Problem 1. Consider the circuit below with $V_{tp} = -0.5$ V, $\mu_pC_{ox}(W/L) = 1$ mA/V$^2$ and $\lambda = 0.05$/V. A) Find $R_S$ such that $I_D = 0.5$ mA. B) Find the circuit gain, $R_i$, $R_o$, and $f_l$.

This is a common-drain amplifier (input at the gate, output at the source) which is biased with two power supplies.

A) Bias: we set $v_{sig} = 0$, and open the capacitor.

$$I_D = 0.5 \times 10^{-3} = 0.5\mu_pC_{ox}(W/L)V_{OV}^2 = 0.5 \times 10^{-3}V_{OV}^2$$

$$V_{OV} = 1 \text{ V} \quad \rightarrow \quad V_{SG} = V_{OV} + |V_{tp}| = 1.5 \text{ V}$$

SG-KVL: $2.5 = R_S I_D + V_{SG} + 10^3 I_G$

$2.5 = 0.5 \times 10^{-3} R_S + 1.5$

$R_S = 2.00 \text{ k}$

SD-KVL: $2.5 = R_S I_D + V_{SD} - 2.5 \quad \rightarrow \quad V_{SD} = 4 \text{ V} > V_{OV} = 1$ (Saturation)

B) Amp parameters: First, we calculate the small-signal parameters:

$$g_m = \frac{2I_D}{V_{OV}} = \frac{2 \times 0.5 \times 10^{-3}}{1} = 1 \text{ mA/V}, \quad r_o = \frac{1}{\lambda I_D} = \frac{1}{0.05 \times 0.5 \times 10^{-3}} = 40.0 \text{ k}$$

In a common-drain amplifier or source follower (Note $R_G = \infty$):

$$\frac{v_o}{v_i} = \frac{g_m(r_o \parallel R_S \parallel R_L)}{1 + g_m(r_o \parallel R_S \parallel R_L)}$$

$$r_o \parallel R_S \parallel R_L = 40.0 \text{ k} \parallel 2.00 \text{ k} \parallel 20 \text{ k} = 1.74 \text{ k}$$

$$g_m(r_o \parallel R_E \parallel R_L) = 1.74 \quad \rightarrow \quad \frac{v_o}{v_i} = \frac{1.74}{1 + 1.74} = 0.635$$

$$R_i = R_G = \infty$$

$$R_o = R_S \parallel (1/g_m) \parallel r_o = 2 \text{ k} \parallel 1 \text{ k} \parallel 40 \text{ k} = 656 \Omega$$

$$\frac{v_o}{v_{sig}} = \frac{R_i}{R_i + R_{sig}} \times \frac{v_o}{v_i} = 1 \times 0.635 = 0.635$$

$$f_l = f_{p2} = \frac{1}{2\pi(R_L + R_o)C_{c2}} = \frac{1}{2\pi(20 \times 10^3 + 656)100 \times 10^{-9}} = 77.1 \text{ Hz}$$
Problem 2. Find $v_o$ in terms of $v_i$ in the circuit below with a Si diode with $V_Z = 2$ V.

**Diode ON:** $v_D = 0.7$ V, $i_D \geq 0$

$v_o = -v_D = -0.7$ V

**KCL:** $i_1 = i + i_D \rightarrow i_D = i_1 - i$

**$\Omega$-law:**

$\frac{v_o}{10^3} = \frac{-0.7}{2 \times 10^3}$ and $i = \frac{v_i - v_o}{2 \times 10^3} = \frac{v_i + 0.7}{2 \times 10^3}$

$i_D = i_1 - i = \frac{-0.7}{2 \times 10^3} - \frac{v_i + 0.7}{2 \times 10^3} = \frac{-2.1 - v_i}{2 \times 10^3}$

$i_D \geq 0 \rightarrow -2.1 - v_i \geq 0 \rightarrow v_i \leq -2.1$ V

**Diode OFF:** $-V_Z = -2 < v_D < 0.7$ V, $i_D = 0$

**KCL:** $i_1 = i + i_D = i$

**KVL:** $v_o = \frac{1}{1 \Omega + 2 \Omega} \rightarrow v_o = v_i/3$

Since $v_D = -v_o = -v_i/3$:

