{\text{sig}} \ll R_i \), \( v_i \approx v_{\text{sig}} \)
  - For \( R_{\text{sig}} = R_i \), \( v_i = 0.5 \, v_{\text{sig}} \)
  - For \( R_{\text{sig}} \gg R_i \), \( v_i \approx 0 \)
- Prefer “large” \( R_i \)

\[ A_v = \frac{v_o}{v_i} = \frac{R_L}{R_L + R_o} A_{vo} \]

- \( A_{vo} \) is the maximum possible gain of the amplifier.
- Value of \( R_o \) is important.
  - For \( R_o \ll R_L \), \( A_v \approx A_{vo} \)
  - For \( R_o = R_L \), \( A_v = 0.5 \, A_{vo} \)
  - For \( R_o \gg R_L \), \( A_v \approx 0 \)
- Prefer “small” \( R_o \)
How to Solve Amplifier Circuits

1. Find Bias and Signal Circuits.

2. Bias circuit (signal = 0):
   - Capacitors are open circuit.
   - Use transistor large-signal model to find the bias point.
   - Use bias parameters to find small-signal parameters ($r_\pi, g_m, r_o$).

3. Signal Circuit (IVS becomes short, ICS becomes open circuit):
   - Assume capacitors are short to find mid-band amplifier parameters.
   - Replace diodes and/or transistors with their small-signal model.
   - Solve for mid-band amplifier parameters ($A_v, R_i, R_o$).
     - For most circuits, we can use fundamental amplifier configurations, elementary R forms instead of solving signal circuits.
   - Include impedance of capacitors to find the lower cut-off frequency of the amplifier.
Example 1: Draw the small-signal equivalent of the circuit below (assume capacitors are short for small signal).

- **IVS → 0**
- **R remains**
- **Caps short**

Replace MOS with its small signal model.

Ground at the bottom.
Example 2: Draw the small-signal equivalent of the circuit below (assume capacitors are short for small signal).

- Flip PMOS
- Ground at the bottom \((100k \ || \ 33k = 24.8 \ k)\)
- Replace MOS with its small signal model
Example 3: Draw the small-signal equivalent of the circuit below (assume capacitors are short for small signal).

ICS → 0
Caps short

IVS → 0
(This makes ICS an open circuit)

Replace MOS with its small signal model
Basic MOS Amplifier Configurations

We are considering only signal circuit here!
Possible MOS amplifier configurations

- **Common-Source**
- **Common-Gate**
- **Common-Drain**
- **Common-Source with $R_s$**
- **Same as Common Gate** ($v_i$ does not change)
- **Not Useful**
PMOS configurations are the same as those of NMOS

Common-Source

Common-Gate

Common-Drain

Since PMOS has the same signal model, configurations and results are exactly the same.
Common Source Configuration (Gain)

Signal Circuit:

\[ v_o = -g_m v_{gs} \left( r_o \parallel R'_L \right) \]
\[ A_v = \frac{v_o}{v_i} = -g_m \left( r_o \parallel R'_L \right) \]
\[ A_{vo} = -g_m r_o \]

Signal Circuit with MOS SSM:

By KCL
Common Source Configuration ($R_i$)

Signal Circuit with MOS SSM:

Relevant circuit for $R_i$ calculation

\[ i_i = 0 \]

\[ R_i = \frac{v_i}{i_i} = \infty \]
Common Source Configuration ($R_o$)

Signal Circuit with MOS SSM:

Relevant circuit for $R_o$ calculation (set $v_i = 0$)

Current source becomes open circuit

$R_o = r_o$
Common Source with Source Resistor

Small Signal Circuit:

Signal Circuit with MOS SSM:

Input Resistance

\[ i_i = 0 \Rightarrow R_i = \frac{v_i}{i_i} = \infty \]
Common Source with Source Resistor (Gain)

Node voltage method:

\[ v_{gs} = v_i - v_S \]

Node \( v_S \):

\[ \frac{v_S}{R_S} + \frac{v_S - v_o}{r_o} - g_m (v_i - v_S) = 0 \]

Node \( v_o \):

\[ \frac{v_o}{R'_L} + \frac{v_o - v_S}{r_o} + g_m (v_i - v_S) = 0 \]

\[ A_v = \frac{v_o}{v_i} = -\frac{g_m r_o R'_L}{r_o + (1 + g_m r_o) R_S + R'_L} \]

\[ A_v \approx -\frac{g_m R'_L}{1 + g_m R_S + R'_L / r_o} \]

\[ A_{vo} = -g_m r_o \]
Common Source with Source Resistor ($R_o$)

