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1. Introduction

The interaction of moving conducting fluids with electric and magnetic fields provides for a rich variety of phenomena associated with electro-fluid-mechanical energy conversion. Effects from such interactions can be observed in liquids, gases, two-phase mixtures, or plasmas. Numerous scientific and technical applications exist, such as heating and flow control in metals processing, power generation from two-phase mixtures or seeded high-temperature gases, magnetic confinement of high-temperature plasmas — even dynamos that create magnetic fields in planetary bodies. Several terms have been applied to the broad field of electromagnetic effects in conducting fluids, such as magneto-fluid-mechanics, magneto-gas-dynamics, and the more common one used here — magneto-hydrodynamics, or “MHD”.

Practical MHD devices have been in use since the early part of the 20th century. For example, an MHD pump prototype was built as early as 1907 [1]. More recently, MHD devices have been used for stirring, levitating, and otherwise controlling flows of liquid metals for metallurgical processing and other applications [2]. Gas-phase MHD is probably best known in MHD power generation. Since 1959 [3,4], major efforts have been carried out around the world to develop this technology in order to improve electric conversion efficiency, increase reliability by eliminating moving parts, and reduce emissions from coal and gas plants. Closed-cycle liquid metal MHD systems using both single-phase and two-phase flows also have been explored.

Still more novel applications are in development or on the horizon. For example, recent research has shown the possibility of seawater propulsion using MHD [5] and control of turbulent boundary layers to reduce drag [6]. Extensive worldwide research on magnetic
confinement of plasmas has led to attainment of conditions approaching those needed to sustain fusion reactions [7].

In the following sections, we review the basic equations describing coupled MHD behavior as well as some basic MHD phenomena in liquids, gases and two-phase mixtures. Much of the underlying physics described is common to many of the applications cited above. Also included are discussions of several of the most important applications, together with their special analysis techniques and examples of equipment involved.

2. Basic Equations

2.1 The Full Set of MHD Equations

The MHD Equations

The complete set of magnetohydrodynamic equations for a Newtonian, constant property fluid flow includes the Navier-Stokes equations of motion (i.e., momentum equation), the equation of mass continuity, Maxwell’s equations, and Ohm’s Law. In differential form they constitute the following system of equations:

\[ \rho \left( \frac{\partial u}{\partial t} + (u \cdot \nabla) u \right) = -\nabla p + j \times B + \mu_f \nabla^2 u + \rho g \] (1)

\[ \frac{\partial \rho}{\partial t} + \nabla \cdot \rho u = 0 \] (2)

\[ \nabla \times E = -\frac{\partial B}{\partial t} \] (3)
\[ \nabla \times B = \mu_m j \]  
(4)

\[ j = \sigma (E + u \times B) \]  
(5)

where the MHD body force \( j \times B \) is included in the Navier-Stokes equation. The displacement current has been neglected from Ampere’s Law, which is a valid approximation for non-relativistic phenomena typical of the response of an inertial liquid. Implicit in Eqs. 1–5 are the following additional relations:

\[ \nabla \cdot B = 0 \]  
(6)

\[ \nabla \cdot j = 0 \]  
(7)

This system of equations is a rich one, describing not only all of the phenomena generally associated with electromagnetics and fluid mechanics, but new phenomena not seen in either discipline. Simplifications usually are required to obtain solutions to physical systems of interest. For instance, for quasi-steady flow problems where \( \frac{\partial B}{\partial t} \) is negligible, the electric field can be represented as the gradient of an electric potential \( \varphi \), which simplifies the problem by eliminating vector Eq. 3.

**Magnetic induction**

The magnetic induction equation is derived easily by taking the curl of Ohm's Law:

\[ \nabla \times j/\sigma = \nabla \times E + \nabla \times (u \times B) \]  
(8)
If $\nabla \times E$ is replaced by Faraday's Law (Eq. 3) and $\nabla \times j$ is replaced by the curl of Ampere’s Law (Eq. 4), then, using the vector identity:

$$\nabla \times (\nabla \times B) = \nabla (\nabla \cdot B) - \nabla^2 B$$ \hspace{1cm} (9)

we obtain:

$$\frac{\partial B}{\partial t} = \nabla \times (u \times B) + \frac{I}{\mu_m \sigma} \nabla^2 B$$ \hspace{1cm} (10)

Equation 10 is known as the induction equation, and suggests that the motion of a conducting liquid in an applied magnetic field will induce a magnetic field in the medium. The total field is the sum of the applied and induced magnetic fields. The relative strength of the induced field is characterized by the magnetic Reynolds number ($Re_m = \sigma \mu_m u L$). The neglect of the induced magnetic field is a valid assumption when $Re_m$ is small.

**Dimensionless parameters**

Fluid mechanics equations typically are cast in dimensionless form so that the relative strengths of the different terms can be inferred by the size of any multiplying factors. The equation of motion (Eq. 1) can be written in dimensionless form by making the substitutions:

$$j^* = \frac{j}{\sigma u_o B_o}$$ \hspace{1cm} (11)

$$p^* = \frac{p}{\sigma u_o B_o^2 a}$$ \hspace{1cm} (12)

$$\nabla^* = \frac{1}{a} \nabla \hspace{1cm} u^* = u/u_o \hspace{1cm} B^* = B/B_o$$ \hspace{1cm} (13)
where \( a, u_o \) and \( B_o \) are characteristic values of length, velocity and applied magnetic field. Characteristic values of the current density and pressure have been selected carefully in order to scale the phenomena of interest; different values could have been selected, leading to different systems of non-dimensionalization. Using this system, the equation of motion (excluding gravity) becomes:

\[
\frac{1}{N} \left( \frac{\partial u^*}{\partial t} + (u^* \mathbf{\nabla}) u^* \right) = - \mathbf{\nabla} p^* + j^* \times B^* + \frac{1}{Ha^2} \mathbf{\nabla}^2 u^*
\]

The characteristic parameters \( Re, Ha \) and \( N \) are the Reynolds number, the Hartmann number, which is an average measure of the ratio of magnetic to viscous forces, and the interaction parameter, which is a measure of the ratio of magnetic to inertial forces. They are defined as:

\[
Re = \frac{\rho u_o a / \mu_f}{15} \\
Ha = \frac{a B_o f}{\mu_f} \\
N = \frac{Ha^2}{Re} = \frac{a B_o^2 \sigma_f / \rho u_o}{17}
\]

Table 1 gives representative values of these characteristic dimensionless parameters for example cases of interest. When the Hartmann number and interaction parameter are both sufficiently large, the momentum equation (Eq. 14) throughout the bulk of the fluid can be reduced to the simple form:

\[
\nabla p = j \times B
\]
2.2 Electrical Equations and Ohm's Law

The Lorentz force

Underlying the MHD body force is the fact that free charges moving in a magnetic field experience a “Lorentz” force perpendicular to both their velocity and the magnetic field induction:

\[ F_q = q (v \times B) \]  

(19)

For collisionless particles, the Lorentz force results in pure harmonic motions in the plane perpendicular to the magnetic field \((B_z)\) with characteristic cyclotron frequency \(\omega_c = qB_z/m\):

\[ m \dot{v}_y = -q v_x B_z \quad \quad m \dot{v}_x = q v_y B_z \]

(20)

\[ \ddot{v}_y = -\left(\frac{qB_z}{m}\right)^2 v_y \quad \quad \dot{v}_x = \frac{qB_z}{m} v_y \quad \quad \ddot{v}_x = -\left(\frac{qB_z}{m}\right)^2 v_x \]

(21)

\[ v_y = A_1 \cos \omega_c t + A_2 \sin \omega_c t \quad \quad v_x = A_3 \cos \omega_c t + A_4 \sin \omega_c t \]

(22)