$-2 < v_D < 0.7 \rightarrow -2 < -v_i/3 < 0.7 \rightarrow -2.1 < v_i < 6$ V

**Diode in Zener:** $v_D = -V_Z = -2$ V, $i_D \leq 0$

$v_o = -v_D = 2$ V

**KCL:** $i_1 = i + i_D \rightarrow i_D = i_1 - i$

**$\Omega$-law:**

$\frac{v_o}{10^3} = \frac{2}{2 \times 10^3}$ and $i = \frac{v_i - v_o}{2 \times 10^3} = \frac{v_i - 2}{2 \times 10^3}$

$i_D = i_1 - i = \frac{2}{2 \times 10^3} - \frac{v_i - 2}{2 \times 10^3} = \frac{4 - v_i + 2}{2 \times 10^3} = \frac{6 - v_i}{2 \times 10^3}$

$i_D \leq 0 \rightarrow 6 - v_i \leq 0 \rightarrow v_i \geq 6$ V

Thus:

For: $v_i \leq -2.1$ V $v_o = -0.7$ V Diode is ON

For: $-2.1 < v_i < 6$ V $v_o = v_i/3$ Diode is OFF

For: $6 V \leq v_i$ $v_o = 2$ V Diode is in Zener

Solution of ECE65 Final A (Winter 2013)
**Problem 3.** Find $v_o$ for $v_i = 0$ V (Si transistor with $\beta = 100$ and $V_A = \infty$).

Assume BJT is ON:

**EB-KVL:** $3 = 2.3 \times 10^3 i_E + v_{EB} + v_i = 2.3 \times 10^3 i_E + 0.7$

$i_E = 1$ mA

Since $i_E > 0$, BJT is ON. Assume BJT in active ($i_E \approx i_C = \beta i_B$):

**EC-KVL:** $3 = 2.3 \times 10^3 i_E + v_{EC} + 3.9 \times 10^3 i_C - 3$

$6 = 6.2 \times 10^3 i_E + v_{EC} \rightarrow v_{EC} = -0.2$ V $< 0.7$ (Not in active)

Assume BJT in saturation: $v_{EC} = 0.2$ V, $i_C/i_B < \beta$:

**EC-KVL:** $3 = 2.3 \times 10^3 i_E + v_{EC} + 3.9 \times 10^3 i_C - 3 = 2.3 + 0.2 + 3.9 \times 10^3 i_C - 3$

$i_C = 0.897$ mA $\rightarrow i_B = i_E - i_C = 1 - 0.897 = 0.103$ mA

Since $i_C/i_B = 8.71 < \beta = 100$ BJT is in saturation with $i_E = 1$ mA, $i_C = 0.897$ mA, and $v_{EC} = 0.2$ V. Then:

$\Omega$-law: $v_o - (-3) = 3.9 \times 10^3 i_C \rightarrow v_o = 0.498$ V

**Problem 4.** Consider the circuit below with a Si transistor with $\beta = 100$ and $V_A = \infty$. Find $V_{CE}$.

Since $I_E = 1$ mA, BJT is ON. Assume it is in active:

$I_C \approx I_E = 1$ mA $\rightarrow I_B = I_C/\beta = 10$ $\mu$A

**BE-KVL:** $0 = 33 \times 10^3 I_B + V_{BE} + V_E$

$0 = 33 \times 10^3 \times 10 \times 10^{-6} + 0.7 + V_E \rightarrow V_E = -1.03$ V

$\Omega$-Law: $3 - V_C = 1.5 \times 10^3 I_C = 1.5 \rightarrow V_C = 1.5$ V

$V_{CE} = V_C - V_E = 1.5 - (-1.03) = 2.53$ V

Since $V_{CE} = 2.53 > 0.7$ V, assumption of BJT in active is justified.
Problem 5. Find the circuit gain, $R_i$, $R_o$, and $f_L$ (Si BJT with $\beta = 100$ and $V_A = 100$ V).

This is a common-emitter amplifier (input at the base, output at the emitter) with $R_E$ which is biased with two power supplies.