- set $v_i = 0$
- Attach $v_x$ and compute $i_x$
- $R_o = v_x / i_x$

Node voltage method:

\[
\begin{align*}
\nu_{gs} &= -\nu_S \\
\text{Node } \nu_S &\quad \frac{\nu_S}{R_S} + \frac{\nu_S - \nu_x - g_m(-\nu_S)}{r_o} = 0 \\
\nu_S &= \frac{\nu_x}{R_S} = \frac{\nu_x}{r_o + (1 + g_m r_o) R_S} \\
i_x &= \frac{\nu_S}{R_S} = \frac{\nu_x}{r_o + (1 + g_m r_o) R_S}
\end{align*}
\]

\[
\frac{1}{R_o} \equiv \frac{i_x}{\nu_x} = \frac{1}{r_o + (1 + g_m r_o) R_S}
\]

\[
R_o = r_o + (1 + g_m r_o) R_S
\]
Common Gate Configuration

Gain

Node voltage method:

\[ v_{gs} = -v_i \]

Node \( v_o \)

\[
\frac{v_o}{R'_L} + \frac{v_o - v_i}{r_o} + g_m (-v_i) = 0
\]

\[
\frac{v_o}{r_o \parallel R'_L} = \frac{1 + g_m r_o}{r_o} v_i
\]

\[
A_v = \frac{v_o}{v_i} = \frac{1 + g_m r_o}{r_o} (r_o \parallel R'_L)
\]

\[
A_v \approx g_m (r_o \parallel R'_L)
\]

\[
A_{vo} \approx g_m r_o
\]
Common Gate Configuration \((R_i \text{ and } R_o)\)

### Input Resistance

![Input Resistance Circuit Diagram]

KVL: \[ v_i = (i_i + g_m v_{gs})r_o + i_i R'_L \]

\[ v_i (1 + g_m r_o) = i_i (r_o + R'_L) \]

\[
R_i = \frac{v_i}{i_i} = \frac{r_o + R'_L}{1 + g_m r_o}
\]

\[
R_i \approx \frac{1}{g_m} + \frac{R'_L}{g_m r_o}
\]

### Output Resistance (set \(v_i = 0\))

![Output Resistance Circuit Diagram]

Current source becomes open circuit

\[
R_o = r_o
\]

---

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Common Drain Configuration (Source Follower)

Gain

Node voltage method:

\[ v_{gs} = v_i - v_o \]

Node \( v_o \)

\[ \frac{v_o}{R'_L} + \frac{v_o}{r_o} - g_m(v_i - v_o) = 0 \]

\[ g_m v_i = \frac{v_o}{r_o \| R'_L} + g_m v_o \]

\[ A_v = \frac{g_m (r_o \| R'_L)}{1 + g_m (r_o \| R'_L)} \]

\[ A_{vo} = \frac{g_m r_o}{1 + g_m r_o} \approx 1 \]
Common Drain Configuration (Source Follower)

Input Resistance

\[
R_i = \frac{v_i}{i_i} = \infty
\]

Output Resistance (set \( v_i = 0 \))

\[
i_x = \frac{v_x}{r_o} - g_m v_{gs} = \frac{v_x}{r_o} + \frac{v_x}{1/ g_m}
\]

\[
R_o = \frac{1}{g_m} \parallel r_o \approx \frac{1}{g_m}
\]
MOS Basic Amplifier Configurations
(PMOS circuits are identical)

**Common Source**

\[ A_v = -g_m (r_o \parallel R'_L) \]

**Common Gate**

\[ A_v = g_m (r_o \parallel R'_L) \]

**Common Source with RS**

\[ A_v = -\frac{g_m R'_L}{1 + g_m R_S + R'_L / r_o} \]

**Common Drain/Source Follower**

\[ A_v = \frac{g_m (r_o \parallel R'_L)}{1 + g_m (r_o \parallel R'_L)} \]
MOS Elementary R forms
A Transistor can be configured to act as a resistor for small signals!

Ex: Output resistance of a CS Amplifier

Set $v_i = 0$, current source becomes open circuit

$$R_o = r_o$$

Notation:
$r_o$ is the small-signal resistance between the point and ground

- If we connect any two terminals of a MOS, we get a two-terminal device.
  - For Small Signals, this two terminal device can be replaced with its Thevenin equivalent circuit.
  - As there is NO independent sources present, the Thevenin equivalent circuit is reduced to a resistor.
Transistor can be configured to act as a resistor for small signals!