In contrast, for collisional particles that are forced to follow the fluid velocity \(u\), the Lorentz force acts on electrons and ions in a direction perpendicular to the flow, but in in opposite directions for positive and negative charges. The net result is charge separation, leading to electric fields. The open circuit voltage between electrodes spaced a distance \(d\) apart in a conducting fluid is:

\[ V_{oc} = \int_{o}^{d} (u \times B) \cdot dl \]

(23)
An electric field $\varepsilon$ arises between the electrodes such that $\varepsilon + uB = 0$, corresponding to the zero-current condition in Ohm’s Law (Eq. 5). If current is allowed as a result of some return current path, then the electric field and electrode voltage are reduced due to the electrical resistance of the fluid:

$$\varepsilon = (u \times B - j/\sigma)$$

(24)

**The Hall effect**

In the regime between collisional and collisionless particles, the Hall effect can be important. Usually the current induced in the fluid is carried predominantly by electrons, which are considerably more mobile than ions. The electron drift velocity, given by:

$$j = n_e e u_e$$

(25)

leads to a second component of velocity, and so, according to Eq. 19, a secondary force and electric field:

$$\varepsilon_H = \beta j \times B$$

(26)

where $\beta = 1/n_e e$ is the Hall constant. The current component created by this electric field, i.e. the “Hall current”, is given by $-\mu_e j \times B$, where $\mu_e = \omega e/B$ is the electron mobility. This leads to a more generalized statement of Ohm’s Law including the Hall effect:

$$j/\sigma = (\varepsilon + u \times B) - \frac{\mu_e}{\sigma} j \times B$$

(27)
Generalized Ohm's Law

Ohm’s Law cited above is a constitutive relationship (for instance, analogous to the equation of state for gases) and as such has a limited range of applicability. Various forms of Ohm’s Law can be obtained depending upon the approximations made in deriving the current-field relationships from the equations of motion and the interactions of the constituent parts of the fluid. For weakly-ionized gases in thermal equilibrium at moderate temperature, Eq. 27 has the equivalent tensor form (neglecting ion current):

\[
j = \hat{\sigma} \cdot (\varepsilon + u \times B) = \hat{\sigma} \cdot \varepsilon^* \tag{28}\]

or

\[
j = \sum_k \sigma_k \varepsilon_k^* \tag{29}\]

When \(B = z_f B_z\),

\[
\hat{\sigma}/\sigma_f = \begin{pmatrix}
1 & 0 & 0 \\
\frac{\omega^2}{1 + \omega^2 \tau^2} & 1 + \omega^2 \tau^2 & 0 \\
\frac{\omega \tau}{1 + \omega^2 \tau^2} & 0 & 1 + \omega^2 \tau^2
\end{pmatrix} \tag{30}
\]

where

\[
\omega = \frac{eB}{m_e} \quad \text{electron cyclotron frequency}
\]

\[
\tau = \frac{\lambda}{c_e} \quad \text{electron collision mean free time}\]
The dimensionless product $\omega \tau$, often called the “Hall parameter”, is an important characteristic number in MHD design. The conductivity tensor is anisotropic due to the Hall component unless $\omega \tau \ll 1$ (typical values for weakly-ionized gases are 1–5). On a microscopic scale, the Hall parameter indicates the average angular travel of electrons between collisions. Typical values are $\lambda \approx 10^{-7}$ m, $c_e \approx 10^5$ m/s, $\tau \approx 10^{-12}$ s, $\omega = 1.76 \times 10^{11} B \approx 10^{12}$ /s for $B \approx 6$ T. Since the mean free path is inversely proportional to pressure, lower pressure and higher values of B give larger values of $\omega \tau$.

On a macroscopic scale, the value of $\omega \tau$ indicates the relative importance of the Hall field and Hall current. When $\omega \tau = 1$, the total current is directed 45° to the left of the $\mathbf{e}^*$ vector (see Figure 1), and for large values of $\omega \tau$ the current vector is nearly perpendicular to $\mathbf{e}^*$ (predominantly Hall current). In weakly-ionized gases, if both the electron ($\omega e \tau_e$) and ion ($\omega i \tau_i$) Hall parameters are large simultaneously then the angle is reduced. In this case, the conductivity is reduced due to a phenomenon called “ion slip”. Even though $\tau_i$ is ordinarily larger than $\tau_e$, $\omega_i$ is much smaller than $\omega e$, such that the product $\omega_i \tau_i$ is usually negligible.

For highly collisional fluids (such as condensed liquids) where $\tau \to 0$, the Hall current is negligible, and the use of Eq. 5 without further modification is satisfactory.

**Circuits with conducting ducts**

For MHD flows in electrically conducting ducts with no external load, a return current can exist in the duct walls (see Fig. 2). For a uniform flow velocity $u$, the loop voltage equation is written:
\[ \oint \mathbf{E} \cdot dl = 2b \left( uB - \frac{j_y}{\sigma_f} \right) - 2b \frac{j_w}{\sigma_w} = 0 \]  

(31)

where the subscript \( w \) denotes values in the wall. Conservation of current dictates:

\[ j_y a = j_w \delta \]  

(32)

so that Eq. 31 becomes:

\[ j_y = \sigma_f uB \frac{\Phi}{1+\Phi} \]  

(33)

where \( \Phi \) is the “wall conductance ratio”:

\[ \Phi = \frac{\sigma_w \delta}{\sigma_f a} \]  

(34)

For this type of uniform flow, Eq. 18 can be used to estimate the pressure required to drive a flow at velocity \( u \) through the duct.

2.3 Basic Flow Characteristics and Power Production

Hartmann flow

Equation 33 predicts zero current when the walls are not electrically conducting, however, the no-slip condition on the fluid at the walls results in a non-uniform channel velocity and
the formation of a boundary layer with a reduced $u \times B$ emf, allowing a conducting return-current path through the fluid itself.

For 1-D, fully-developed (hence inertialess) flow, the momentum equation for $u_x(z)$ becomes:

$$\frac{dp}{dx} = j_y B z + \mu_f \frac{d^2 u_x}{dz^2}$$

(35)

where $\frac{dp}{dx}$ is constant, and Ohm’s Law is:

$$j_y = \sigma (\varepsilon - u_x B z)$$

(36)

The electric field also is constant. Substituting Ohm’s Law into the momentum equation, we obtain a simple differential equation for $u_x$:

$$\mu_f \frac{d^2 u_x}{dz^2} - \sigma B^2 u_x = \left( \frac{dp}{dx} - \sigma B \varepsilon \right)$$

(37)

The solution to this equation is:

$$\frac{u_x}{u_b} = \frac{Ha \cosh Ha}{Ha \cosh Ha - \sinh Ha} \left( 1 - \frac{\cosh Ha \frac{z}{a}}{\cosh Ha} \right)$$

(38)

where $u_b$ is the bulk average velocity. For large values of the Hartmann number, Eq. 38 simplifies to:

$$\frac{u_x}{u_b} \approx 1 - \exp \left( Ha \left( \frac{z}{a} \right) - 1 \right)$$

(39)
Equation 39 describes a velocity profile that is nearly flat throughout the duct, with thin boundary layers at the walls where viscous drag forces the flow to zero. The thickness of the Hartmann boundary layer scales as a/Ha. The shape of the Hartmann profile is shown in Fig. 3 for a range of Hartmann numbers.

**Channel power and conversion efficiency**

The total electric power generated internally in a channel is equal to the mechanical work against the MHD body force, which is given by the product of the volume flow rate and pressure drop.

