1. Review of Circuit Theory Concepts
Circuit Theory is an Approximation to Maxwell’s Electromagnetic Equations

- A circuit is made of a bunch of “elements” connected with ideal (i.e., no resistance) wires.

- Circuit Theory is an Approximation to Maxwell’s Electromagnetic Equations:
  - Speed of light is infinite (or dimension of the circuit is much smaller than wave-length of voltage/current waveforms:
    For each electron that enters an element, an electron leaves that element instantaneously.
  - Electric and magnetic fields are confined within each element:
    1) Internal of an element manifests itself as an \( iv \) characteristic eq.
    2) Elements communicates with each other only through the wires!

- Since the rest of the circuit only sees the \( iv \) characteristics of an element, different physical elements with similar \( iv \) characteristics are identical!
Currents and voltages are circuit variables

- Equations governing the circuits are:
  - Internal of each element:
    - $iv$ characteristic equation of each element: $v = f(i)$
  - How the elements are connected:
    - KCL: (conservation of charge), and KVL: (topology)

- A circuit with $N$ two-terminal element has $2N$ variables and need $2N$ equations:
  - $N$ $iv$ characteristic equation
  - $N$ KCL/KVL

- Node-voltage (or mesh current methods) reduce the number of equations to be solved by atomically satisfying all KVLs (or KCLs).
Linear circuits have many desirable properties

- A linear circuit element has a linear $iv$ characteristic equation $(Av + Bi + C = 0)$.
- If all elements in a circuit are linear, the circuit would be linear and has many desirable properties (e.g., proportionality and superposition) which are essential for many functional circuits.
- Circuit theory has “symbols” for ideal linear elements:
  - five two-terminal elements: resistors, capacitors, inductors, independent voltage and independent current sources
  - Four four-terminal elements: controlled voltage and current sources.
- It is essential to remember that the above ideal element are NOT real components. Rather they are representative of elements with a certain $iv$ characteristic equation.
Practical elements can only be approximated by “ideal” circuit theory elements.

Is a symbol for

\[ v = R \cdot i \]

Is NOT exactly this
Practical elements can only be approximated by “ideal” circuit theory elements

At high enough current, the resistor “burns” up

As the current increases, resistor heats up and its resistance increases

A Lab resistor can be approximated as an ideal circuit theory resistor for a range of current or voltage (identified by its rated maximum power)
We will analyze many functional circuits

Two-terminal Networks

Function is defined by the $i v$ equation

Two-port Networks

Function is defined by the transfer function (e.g., $v_o$ in terms of $v_i$)
A linear two-terminal network can be represented by its Thevenin Equivalent

- **Thevenin Theorem:**
  - If all elements inside a two-terminal network are linear, the $iv$ equation of the two-terminal network would be linear: $Av + Bi + C = 0$
  - A linear two-terminal network can be modeled with two ideal circuit theory elements ($v_T = -C/A$, $R_T = -B/A$)

\[ v = v_T - R_Ti \]

- If the two-terminal network does NOT contain an independent source, $v_T = 0$ and it reduces to a resistor.
- See Lecture note for examples of computing/measuring Thevenin equivalent circuit
A Functional circuit contains several two-terminal and two-port networks

We divide the circuit into building blocks to simplify analysis and design
Source only sees a load resistor

We only need to analyze the response of a source ONCE with RL as a parameter.

In fact, we only need to find the Thevenin parameters.

A two-terminal network containing NO independent source
A two-terminal network containing AN independent source

A two-terminal network containing NO independent source

Transfer function of a two-port network can be found by solving the above circuit once.
Accuracy

Mathematical precision is neither possible nor required in practical systems!
Accuracy (or tolerance) in practical systems

- **Measurement Accuracy:**
  - Measuring instruments have a finite accuracy.
  - When a scope with an 2% read a voltage of 1.352 V, it means that the real voltage is in the range of $1.352 \pm 0.02 \times 1.352$ (or between 1.325 and 1.379 V).

- **Component Accuracy:**
  - Components are manufactured with a finite accuracy (tolerance).
  - A 1k resistor with 5% accuracy has a resistance between 0.950 and 1.050k.

- **Modeling/Analysis Accuracy:**
  - We “approximate” practical circuit element with ideal circuit theory element.
  - We make approximation in the analysis by ignoring terms.
How accuracy affect analysis:

- When a number has, $A$, has a relative accuracy of $\varepsilon$, it means that its value is between $A (1 - \varepsilon)$ and $A (1 + \varepsilon)$.

- Alternatively, we are saying that all numbers in that range are approximately equal to each other.

\[
B \approx A \iff A (1 - \varepsilon) \leq B \leq A (1 + \varepsilon)
\]

- When we assume $a \ll A$, we mean:

\[
A + a \approx A \implies A (1 - \varepsilon) \leq A + a \leq A (1 + \varepsilon)
\]

\[
A - \varepsilon A \leq A + a \leq A + \varepsilon A
\]

\[
- \varepsilon A \leq a \leq \varepsilon A
\]

\[
a \ll A \implies |a| \leq \varepsilon |A|
\]
How accuracy affects modeling (1)

$iv$ equation of an element

Accuracy of 5%:
Shaded region: $1 \text{ V} \pm 5\%$

$v \approx 1 \text{ V}$ for all currents

This element can be modeled with an independent voltage source with $v_s = 1 \text{ V}$ with an accuracy of 5%
Accuracy of 2%:
Shaded region: 1 V ± 2%

Voltage is NOT constant. So the element CANNOT be modeled as independent voltage source with 2% accuracy

Accuracy of 2%:

However, it can be modeled with a linear $\textit{iv}$ equation corresponding to $v_T = 1.05 \text{ V}$ and $R_T = 1.2 \text{ } \Omega$