- But, MOS should be in saturation for small signal model to work!
  - Connection between MOS terminals are, therefore, made through ground for small signals.
  - In fact, one or both MOS terminals have to be connected to bias power supplies to ensure that MOS is in saturation.

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MOS Elementary R forms
(PMOS circuits are identical)

\[ r_o (1 + g_m R) + R \approx r_o (1 + g_m R) \]

Output resistance of CS Amp with \( R_s \)

\[ \frac{r_o + R}{1 + g_m r_o} \]

Input resistance of CG Amp

\[ \frac{1}{g_m} || r_o \approx \frac{1}{g_m} \]

Diode-connected Transistor
Always in saturation!

Above configurations are for Small Signal. Typically one or both “signal” grounds are actually connected to bias voltage sources to ensure that MOS is in saturation!
Basic BJT Amplifier Configurations

We are considering only signal circuit here!
Possible BJT amplifier configurations

- **Common-Emitter**
- **Common-Base**
- **Common-Collector**

- **Common-Emitter with $R_E$**
  - $v_i$ does not change

- **Same as Common Base**

- **Not Useful**
Common Emitter Configuration (Gain)

Signal Circuit:

\[ v_o = -g_m v_\pi (r_o \parallel R'_L) \]

\[ A_v = \frac{v_o}{v_i} = -g_m (r_o \parallel R'_L) \]

\[ A_{vo} = -g_m r_o \]

Signal Circuit with MOS SSM:

By KCL
Common Emitter Configuration ($R_i$)

Signal Circuit with MOS SSM:

Relevant circuit for $R_i$ calculation

\[ i_i = \frac{v_i}{r_\pi} \]

\[ R_i = \frac{v_i}{i_i} = r_\pi \]
Common Emitter Configuration ($R_o$)

Signal Circuit with MOS SSM:

Relevant circuit for $R_o$ calculation (set $v_i = 0$)

Current source becomes open circuit

$$R_o = r_o$$
BJT Basic Amplifier Configurations
(PNP circuits are identical)

Common Emitter

\[ A_v = -g_m (r_o \parallel R'_L) \]

Common Base

\[ A_v = g_m (r_o \parallel R'_L) \]

Common Emitter with \( R_E \)

\[ A_v = -\frac{g_m R'_L}{1 + g_m R_E + (R'_L / r_o)(1 + R_E / r_n)} \]

Common Collector/Emitter Follower

\[ A_v = \frac{g_m (r_o \parallel R'_L)}{1 + g_m (r_o \parallel R'_L)} \]
BJT Elementary $R$ forms
(PNP circuits are identical)
Discrete Amplifier Configurations

We focus on biasing with Emitter/Source degeneration!
Emitter-degeneration bias circuits

Bias with **one** power supply (voltage divider)

\[ V_{BB} = I_B R_B + V_{BE} + I_E R_E \]

Bias with **two** power supplies

\[ V_{EE} = I_B R_B + V_{BE} + I_E R_E \]
Emitter-degeneration bias circuits have similar signal circuits

Bias with **one** power supply (voltage divider)

Bias with **two** power supplies

\[ R_B = R_{B1} \parallel R_{B2} \]
By-pass capacitors

- There is no $R_E$ in the basic Common-Emitter configuration.
- However, $R_E$ is necessary for bias in discrete circuits.
- Use a **by-pass capacitor**

**Real Circuit**

**Bias Circuit:**
Cap is open, $R_E$ stabilizes bias

**Signal Circuit:**
Capacitor shorts $R_E$
Discrete Common-Emitter Amplifier

Real Circuit

CE amplifier:
Input at the base
Output at the collector

Standard Bias Circuit:*
Caps are open circuit

* Bias calculations are NOT done here as we have done them before.
Signal circuit of the discrete CE Amplifier

Real Circuit

Short caps
Zero bias supplies

Rearrange

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Discrete CE Amplifier (Gain)

Basic CE configuration

- Signal input at the base
- Signal output at the collector
- No $R_E$

$R'_L = R_C \parallel R_L$

\[
\frac{v_o}{v_i} = -g_m \left( r_o \parallel R_C \parallel R_L \right)
\]

\[
\frac{v_o}{v_{sig}} = \frac{R_i}{R_i + R_{sig}} \times \frac{v_o}{v_i}
\]

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Discrete CE Amplifier ($R_i$)

- Replace transistor with its equivalent resistance

**Elementary R form**

\[ R = r_\pi \]

\[ R_i = R_B \parallel r_\pi \]
Discrete CE Amplifier \( (R_o) \)