For a distributed medium, the Lorentz force leads to the pressure gradient:

\[
\nabla p = nF_q = nqv \times B = j \times B
\]  

(40)

so that the power density is simply:

\[
P_i = u \cdot \nabla p = u \cdot (j \times B)
\]

(41)

The amount of power delivered to the external load is

\[
P_L = j \cdot \varepsilon
\]

(42)

so that we can write the local electrical efficiency as:

\[
\eta_e = \frac{P_i}{P_L} = \frac{j \cdot \varepsilon}{u_x j_y B_z}
\]

(43)
3. Liquid MHD

3.1 Introduction

As seen above, the presence of the $j \times B$ force on the flow of conducting liquids can alter the velocity and pressure characteristics of the flow. The interaction with a magnetic field also can significantly delay the onset of turbulent fluctuations. These two effects together or individually can dramatically alter the heat transfer characteristics and fluid drag in closed or open channel flows. Technological applications of such phenomena include cooling systems for magnetic fusion reactors and reduced-drag ship hulls and airplane fuselages.

The MHD force can be applied in such a way that useful work can be done. For example, EM pumps can be designed to precisely control liquid flows – liquid metal flows, in particular – where high temperature and corrosive tendencies prohibit the use of seals in standard mechanical pumps. Such pumps have no moving parts and are extremely reliable. The converse is also possible; MHD generators can produce high currents at low voltages.

This section is concerned with exploring the interaction of the magnetic field with liquid flows both with and without applied electrical currents. For incompressible liquids, the equations of Section 2.1 are valid, with Eq. 2 reduced to:

$$\nabla \cdot \mathbf{u} = 0$$  \hspace{1cm} (44)

A discussion of various applications where such phenomena are encountered is included as well. More comprehensive discussions of many of these subjects can be found in textbooks [12–14] as well as the numerous other references provided throughout this section.
3.2 Closed Channel Flows

3.2.1 Fully Developed Channel Flow

The term fully developed, used to describe Hartmann flow in Section 2, denotes a condition where the velocity profile is no longer changing (zero derivative) in the main flow direction, i.e., a flow that has reached a stable steady state driven by a constant pressure gradient, \( P = -\frac{dp}{dx} \). The study of fully-developed flow with a constant applied magnetic field is useful because the equations can be solved analytically for a variety of cases, and then can be used as benchmark problems for complete numerical algorithms. In addition, the fully developed solutions predict some phenomena of general interest, especially the existence of different boundary layers, which are important for a general understanding of MHD flows. By controlling the amount of current that can flow in the main body of the liquid, these boundary layers and the MHD boundary conditions exert a significant influence on the velocity profile and pressure drop.

*Equations and boundary conditions for 2D fully-developed flow*

In a rectangular channel (a round pipe is fundamentally the same), we denote the flow direction as \( x \) and restrict the applied magnetic field \( B_o \) to be constant and aligned with \( z \), as seen in Fig. 2. The MHD equations, Eqs. 1, 3–5 and 44, can be simplified to the following form:

\[
\mu_f \left( \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right) + \frac{B_o}{\mu_m} \frac{\partial B}{\partial z} = -P \tag{45}
\]

\[
\frac{I}{\sigma_f \mu_m} \left( \frac{\partial^2 B}{\partial y^2} + \frac{\partial^2 B}{\partial z^2} \right) + B_o \frac{\partial u}{\partial z} = 0 \tag{46}
\]
where the velocity vector has only one component in the x-direction, and the magnetic field is the sum of the constant applied field in the z-direction and the small field induced in the x direction.

\[ u = [u(y,z),0,0] \quad \quad B = [B(y,z),0,B_0] \] (47)

Terms quadratic in \( B_x \) have been discarded as small, an assumption equivalent to assuming small \( \text{Re}_m \), so that \( B \) really represents a stream function for the electric current, where:

\[ \mu m j_y = \frac{\partial B_x}{\partial x} \quad \quad \mu m j_z = -\frac{\partial B_x}{\partial z} \] (48)

Walls parallel to the magnetic field (i.e., “side walls”) are located at \( y = \pm b \), and walls perpendicular to the field (i.e., “Hartmann walls”) are located at \( z = \pm a \). The boundary condition for the velocity is the standard fluid-mechanical no-slip condition:

\[ V = 0 \quad (\text{at all walls}). \] (49)

For certain boundary conditions on the induced field \( B \), analytic solutions exist in the form of infinite series to the system. Much of the classical work in the 1950’s and 1960’s focused on solving these equations either exactly or approximately by expanding one dimension in an appropriate set of eigenfunctions. Some boundary conditions for the induced field are summarized in Eqs. 50–52. These conditions are derived by considering the behavior of the normal \( n \) and tangential \( s \) current at the wall, and using Eq. 48 to phrase these conditions in terms of the induced magnetic field.

\[ B = 0 \quad \text{electrically insulated wall} \quad [15], [16] \] (50)

\[ \frac{\partial B}{\partial n} = 0 \quad \text{perfectly conducting wall} \quad [17] \] (51)

\[ a\Phi \frac{\partial B}{\partial n} - B = 0 \quad \text{thin conducting walls} \quad [18], [19], [20] \] (52)
**Invicid core flow and boundary layers**

When Ha is large, the infinite series solutions cited above show (see Fig. 4) the existence of a flat, inviscid core region in the central section of the duct, bordered by different types of viscous boundary layers near the Hartmann walls and side walls[21]. In this core region, the driving force of pressure is balanced entirely by the MHD force, \(\nabla p = j \times B\). The curl of this equation implies \( (B \cdot \nabla)j = 0\), indicating that the current density is constant along (applied) field lines in the core. A constant pressure and a constant current produce a flat constant velocity in the core region.

On the Hartmann walls at \(z = \pm 1\), a Hartmann layer forms (similar to that seen in the 1D example of Section 2.3), as shown in Figure 5. Hartmann layers have thickness of \(a/Ha\), and join smoothly to the core value of the velocity. The Hartmann layer serves as a region where electric currents induced in the core flow in the y-direction can return and complete the current loop. This role as current return path makes the Hartmann layer an active boundary layer, one whose properties controls the amount of flow possible in the core region. If the Hartmann wall is electrically conducting, the electric current will flow preferentially in the wall, and the influence of the Hartmann layer on the core flow will be accordingly reduced.

In most cases, it is the combined conductivity of the Hartmann layer and Hartmann wall that determines the MHD resistance to the fluid flow in the core, and so governs the pressure gradient \(P\) (i.e., the pressure gradient required to drive the flow at a given average velocity). In an electrically insulated channel, the increase in \(P\) is due to increased shear friction at the walls as a result of the modification of the parabolic laminar velocity profile. The average electromagnetic force in this case is zero since all the current induced in the flow closes through boundary layers in the fluid itself. For large Ha, \(P\) increases linearly.
with Ha. For electrically conducting channels, the net electromagnetic force is no longer zero, but can in fact be quite large. For perfectly conducting channel with large Ha, P increases with Ha^2.

On the side walls at \( y = \pm 1 \), one observes the formation of a different type of boundary layer, alternately known as a “side layer”, “shear layer”, or “Shercliff layer”. The interpretation of these layers is a region where mismatched electric potentials equalize, sometimes with a significant jet of liquid when the Hartmann walls are electrically conducting (this is the case pictured in Fig. 4). These jets can carry an appreciable portion of the net mass flux under certain cases. The thickness of side layers is of order \( a/\text{Ha}^{1/2} \), which is much greater than that of the Hartmann layer. Thus, in most cases (except when the Hartmann walls are highly conducting but the sidewalls are not), the electrical resistance of the side layers does not add significantly to the total resistance of the return current path, and so does not influence the core velocity. For larger Hartmann numbers on the order of \( 10^3 \) or \( 10^4 \), the flow in the core region drops almost to zero, and the velocity jets can be up to a factor of Ha times the core flow velocity. Obviously, such velocity structures can be very important in determining heat transfer in liquid metal coolant pipes, as well as affecting corrosion, mass diffusion and other important physical processes.