Bias: we set $v_{sig} = 0$, and open the capacitor.

EB-KVL: $2 = 1.3 \times 10^3 I_E + V_{EB}$
$I_C \approx I_E = 1.00$ mA $\rightarrow I_B = I_c/\beta = 10.0$ $\mu$A
EC-KVL: $2 = 1.3 \times 10^3 I_E + V_{EC} + 5.1 \times 10^3 I_C - 6$
$V_{EC} = 8 - 1.3 - 5.1 = 1.60$ V $> V_{D0}$ (active)

$$g_m = \frac{I_C}{V_T} = \frac{10^{-3}}{26 \times 10^{-3}} = 38.5$ mA/V, $$
$$r_o = \frac{V_A}{I_C} = \frac{100}{10^{-3}} = 100$ k $r_\pi = \frac{\beta}{g_m} = 2.60$ k

In a common-emitter amplifier with $R_E$ (note $R_B = \infty$, $R_{sig} = 0$):

$$\frac{v_o}{v_i} = -\frac{g_m(R_C || R_L)}{1 + g_m R_E + (1 + R_E/r_\pi)(R_C || R_L)/r_o}$$
$R_C || R_L = 5.1$ k $|| 20$ k $= 4.06$ k

$$\frac{v_o}{v_i} = -\frac{156}{1 + 50.1 + 0.061} = -3.05$$

$$R_i = R_B || \left[ r_\pi + R_E + \frac{\beta R_E}{1 + (R_C || R_L)/r_o} \right] = 2.60$ k $+ 1.3$ k $+ 125$ k $= 129$ k

$$R_o = R_C || \left[ r_o \left( 1 + \frac{\beta R_E}{r_\pi + R_E + R_B || R_{sig}} \right) \right] = 5.1$ k $|| 3.43$ M $= 5.09$ k

$$\frac{v_o}{v_{sig}} = \frac{R_i}{R_i + R_{sig}} \times \frac{v_o}{v_i} = 1 \times (-3.05) = -3.05$$

$$f_l = f_{p2} = \frac{1}{2\pi(R_L + R_o)C_{c2}} = \frac{1}{2\pi(20 \times 10^3 + 5.09 \times 10^3)100 \times 10^{-9}} = 63.4$ Hz
Problem 6. Find the circuit gain, $R_i$, and $R_o$. ($\mu_n C_{ox}(W/L) = 8 \text{ mA/V}^2$, $V_t = 0.5 \text{ V}$, and $\lambda = 0$).

This is a two-stage amplifier. The first stage is a common-source amplifier with no $R_S$ (input at the gate, output at the drain, $R_S$ is by-passed by the 47 $\mu$F capacitor). The second stage is a common-drain amplifier (input at the gate, output at the source). Both are biased with source degeneration. Q1 is biased with a voltage divider and Q2 is directly biased from Q1 with $V_{G2} = V_{D1}$.

Bias: Bias circuit (capacitors open) is shown above. Because $I_G = 0$:

$$V_{G1} = \frac{200 \text{ k}}{200 \text{ k} + 300 \text{ k}} \times 5 \text{ = 2 V}$$
$$I_{D1} = 0.5\mu_n C_{ox}(W/L)V_{OV1}^2 = 0.5 \times 8 \times 10^{-3}V_{OV1}^2 = 4 \times 10^{-3}V_{OV1}$$

GS1-KVL: $V_{G1} = V_{GS1} + 10^3I_{D1} = V_{OV1} + V_t + 10^3I_{D1}$

$$2 = V_{OV1} + 0.5 + 10^3 \times 4 \times 10^{-3}V_{OV1}$$

$$4V_{OV1}^2 + V_{OV1} - 1.5 = 0 \rightarrow V_{OV1} = 0.500 \text{ V} \rightarrow I_{D1} = 1.00 \text{ mA}$$

$$V_{GS1} = V_{OV1} + V_t = 1.00 \text{ V} \rightarrow V_{S1} = V_{G1} - V_{GS1} = 2.0 - 1.0 = 1.00 \text{ V}$$