- Set \( v_{sig} = 0 \)
- Replace transistor with its equivalent resistance

Elementary R form

\[
R = r_o
\]

\[
R_o = R_C \parallel r_o
\]
Discrete CE and CS Amplifiers

\[
\frac{v_o}{v_i} = -g_m \left( r_o \parallel R_C \parallel R_L \right)
\]

\[
R_i = R_B \parallel r_\pi
\]

\[
R_o = R_C \parallel r_o
\]

\[
\frac{v_o}{v_{\text{sig}}} = \frac{R_i}{R_i + R_{\text{sig}}} \times \frac{v_o}{v_i}
\]
Discrete CS Amplifier with $R_S$

Real Circuit

Input at the gate
Output at the drain

Bias Circuit
Caps open

Signal Circuit
Short caps
Zero bias supplies

CS amplifier with $R_S$
Discrete CS Amplifier with $R_S$ (Gain)

- Signal input at the gate
- Signal output at the drain
- $R_S$

Basic CS configuration with $R_S$

$$R'_L = R_D \parallel R_L$$

$$\frac{v_o}{v_i} = \frac{g_m (R_D \parallel R_L)}{1 + g_m R_S + (R_D \parallel R_L) / r_o}$$

$$\frac{v_o}{v_{sig}} = \frac{R_i}{R_i + R_{sig}} \times \frac{v_o}{v_i}$$

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Discrete CS Amplifier with $R_S (R_i)$

✓ Replace transistor with its equivalent resistance

$R = \infty$

Elementary R form

$R_i = R_G$
Discrete CS Amplifier with $R_S$ ($R_o$)

- Set $v_{sig} = 0$
- Replace transistor with its equivalent resistance
- Since $i_g = 0$, $R_{sig}$ and $R_G$ can be removed ($v_g = 0$)

$$R = r_o(1 + g_m R_S) + R_S$$

Elementary R Configuration

$$R_o = R_D \parallel \left[ r_o (1 + g_m R_S) + R_S \right]$$
Discrete CE and CS Amplifiers with $R_E / R_S$

\[ \frac{v_o}{v_{sig}} = \frac{R_i}{R_i + R_{sig}} \times \frac{v_o}{v_i} \]

\[ v_o = -g_m \left( \frac{R_D}{R_L} \right) \]

\[ v_i = 1 + g_m R_E + \left[ \left( \frac{R_D}{R_L} \right) / r_o \right] (1 + R_E / r_\pi) \]

\[ v_o \approx -g_m \left( \frac{R_D}{R_L} \right) \]

\[ v_i = 1 + g_m R_E \]

\[ R_i \approx R_B \left[ r_\pi + (1 + \beta) R_E \right] \]

\[ R_o \approx R_C \left[ r_o \left( 1 + \frac{\beta R_E}{r_\pi + R_E + R_B / R_{sig}} \right) \right] \]

\[ \frac{v_o}{v_{sig}} = \frac{R_i}{R_i + R_{sig}} \times \frac{v_o}{v_i} \]
Discrete CB Amplifier

Real Circuit

Bias Circuit
Caps open

Signal Circuit
Short caps
Zero bias supplies

CS amplifier with $R_S$
Input at the gate
Output at the drain

Capacitor $C_B$ is necessary.
Otherwise, Amp gain drops substantially.
Discrete CB Amplifier (Gain)

- Signal input at the source
- Signal output at the drain

Basic CB form

\[ R_{L}' = R_C \parallel R_L \]

\[ \frac{v_o}{v_i} = +g_m (r_o \parallel R_C \parallel R_L) \]

\[ \frac{v_o}{v_{\text{sig}}} = \frac{R_i}{R_i + R_{\text{sig}}} \times \frac{v_o}{v_i} \]
Discrete CB Amplifier ($R_i$)

✓ Replace transistor with its equivalent resistance

\[
R = \frac{r_o + (R_C \parallel R_L)}{1 + g_m r_o}
\]

Elementary R Configuration

\[
R_i = R_E \parallel \left[ \frac{r_o + (R_C \parallel R_L)}{1 + g_m r_o} \right]
\]
Discrete CB Amplifier ($R_o$)

- Set $v_{sig} = 0$
- Replace transistor with its equivalent resistance

\[
R \approx r_o [1 + g_m (R_E \parallel R_{sig})]
\]