Hartmann layers will form on any wall that has a normal component of \( B_0 \), but shear layers form only along the magnetic field lines. Thus, if the channel is not perfectly aligned with \( B_0 \), then all walls will have Hartmann layers, and the shear layers will detach from the wall and form about the magnetic field line that intersects the corner of the duct (see Fig. 5b). Shear layers that extend into the fluid are known as free shear layers [22].
3.2.2 Developing flows, variable fields, variable duct sizes and entrance effects

Few practical MHD flows are fully developed over their entire length. Developing flows are inherently 3D since the motion, electric currents, and magnetic field and its gradients invariably are oriented in different directions. Sudden expansion and other change in the magnetic field, channel shape or channel electrical conductivity can result in significant changes in velocity profiles and additional “3D” pressure drops. Even local regions of reversed flow are possible in different MHD duct flow configurations, as in the case of a locally conducting crack in a pipe wall covered with an electrical insulator coating.

Only when the flow has advanced sufficiently far from these disturbances will it again become fully-developed and assume the characteristic velocity profiles and pressure drops discussed above. A rigorous treatment of the effects of changes in field and geometry possible in MHD machines are available in the literature [12,13]. One example of practical interest is the entrance of a rectangular duct into a magnetic field (typical of MHD conduction pumps discussed later), and the so-called M-shaped velocity profile. The use of the core flow approximation is a powerful tool for analyzing these MHD phenomena at large Ha.

Duct flows in varying magnetic fields

Consider a rectangular duct with the orientation shown in Fig. 2, but assuming that at x=0 the magnetic field changes abruptly from zero to some value B\(_z\) (the case of a more gradual transition does not fundamentally change the description). As in the fully developed case, the \(\mathbf{u} \times \mathbf{B}\) emf induces a current in the negative y-direction. At the side walls, the current can turn into the z-direction and then flow to the Hartmann walls, but now it can also turn
(more easily) into the x-direction and return to the other side wall through the region of no field directly adjacent to the region of high field, see Fig. 6. The same electric field set up by the charge separation that drives return current through the Hartmann layers (walls) will drive this current in the no-field region. Thus, an additional current closure path exists, and so more core current can flow in the high field region as compared to the corresponding fully developed flow in a region of constant $B_z$. This additional current results in a greater pressure drop, usually denoted $\Delta p_{3D}$.

The current in the x-direction near the side walls also causes a distortion of the velocity profile near the region of changing $B_z$. The $j_x$ current, which is positive in the upper part of the channel ($y > 0$), will induce a force in the negative y-direction near the sidewall. This force will essentially pressurize the sidelaye, and cause a velocity jet to form in the sidelaye. A similar result occurs in the lower half ($y < 0$) plane. The result is a velocity profile called “M-shaped”, where the velocity is reduced in the core due to increased $j \times B$ forces, but increases in the side layers. This looks very similar to the side layer jets that can occur in fully-developed flow when the Hartmann walls are electrically conducting, but the M-shaped profile forms in the developing region even when the entire channel is non-conducting. Like the fully-developed side layer jets, the M-shaped velocity profile is shorted out when the side walls are highly conducting, since the $j_x$ current preferentially flows in the walls in this case, and no force is induced in the liquid itself. The same effect occurs at the exit of a magnetic field, and even if the field is more gradually varied.

Mathematically, the formation of the M-shaped velocity profile can be understood by taking the curl of the steady invicid equation of motion:

$$\nabla \times (\rho(u \cdot \nabla)u = -\nabla p + j \times B$$  \hspace{1cm} (53)
and considering the $z$ component of vorticity:

$$\rho u \frac{\partial \omega_z}{\partial x} \approx - j_x \frac{\partial B_z}{\partial x} - B_z \frac{\partial j_y}{\partial y} \tag{54}$$

Both terms on the right hand side of Eq. 54 will be negative in the upper right quadrant of Fig. 6, and positive in the lower right quadrant. These sources of $z$-directed vorticity can be thought of as swirling motion around $z$ that decelerates the center and shifts fluid to the side layers causing the formation of the velocity jets. It is easily seen that the formation of side layer jets in fully-developed flow is also governed by the last term in the above equation, which is present in regions of constant $B_z$ as well.

**General core flow equations**

For high conductivity liquids like liquid metals, calculations can become very difficult unless simplifying approximations are made. One such approximation indicated by the above discussion is the so called core flow approximation, where the momentum equation is simplified to:

$$\nabla p = j \times B \tag{55}$$

In 1968, Kulikovskii [23] showed that the core equations (Eqs. 5, 7, 44 and 60) could be manipulated in such a way as to reduce the solution for any flow geometry without side walls or internal shear layers to, at most, four two-dimensional partial differential equations. In addition, the magnetic field is assumed to arise from external currents only ($Re_m=0$). The unique features of these equations allow us to separate components of velocity and current into components parallel and perpendicular to the magnetic field:
\[ J_\perp = \frac{B}{B^2} \times \nabla p \]  
(56)

\[ J \parallel = \int \left( B \times \frac{\nabla}{B^2} \right) \cdot \nabla p \ dl + A_1 \]  
(57)

\[ \mathbf{v}_\perp = -\frac{1}{\sigma B^2} \nabla p + \frac{B}{B^2} \times \nabla \varphi \]  
(58)

\[ \mathbf{v}_\parallel = \int \left( \left( B \times \frac{\nabla}{B^2} \right) \cdot \nabla \varphi + \left( \nabla p \cdot \frac{\nabla}{\sigma B^2} \right) + \frac{\nabla^2 p}{\sigma B^2} \right) \ dl + A_2 \]  
(59)

\( J_\perp \) is obtained by taking the cross product of \( B \) with the momentum equation (Eq. 55). Similarly, \( \mathbf{v}_\perp \) is obtained by taking the cross product of \( B \) with Ohm’s Law (Eq. 5). \( J \parallel \) and \( \mathbf{v}_\parallel \) are obtained by integrating the conservation laws (Eqs. 7 and 44) along the magnetic field direction, \( dl \), defined by \( \nabla \parallel = \frac{d}{dl} \).

Finally, the electric potential can be related to the parallel current:

\[ \sigma \frac{\partial \varphi}{\partial l} = J \parallel \]  
(60)

\[ \sigma \varphi = \int \int \left( B \times \frac{\nabla}{B^2} \right) \cdot \nabla p \ dl \ dl + A_1 l + A_3 \]  
(61)

The boundary conditions at the walls completely determine the unknown functions (\( A_1 \), \( A_2 \), \( p \), and \( \varphi \)). There are four boundary conditions. Zero mass flux into the walls and conservation of current are applied twice for each field line: once where the field line enters the fluid and once where it exits.

\[ \mathbf{v} \cdot \hat{n} = 0 \]  
(62)

\[ \Phi \nabla^2 \varphi = \mathbf{J} \cdot \hat{n} \]  
for conducting ducts  
(63)

\[ \frac{l}{H \alpha} (\nabla \times \mathbf{v}) \cdot \hat{n} = \mathbf{J} \cdot \hat{n} \]  
for nonconducting ducts  
(64)
This method has been successfully applied to the solution of a number of basic geometries [24]. For symmetric problems, the constants $A_1$ and $A_2$ can be eliminated. The constant $A_3$ can be replaced by the evaluation of $\phi$ at any location along $l$. (For example, $A_3$ can be replaced by $\phi_w$ – the potential at one wall.) In this case, only two partial differential equations remain for $p$ and $\phi_w$.

The resulting set of linear partial differential equations can be solved using any appropriate numerical technique. In the work described in [24], a finite difference representation was applied and SOR was used to solve the resulting system of algebraic equations. Corrections for side layers and internal shear layers also have been developed [25].

### 3.3 EM pumps and Flow Meters

One of the more practical uses of the MHD force is in pumping systems, where electrical energy is converted directly into force on the working liquid. “EM pumps” (as they are commonly known) have been in existence for many years, and many different designs have been successfully developed and employed. A generic conduction style pump is shown in Fig. 7. Another common MHD device is the EM flow meter, where the potential induced by fluid motion is measured and used to infer the average flow rate.