$\Omega$-Law: $5 - V_{D1} = 2 \times 10^3I_D = 2 \rightarrow V_{D1} = 3.00 \text{ V}$

$$V_{DS1} = V_{D1} - V_{S1} = 3 - 1 = 2.00 \text{ V} \overset{>}{V_{OV1} = 0.5} \text{ (Saturation)}$$

$$V_{G2} = V_{D1} = 3 \text{ V} \quad I_{D2} = 4 \times 10^{-3}V_{OV2}^2$$

GS2-KVL: $V_{G2} = V_{GS2} + 2 \times 10^3I_{D2} = V_{OV2} + V_t + 2 \times 10^3I_{D2}$

$$3 = V_{OV2} + 0.5 + 2 \times 10^3 \times 4 \times 10^{-3}V_{OV2}^2$$

$$8V_{OV2}^2 + V_{OV2} - 2.5 = 0 \rightarrow V_{OV2} = 0.500 \text{ V} \rightarrow I_{D2} = 1.00 \text{ mA}$$

$$V_{GS2} = V_{OV2} + V_t = 1.0 \text{ V} \rightarrow V_{S2} = V_{G2} - V_{GS2} = 3 - 1 = 2.00 \text{ V}$$

$$V_{DS2} = 5 - V_{S2} = 3.00 \text{ V} \overset{>}{V_{OV2} = 0.5} \text{ (Saturation)}$$
\[ g_{m1} = \frac{2I_{D1}}{V_{OV1}} = 4.00 \text{ mA/V}, \]
\[ r_{o1} = \frac{1}{\lambda_1 I_{D1}} = \infty \]
\[ g_{m2} = \frac{2I_{D2}}{V_{OV2}} = 4.00 \text{ mA/V}, \]
\[ r_{o2} = \frac{2}{\lambda_2 I_{D2}} = \infty \]

Signal circuit is shown above. The first stage is a common-source amplifier (no \( R_S \)) and the second stage is a common-drain amplifier. To find the circuit gain and \( R_i \), we start from the load side (\( R_{L2} = R_L = 20 \text{ k} \)). Note that \( R_{G1} = 200 \parallel 300 = 120 \text{ k} \) and \( R_{G2} = \infty \).

\[ \frac{v_{o2}}{v_{i2}} = \frac{g_{m2}(r_{o2} \parallel R_{S2} \parallel R_{L2})}{1 + g_{m2}(r_{o2} \parallel R_{S2} \parallel R_{L2})} \]
\[ r_{o2} \parallel R_{S2} \parallel R_{L2} = 1.82 \text{ k} \quad \rightarrow \quad \frac{v_{o2}}{v_{i2}} = \frac{7.28}{1 + 7.28} = 0.879 \]
\[ R_{i2} = R_{G2} \quad \rightarrow \quad R_{i2} = \infty \]

Setting \( R_{L1} = R_{i2} = \infty \):

\[ \frac{v_{o1}}{v_{i1}} = -g_{m1}(r_{o1} \parallel R_{D1} \parallel R_{L1}) = -g_{m1}R_{D1} = -8.00 \]
\[ R_{i1} = R_{G1} = 120 \text{ k} \]

To find \( R_o \), we start from the source side (\( R_{s1} = R_{s2} = 10 \text{ k} \)).

\[ R_{o1} = R_{D1} \parallel r_{o1} = R_{D1} = 2 \text{ k} \]
\[ R_{s2} = R_{o1} = 2 \text{ k} \]
\[ R_{o2} = \frac{1}{g_{m2}} \parallel r_{o2} \parallel R_{S2} = 250 \text{ } \Omega \parallel \infty \parallel 2 \text{ k} = 222 \text{ } \Omega \]

Parameters of the two-stage amplifier are:

\[ R_i = R_{i1} = 120 \text{ k} \quad R_o = R_{o2} = 225 \text{ } \Omega \]
\[ \frac{v_o}{v_{sig}} = \frac{R_i}{R_i + R_{s1}} \times \frac{v_{o1}}{v_{i1}} \times \frac{v_{o2}}{v_{i2}} = \frac{120 \text{ k}}{120 \text{ k} + 10 \text{ k}} \times (-8) \times 0.879 = -6.49 \]