Elementary R Configuration

\[
R_o = R_C \parallel \left\{ r_o [1 + g_m (R_E \parallel R_{sig})] \right\}
\]
Discrete CB and CG Amplifiers

\[ \frac{v_o}{v_i} = +g_m \left( r_o \parallel R_C \parallel R_L \right) \]

\[ R_i = R_E \parallel \left[ \frac{r_o + (R_C \parallel R_L)}{1 + g_m r_o} \right] \]

\[ R_o = R_C \parallel \left\{ r_o \left[ 1 + g_m \left( R_E \parallel R_{sig} \right) \right] \right\} \]

\[ \frac{v_o}{v_{sig}} = \frac{R_i}{R_i + R_{sig}} \times \frac{v_o}{v_i} \]

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Discrete CD Amplifier (Source Follower)

Real Circuit

CS amplifier with $R_S$
Input at the gate
Output at the drain

Bias Circuit
Caps open

Signal Circuit
Short caps
Zero bias supplies
Discrete CD Amplifier (Gain)

✓ Signal input at the gate
✓ Signal output at the source

Basic CD form

\[
R'_L = R_S \parallel R_L
\]

\[
\frac{v_o}{v_i} = \frac{g_m (r_o \parallel R_S \parallel R_L)}{1 + g_m (r_o \parallel R_S \parallel R_L)}
\]

\[
\frac{v_o}{v_{\text{sig}}} = \frac{R_i}{R_i + R_{\text{sig}}} \times \frac{v_o}{v_i}
\]
Discrete CD Amplifier ($R_i$)

✓ Replace transistor with its equivalent resistance

Elementary R Configuration

$$R_i = R_G$$
Discrete CD Amplifier ($R_o$)

- Set $v_{sig} = 0$
- Replace transistor with its equivalent resistance
- Since $i_g = 0$, $R_{sig}$ and $R_G$ can be removed ($v_g = 0$)

\[
R_m \approx \frac{1}{g_m} \parallel r_o \approx \frac{1}{g_m}
\]

Elementary R Configuration

\[
R_o = R_S \parallel \frac{1}{g_m}
\]
Discrete CC and CD Amplifiers

\[
\begin{align*}
    v_o &= \frac{g_m (r_o \parallel R_E \parallel R_L)}{1 + g_m (r_o \parallel R_E \parallel R_L)} \\
    v_i &= \frac{R_i}{1 + g_m (r_o \parallel R_E \parallel R_L)} \\
    R_i &= R_B \parallel \left[ r_\pi + (1+\beta) (r_o \parallel R_E \parallel R_L) \right] \\
    R_o &\approx \frac{r_\pi + R_B \parallel R_{\text{sig}}}{\beta} \\
    v_o &= \frac{g_m (r_o \parallel R_S \parallel R_L)}{1 + g_m (r_o \parallel R_S \parallel R_L)} \\
    v_i &= \frac{R_i}{1 + g_m (r_o \parallel R_S \parallel R_L)} \\
    R_i &= R_G \\
    R_o &= R_S \parallel \frac{1}{g_m} \\
    \frac{v_o}{v_{\text{sig}}} &= \frac{R_i}{R_i + R_{\text{sig}}} \times \frac{v_o}{v_i}
\end{align*}
\]
Impact of Coupling and Bypass Capacitors (1)

\[ \frac{v_i}{v_{\text{sig}}} = \frac{R_i}{R_i + R_{\text{sig}} + 1/(j \omega C_{c1})} \]

\[ \frac{v_i}{v_{\text{sig}}} = \frac{R_i}{R_i + R_{\text{sig}}} \times \frac{1}{1 - j \omega p_1 / \omega} \]

\[ \omega_{p_1} = \frac{1}{(R_i + R_{\text{sig}})C_{c1}} \]

High Pass filter with pole at \( \omega_{p_1} \)

\[ \frac{v_o}{v_{\text{sig}}} = \frac{R_i}{R_i + R_{\text{sig}}} \times \frac{v_o}{v_i} \times \frac{1}{1 - j \omega p_1 / \omega} \]
Impact of Coupling and Bypass Capacitors (2)

Each capacitor introduces a pole!

Poles can be found by inspection:
1) Set $v_{\text{sig}} = 0$
2) Consider each capacitor separately (i.e., assume all others are short).
3) Find $R$, the total resistance seen between capacitor terminals
4) Pole is given by

$$f_p = \frac{1}{2\pi RC_{c_1}}$$

The lower cut-off frequency of amplifiers can be found from

$$f_p \approx f_{p1} + f_{p2} + \ldots$$