These devices can be constructed with no moving parts and no direct contact with the working liquid. This is a distinct advantage if high temperature and/or corrosive liquids must be handled. The absence of seals or moving parts leads to a highly reliable system. In addition, EM pumps are typically controllable, and even reversible, by varying the magnitude and direction of the applied current.
3.3.1 MHD flow meters

Eq. 23 suggests that the voltage induced by the \( u \times B \) emf would provide an ideal method by which one could measure the average flowrate of a conducting liquid. However, it is impossible to achieve a completely open-circuit configuration as described by Eq. 23, as return current will flow in pipe walls and boundary layers of all finite size channels. Some accounting for these return currents must be included when determining a relation for the voltage signal as a function of the flow velocity. Ohm’s Law, as written in Eq. 24, shows the effect of return currents on the measured voltage signal in a rectangular channel like that in Fig. 2:

\[
-\varepsilon = \varphi/2b = uB - j/\sigma
\]  

(65)

where \( \varphi \) is the voltage signal. Any core current allowed to flow, then, reduces the measured voltage signal by some amount. Using the result of Eq. 33 for the core current density we see that the voltage signal is:

\[
\varphi = \frac{2buB}{1+\Phi} = \frac{QB}{2a(l+\Phi)},
\]  

(66)

which is independent of the electrical conductivity of the liquid, except as it appears in the wall conductance ratio \( \Phi \). This means that many moderately conducting liquids can also be measured by this method, especially when an electrically insulated channel can be used (for insulated channels, substitute \( Ha^{-1} \) for \( \Phi \) in Eq. 66). For common laboratory and industrial values of the flowrate \( Q \), magnetic field induction and duct dimensions, the measured voltage is typically in the range of hundreds of microvolts to several millivolts.
Most MHD flowmeters do not use rectangular channels, but instead use round pipes that can fit easily into standard piping systems, like that shown in Fig. 8. Ref. [26] suggests the following relation for the volumetric flowrate in a pipe made of an electrically conducting material:

$$Q = \frac{3162}{k_1 k_2 k_3} \frac{d\varphi}{B}$$  \hspace{1cm} (67)

where $Q$ is in units of gpm, $B$ is in Gauss, the inside pipe diameter $d$ is in inches and the electric leads to measure $\varphi$ are attached to the outside of the pipe wall.

Equation 71 takes into account, in the form of semi-empirical multiplicative constants $k_{1-4}$, several non-ideal effects that were ignored in Eq. 65. $k_1$ is the pipe-wall current shunting correction factor defined as:

$$k_1 = \frac{2dD}{D^2 + d^2 + \frac{\sigma_w}{\sigma_f} (D^2 - d^2)}$$  \hspace{1cm} (68)

where $D$ is the outside pipe diameter, also in inches.

Poorly conducting liquids tend to have a $k_1$ correction factor that deviates significantly from unity, meaning that the electrical signal is lower for the same flowrate and so more difficult to measure accurately. $k_2$ is the magnetic field end-effect correction factor. It is empirically obtained as a function the magnet pole piece length $L$. Typical values are given in Table 2. $k_3$ is the magnetic material, temperature correction factor, which accounts for changes in the magnetic field as a function of temperature of the permanent magnetic material or electromagnet windings. The manufacturer of such materials typically provides the appropriate temperature correction factors. $k_4$ is the pipe thermal expansion correction factor which
accounts for changing pipe sizes as a function of temperature, and can be expressed as $k_4 = 1 + \gamma (T - T_o)$, where $\gamma$ is the thermal expansion coefficient for the pipe material. For 304 stainless steel the value of $\gamma$ is 9.6 $\mu$-in/in F

The idea behind these simple DC flowmeters has evolved into more complicated implementations where pulsed DC or pulsed AC electromagnets are used to sample the flowrate at some pulse rate. The pulsed electromagnet devices have lower power consumption and lower heat generation in the electric coils. These devices are available commercially \[27\] with all manner of pipe materials and sizes, and can be inserted easily into existing piping configurations. Small units ($d \approx 1$ inch) have accuracies of around 1% at full scale. The accuracy tends to improve as pipe sizes become larger.

### 3.3.2 Conduction pumps

Liquid metal conduction pumps, or “Faraday” pumps, consist of a rectangular channel in the gap of a magnet (either a permanent or electromagnet) where an electric current is passed through the conducting liquid perpendicular to the field. The resulting $j \times B$ force drives the flow. This type of pump is a conceptually simple extension of the rectangular duct flows discussed above with side walls replaced by electric bus bars connected to an outside voltage source which drives current in the direction opposite to the $v \times B$ emf.

To understand the pressure and flow rate behavior of a simplified conduction pump, pictured in Fig. 7, it is helpful to represent the pump by an equivalent electrical network (Fig. 9). Here $R_{LM}$ is the effective resistance of the liquid in the channel, $R_{Loss}$ is the resistance of any loss paths such as the channel walls and fringing paths outside of the magnetic field area, and $E_i$ is the voltage induced in the flow which works against the applied current, $I_a$. Assuming slug flow of flow rate $Q$ in a rectangular pumping channel of
flow length $L$ within the field, with walls of thickness $\delta_w$ and electrical conductivity $\sigma_w$, the following approximations can be applied:

$$R_{LM} = \frac{b}{La\alpha_f}$$
$$R_{Loss} = \frac{b}{L\delta_w\sigma_w}$$  \hspace{1cm} (69)

$$I_{LM} = \frac{2a\Delta p}{B}$$
$$E_i = 2bvB = \frac{QB}{2a}$$  \hspace{1cm} (70)

The duct geometry used is the same as in Fig. 2, and the $R_{Loss}$ term only takes into account current losses through conducting Hartmann walls. The circuit then can be solved to give the following linear relationship between pressure and flow rate:

$$\Delta p = \frac{BAI/2a - B^2LQ\Phi\sigma_f}{A(1+\Phi)}$$  \hspace{1cm} (71)

where $A = 4ab$ is the flow cross sectional area, and $\Phi$ is the wall conductance ratio defined in the usual way as in Eq. 34. This relationship is plotted in Fig. 10 for a small conduction pump with various values of $\Phi$. We see the obvious need for thin and/or low conductivity walls in the pumping channel structure in order to maximize the pumping power. In the case of $\Phi = 0.1$, all of the applied current is simply shunted through the walls, as the induced emf is larger than the applied voltage. Current flow in the core is reversed and MHD drag, instead of pumping, results.

The applied voltage for this particular pump, assuming $Q=10$ l/s and $\Phi=0.01$, is only 213 mV. The inherently low voltage and high current of conduction pumps is one of their disadvantages, since they require special power supplies capable of coupling efficiently to such a load. Possible power supplies which are more efficient than standard transformer-rectifier systems include homopolar and unipolar generators, which can be up to 80%
efficient. To increase the voltage of the load somewhat, it is common to run the field coil windings of the electromagnet in series with the LM section, so that only one supply (albeit at a greater power level) is needed.

The power dissipated in the conduction pump system for our example above is:

\[ P_a = 0.215 \, \text{V} \cdot 1500 \, \text{A} = 322 \, \text{W} \]  \hspace{1cm} (72)

The power delivered to the fluid \((Q \cdot \Delta p)\) is \(P_f = 0.01 \, \text{m}^3/\text{s} \cdot 25.7 \, \text{kPa} = 257 \, \text{W}\). This gives an efficiency of 81%. A formula for the electrical efficiency in the general case is easily constructed in terms of either the applied voltage or the applied current:

\[ \eta_e = \frac{Q\Delta p}{V_a I_a} = \frac{BQ}{2aV_a \left( 1 + \frac{\Phi}{l-BQ/2aV_a} \right)} = \frac{2bI_a - BLQ \sigma_f \Phi}{2bI_a \left( 1 + \frac{2bI_a}{BLQ \sigma_f} \right)} \]  \hspace{1cm} (73)

However, the losses in the wall included in this simple calculation are not the only losses the conduction pump experiences. Applied current usually fringes outside the area of the applied magnetic field where it induces no \(j \times B\) force. Energy is lost to the damping of turbulence and restructuring of the average velocity profile as the liquid enters the magnetic field area. Energy losses occur due to friction of the flow on the channel walls and the applied magnetic field is altered by the field induced by the applied current itself. All of these effects decrease the net efficiency of conduction pumps to the order of 15–20% for small pumps, and 40–50% percent for larger pumps. Including losses in generators and field windings, the bus bar efficiency for liquid metal conduction pumps varies from 10% to 40%, increasing with pump size [28].
As seen from Eq. 73, even in the absence of losses other than the resistance of the LM itself, the electrical efficiency of the system is equal to $\eta_e = \eta_{\text{ind}} = \frac{2bvB}{V_a}$, which is the induction efficiency, i.e., the ratio of induced voltage due to the $\mathbf{v} \times \mathbf{B}$ emf to the applied voltage $V_a$.

### 3.3.3 Induction Pumps

An alternative to the conduction pump is the induction pump, where electric currents are induced in the liquid metal by means of a time-varying magnetic field, producing a $\mathbf{j} \times \mathbf{B}$ force with the instantaneous field to drive the flow. Many types of induction pumps are possible. Here we focus on the flat linear induction pump and the annular linear induction pump. The advantage of induction pumps is that they can be driven easily by single-phase or 3-phase AC power sources, possibly with a step transformer for control of the flow rate. Typical disadvantages are greater power losses and the need for electrical insulation at high temperatures close to the working liquid.

The flat linear induction pump, or FLIP, is conceptually similar to an AC induction motor. The 3-φ winding, however, produces a sliding, rather than rotating magnetic field, which tends to pull the fluid along. The action of this class of pump is easily pictured by contemplating the simplified induction pump shown in Fig. 11. If the peak vertical magnetic field is sliding to the right, leaving behind a slightly reduced magnetic field, a current loop will be induced. This induced current tries to maintain the field at its original strength. The induced current into the page will be in a region of stronger magnetic field than the current coming out of the page, since the field peak is propagating to the right, so the net $\mathbf{j} \times \mathbf{B}$ force will to the right.
The disadvantage to these systems is that in order to have a relatively wide channel for liquid flow, the gap between the stators must be larger than that in induction motors, and the field losses are relatively higher. In addition, field fringing occurs at the boundaries of the wide flat channel where the magnetic core stops. One-sided stator induction pumps are also possible, but losses will be even higher. Such systems have uses as EM stirrers and for ship propulsion, as will be discussed later.

The annular linear induction pump (ALIP), sometimes called an Einstein-Szilard pump, is a modification of the FLIP so that end effects do not cause additional field losses. The ALIP consists of an annular flow region with an internal magnetic core. Induced currents flow in a continuous loop through the liquid and no core edge exists off which magnetic fields fringe. In essence, an ALIP is like a FLIP that has been bent around until the free ends are joined.

Design guidelines and photographs of various styles of EM induction pumps are available in Ref. [29].

3.3.4 MHD Ship Propulsion

Another potential use of EM pumping technology is MHD thrusters for ship propulsion. Seawater has a moderate electrical conductivity, of the order of $5 \, \Omega^{-1} \, m^{-1}$, and under the appropriate set of conditions can be pumped by the Lorentz force. Care must be taken to avoid large losses in conducting walls in this application, but this is more easily done when the working fluid is seawater, rather than high temperature liquid metals.

Conduction pump thrusters [30] are more commonly envisioned for MHD ship propulsion because of the difficulty inducing large currents in poorly conducting water. Using the
above equations for the conduction pump, we find that the ideal conduction pump thruster will deliver a power to the liquid equal to:

\[
P_w = F_{EM} v = (1-e)\eta_e \frac{\sigma V_d L a}{b}
\]

(74)

For a given size channel (usually limited by the size of the craft under consideration), a given applied voltage (usually limited by the power supply aboard the craft, e.g. a battery) and a given liquid (seawater), the mechanical power is maximized at \(\eta_e=50\%\). This means one half of the electrical power supplied is lost as Ohmic heating. Thruster designers must decide whether their goal is to maximize mechanical power, or to minimize energy consumption.

For a moderately sized submarine (10-m diameter, 83-m length) using four conduction thrusters with length \(L=55\) m, \(b=5\) cm and \(a=15\) cm, a 5 T field is sufficient to generate reasonable thrust and efficiency. At a speed of 36 knots, the thrusters will consume about 66 MW of electric power, requiring a 200 MW thermal nuclear plant with a typical thermal conversion efficiency (excluding power needed for other boat systems). This level of power is not unreasonable for a submarine of this size. Superconducting magnets are necessary for this field strength and core size, since the Ohmic losses in resistive magnets would be unacceptable.

At least one design using an induction pump thruster has been advanced. The “ripple motor” described by Mitchell and Gubser [31] utilizes a 3-ϕ AC solendoidal winding around a core of sea water. An annulus of liquid sodium or other liquid metal serves as an intermediate layer separated from the sea water by a flexible membrane. The thickness of the sodium layer is matched to the skin depth of the AC field. The traveling magnetic field sets up a traveling pressure wave in the sodium, and thus a traveling wave on the flexible
membrane. This wave pushes along the seawater and eventually ejects it out of the trailing end of the thruster, providing the thrust.

### 3.4 Turbulence in Liquid MHD Flow

MHD forces can have a large effect on the turbulence structure of liquid flows. Not only does the induction of a current density result in Ohmic dissipation of energy, a new energy loss mechanism that augments the viscous dissipation, but the field also changes the average velocity profiles as discussed in previous sections, resulting in new turbulence creation scenarios as compared to non-MHD flows. The magnetic field is typically thought to laminarize already turbulent flows, or to prevent the transition to turbulence in laminar flows. In theory, it is possible to laminarize any flow with a sufficiently strong magnetic field.

In electrically conducting channels, core velocity fluctuations are damped for values of $\text{Ha}/\text{Re} > 0.008$ [12]. Near the side layer jets, though, turbulent fluctuations increase, indicating that the strong velocity jets, like those depicted in Fig. 4, are unstable and periodically break down. For $\text{Ha}/\text{Re} > 0.02$ these fluctuations are also damped (or at least unresolved due to boundary layer thinning), and the liquid flow becomes effectively laminar.

Electrically insulating channels exhibit a change, as the field is applied, from standard turbulence to a quasi-2D turbulence, where the vorticity of the turbulent fluctuations is predominately aligned with the direction of the field. Turbulence fluctuations of this type can be quite long lived and probably result from the reorganization of the flow as it enters into a magnetic field. The formation of M-shaped velocity structures, which then decay
into rotating vortices, is the source of such fluctuations. In an infinitely long, electrically insulated channel, all turbulence fluctuations are eventually damped when $Ha/Re > 0.008$.

Control of turbulence near the wall of a ship or submarine can in theory reduce the overall drag on the structure. Early work on MHD channels flows [32] showed that the pressure drop in an initially turbulent LM duct flow could be reduced by the judicious application of a magnetic field (too strong a field will result in increased MHD drag, as discussed above). For the control of turbulence near ships, one must contend with the fact that sea water is a poor electrical conductor, and that induced currents alone will not dissipate enough energy to stabilize a turbulent boundary layer. Instead, currents must be generated by an applied voltage.

One such scheme to reduce drag on, and radiated noise from, a flat plate is to construct the surface with alternating north and south pole magnets interspersed between positive and negative electrodes (see Fig. 12). The criss-crossing lines of magnetic field and current induce a $j \times B$ force in the streamwise direction, acting as a sort of one sided conduction pump. Preliminary experiments [33] have shown that turbulent fluctuations can be reduced over much of the boundary layer when the modified interaction parameter ($N^*=J_o B_o \theta / 0.5 \rho u_\tau^2$ where $J_o$, $B_o$ are the current, field at the electrode, magnet surface and $\theta$ and $u_\tau$ are the standard momentum thickness and friction velocity of the boundary layer) is order one or larger. The boundary layer is found to approach an asymptotic value, rather than growing indefinitely and breaking down due to instability. Work in this area by a number of researchers is continuing.
3.5 Open Channel Flows

Open channel flows of liquids in magnetic fields are of interest for metallurgical and welding applications where melts and melt layers are influenced by electric currents and applied magnetic fields. There is also interest in open channel MHD flows in magnetic fusion energy reactors were it might be advantageous to have high heat flux surfaces facing the burning plasma be covered with a flowing liquid metal layer. When the problem of open channel MHD flows is examined closely, one finds that the complicated motion of closed channel duct flows described above become even more complicated when the liquid interface (free surface) is allowed to move in response to MHD forces.

The interfacial boundary condition for open channel flows requires that the tangential component of the viscous stress must be continuous. The term “free surface” implies the less general case where the liquid surface is unhampered by friction with a gas phase outside the liquid region, and so the tangential component of the stress vanishes. However, this condition is changed in MHD flows where the total stress, the sum of the tangential viscous and magnetic stresses, must be continuous. The magnetic stress is represented by the Maxwell stress tensor (found elsewhere in this handbook), and can exist in vacuum as well as in conducting media. This EM stress vanishes for temporally and spatially uniform magnetic fields, but needs to be considered in the general case.

Some simple cases of flow down an inclined plate are analyzed by Alpher [34], Aitov [35], Morley [36] and others. It is seen that for a magnetic field normal to the surface of a very wide, long plate, that a half-Hartmann velocity profile forms in the surface normal direction. The flow is essentially that shown in Fig. 3 split open, where $z = 0$ is the free surface, and $z=1$ is the back plate. The Hartmann layer on the plate is exactly the same as one would expect in channel flow, and provides a return current path for currents generated
in the core. Also similar to flow in ducts, the modification of the velocity profile and the unbalanced \( j \times B \) force can cause an increase in drag. For flow down an inclined plate, where there can be no applied pressure driving the flow, this results in a thickening and slowing down of the flow.

Similarly to the effect of magnetic fields on turbulence, is the stabilizing effect of magnetic fields at the free surface. It has been shown both theoretically and experimentally that a constant strong magnetic field can stabilize an otherwise wavy free surface, resulting in a smooth flow. For the flow described in the preceding paragraph, Hsieh [37] found that for high \( Ha \), the surface is stable to long wavelengths provided that:

\[
Re < \frac{\exp(2 Ha)}{4} \cot \theta
\]

where \( Re \) is the Reynolds number of the flow and \( \theta \) is the angle of inclination of the plate to gravity. This is a much greater range of Reynolds number than the classical non-MHD result of \( Re < \frac{5}{4} \cot \theta \).

**Applications in Metals Processing**

Metals processing requires the handling of large amount of liquified metals in a controlled manner. Certainly the MHD devices discussed above, *e.g.* pumps and flowmeters, will have manifold applications in this industry. But it is also possible to actively control the shape of a free surface by use of high frequency AC magnetic fields.

A high frequency magnetic field in the region around an electric conductor, like a liquid metal, takes a finite time to penetrate into the conductor. It is easily seen from Eq. 10 in the
limit of slow motion of the liquid $u$ as compared to the sinusoidal oscillation frequency $\omega$, that:

$$\frac{B}{\tau} \equiv \frac{1}{\sigma \mu_m} \frac{B}{\delta^2}$$

If the characteristic time $\tau$ is taken as $2 \omega^{-1}$, then the skin depth $\delta \equiv (2/\sigma \mu_m \omega)^{1/2}$. This means that if $\delta$ is small, the field cannot penetrate far into the conducting medium during the oscillation period of the AC field. In reality, currents induced in the skin region act to nullify the applied field variation. The resulting $\mathbf{j} \times \mathbf{B}$ force can have both a pressure-like component and a tangential stress component. The pressure component can be applied to the free surface in such a way to deflect and shape jets of liquids issuing from a nozzle and even levitate an entire melt. The tangential stress components can be used to induce motion and stir the melt as desired. The induced currents can also provide significant joule heating in the skin region.

Using a combination of all these effects it is possible to design various MHD devices like levitating, self-stirred, induction furnaces where the LM never comes in contact with a solid crucible surfaces, and MHD granulators where free LM jets or sheets decay into droplets that then solidify into powders. The interested reader is referred to Moreau [12] and Kolesnichenko [2] for more detailed mathematical descriptions of these problems with more complete bibliographies.
4. Gaseous MHD

4.1 Introduction

Since 1959, substantial effort has been devoted to exploring the conditions under which a conducting gas moving through a magnetic field might generate useful electrical power. The primary motivation for the development and use of MHD generators in central-station power plants is the production of power at lower cost through reduced fuel costs per unit of energy produced, traded off against additional capital and operating costs. Operation at high thermal conversion efficiency provides the added benefit of reduced thermal discharge from the plant, thus reducing thermal pollution as compared with conventional steam plants with $\eta \sim 40\%$ or nuclear plants with efficiency as low as 30%.

As originally envisioned, the MHD generator was a “topping” unit on an otherwise conventional steam turbine-generator station. In this case, electric power is generated in the MHD unit, and its exhaust heat, with temperature as high as 2200 K, is used to generate steam. The limiting Carnot efficiency for such a station might be raised from a maximum of about 65% ($T_1=850$ K, $T_2=300$ K) upward toward 85% ($T_1=2600$ K, $T_2=420$ K). The net efficiency of the combined cycle can be expressed as $\eta_1 + \eta_2 (1-\eta_1)$, where $\eta_1$ is the efficiency of the MHD generator and $\eta_2$ is the efficiency of the “bottom” steam plant. Typical values are $\eta_1=0.25$ and $\eta_2=0.4$, for an overall efficiency of 0.55.

Perhaps the greatest importance of the MHD steam plant, as now envisioned, is its potential for very low air pollution while burning high-sulfer coal [38]. The $SO_2$, $NO_2$ and particulate emissions are all reduced to very low levels by their interaction with the MHD “seed” material. In pilot plant tests, 2.2 wt.% sulfur coal was burned in a cyclone furnace at 2200°C with seed concentration of 1 g•mole $K_2CO_3$/kg coal with 99.8% removal of
SO₂, leaving only 5 ppm SO₂ in the gaseous effluent. This occurs because of an affinity that the potassium seed material has for SO₂. So seed recovery in the MHD system, which is necessary for economic reasons, also removes the SO₂. The seed removal costs are calculated as approximately 1/5 of the SO₂ removal costs in a conventional coal-fired plant. In the same tests, through the use of 2-stage combustion, NOₓ emissions were reduced below 150 ppm, and complete combustion of CO was achieved [38,39].

Table 3 summarizes the main features of both open and closed cycle systems which remain the subject of both theoretical and experimental investigation. Generally, practical designs have DC output taken from electrodes at the sides of the MHD channel. AC power then is obtained using electronic inverters.

In the remainder of this section, we first review the main generator configurations used in MHD power generation, together with their governing electrical equations. Flow behavior is described for an example case, using the segmented-electrode Faraday type of generator. Following this, properties of seeded gases are given, and finally the major engineering issues for electrodes and magnets are summarized.

4.2 Generator Configurations

Figure 13 depicts the basic elements of a generic MHD generator, having quasi-one-dimensional flow of partially-ionized gas channeled through a perpendicular, static magnetic field. For this geometry, \( \mathbf{u} = \hat{x} \ u_x \) and \( \mathbf{B} = \hat{z} \ B_z \). In the general case, the components of Ohm’s Law (Eq. 9) are:

\[
    j_x = \sigma \varepsilon_x - \mu_e j_y B_z
\]

(77)
\[ j_y = \sigma \varepsilon_y - \sigma u_x B_z + \mu_e j_x B_z \quad (78) \]

\[ j_z = \varepsilon_z = 0 \quad (79) \]

Four alternative generator configurations are considered here and depicted in Fig. 14:

(a) segmented-electrode Faraday generator,
(b) continuous-electrode Faraday generator,
(c) Hall generator, and
(d) diagonally-connected generator.

(a) **Segmented-electrode Faraday generator.** Configurations in which the circuit for \( j_y \) is completed through the external load are called “Faraday generator” configurations. If the electrodes are segmented along the x-direction in order to electrically isolate each pair, then the Hall current is suppressed \((j_x=0)\).

Assuming uniform conditions across the channel, the open circuit voltage \((j_y=0)\) is given by Eq. 78:

\[
V_{oc} = \int_{o}^{d} \varepsilon_y \, dy = - \int_{o}^{d} u_x B_z \, dy = - u_x B_z \, d \quad (80)
\]

and the load power generated is:

\[
P_L = j_y \varepsilon_y = - \sigma (u_x B_z - \varepsilon_y) \varepsilon_y = - \sigma u_x^2 B_z^2 (1-K) \quad K \quad (81)
\]

where the dimensionless loading parameter \( K \) is defined by \( K = \varepsilon_y / u_x B_z \). Using Eq. 43, we find that the conversion efficiency is:
\[ \eta_e = \frac{j_y \epsilon_y}{u_x j_y B_z} = \frac{\epsilon_y}{u_x B_z} = K \]  \hspace{1cm} (82)

(b) **Continuous-electrode Faraday generator.** In a continuous-electrode generator, \( \epsilon_x = 0 \) and the Hall current is finite, \( j_x \neq 0 \). The x-component of current (in the direction of the fluid flow) has its circuit completed through the electrode walls. The \( j_y \) component is reduced due to this effect:

\[ j_x = -\mu_e j_y B_z \]

\[ j_y = \frac{\sigma}{l + \omega^2 \tau^2} (\epsilon_y - u_x B_z) = \frac{\sigma u_x B_z}{l + \omega^2 \tau^2} (1 - K) \]  \hspace{1cm} (83)

The load power is:

\[ P_L = j_y \epsilon_y = \frac{\sigma}{l + \omega^2 \tau^2} (\epsilon_y - u_x B_z) \epsilon_y = -\frac{\sigma u_x^2 B_z^2}{l + \omega^2 \tau^2} (1 - K) K \]  \hspace{1cm} (84)

The open-circuit terminal voltage is the same for both types of Faraday generator, as is the local electrical efficiency, \( \eta_e = K \). However, the power density is reduced by \( 1 + \omega^2 \tau^2 \) when the electrodes are continuous.

(c) **Hall generator.** In the Hall generator configuration, opposing electrode pairs are short circuited (\( \epsilon_y = 0 \)) and the circuit for \( j_x \) is completed through the external load. In this case, the current components derived from Eqs. 77–79 are:
\[ j_x = \frac{\sigma}{l + \omega^2 \tau^2} (\varepsilon_x + \omega \tau u_x B_z) \]  
(85)

\[ j_y = \frac{\sigma}{l + \omega^2 \tau^2} (\omega \tau \varepsilon_x - u_x B_z) \]  
(86)

The open circuit voltage \((j_x=0)\) across the full channel length is given by:

\[ V_{oc} = -\int_o^L \varepsilon_x \, dy = \int_o^L \omega \tau u_x B_z \, dx = \omega \tau u_x B_z L \]  
(87)

In this case, the Hall loading parameter is written \(K_H = -\varepsilon_y / \omega \tau u_x B_z\), so that the load power generated is:

\[ P_L = j_x j_y = \frac{\sigma}{l + \omega^2 \tau^2} (\varepsilon_x + \omega \tau u_x B_z) \varepsilon_y = \frac{\sigma}{l + \omega^2 \tau^2} \left( \frac{\omega^2 \tau^2 u_x^2 B_z^2}{l + \omega^2 \tau^2} \right) (1-K_H) K_H \]  
(88)

The conversion efficiency is:

\[ \eta_c = \frac{j_x \varepsilon_x}{u_x j_y B_z} = \frac{(1-K_H) K_H}{K_H + 1/\omega^2 \tau^2} \]  
(89)

A comparison between Faraday and Hall generator efficiencies is given in Fig. 15. Good efficiency in a Hall generator requires high \(\omega \tau\) and a small loading parameter, whereas the Faraday generator efficiency is independent of \(\omega \tau\) and improves with higher values of the loading parameter.

\(d)\) **Diagonally-connected generator.** A configuration that has been favored recently is diagonal connection of electrodes along equipotential surfaces at the angle \(\theta = \tan^{-1} \varepsilon_y / \varepsilon_x\)
with respect to the vector $\mathbf{u} \times \mathbf{B}$. Ideally with this configuration, the Hall current is zero and the electrode voltage between opposite pairs is the same as for the Faraday generator. With the diagonal connections, the overall circuit is a series connection of multiple Faraday-generator electrodes. The output, at comparatively high voltage, is taken between the first and last electrodes in the series.

The characteristics of the diagonally connected generator are intermediate between those of the Hall and Faraday generators. Allowing for a finite Hall current, the current components in their most general form are:

$$j_x = \frac{\sigma}{1 + \omega^2 \tau^2} \left( \varepsilon_x - \omega \tau (\varepsilon_y - u_x B_z) \right)$$

$$= \frac{\sigma u_x B_z}{1 + \omega^2 \tau^2} \left( \omega \tau (1 - K) - \frac{K}{\tan \theta} \right)$$

(90)

$$j_y = \frac{\sigma}{1 + \omega^2 \tau^2} (\omega \tau \varepsilon_x + \varepsilon_y - u_x B_z)$$

$$= -\frac{\sigma u_x B_z}{1 + \omega^2 \tau^2} \left(1 - K + \frac{\omega \tau K}{\tan \theta} \right)$$

(91)

The condition $j_x = 0$ is satisfied if:

$$K = \frac{\omega \tau \tan \theta}{1 + \omega \tau \tan \theta}$$

(92)

In this case, the local electrical efficiency is $\eta = K$. 
4.3 Energy Extraction and Flow Relations

In general, numerical solutions are needed to solve the complete set of equations governing power generation and flow. The simple case of a quasi-one-dimensional constant-velocity ideal gas flow is examined here in closed form in order to provide insight into the flow behavior. The velocity is maintained constant as the gas expands by adjusting the flow area $A_c$ such that mass conservation leads to:

\[ \frac{d}{dx} (\rho A_c) = 0 \]  \hspace{1cm} (93)

A Faraday generator configuration with segmented electrodes is assumed. In this case, the Hall current vanishes and Ohm’s Law can be written:

\[ j = \sigma (\epsilon - uB) = -\sigma uB (1-K) \]  \hspace{1cm} (94)

The pressure variation is linear for a constant field:

\[ \frac{dp}{dx} = jB = -\sigma uB^2 (1-K) \]  \hspace{1cm} (95)

The solution can be written in terms of the pressure at the inlet ($x=0$):

\[ \frac{p}{p_0} = 1 - \frac{x}{L_i} \]  \hspace{1cm} (96)

\[ L_i = \frac{1}{1-K} \frac{p_0}{\sigma uB^2} \]  \hspace{1cm} (97)

where $L_i$ is the “interaction length” [40], which is an approximate measure of the channel length required to extract an appreciable amount of the gas energy.
The energy equation can be used to determine the temperature variation along the duct. The total fluid enthalphy (per unit mass) is the sum of the kinetic energy, internal energy and static pressure:

\[ H = \frac{u^2}{2} + e + \frac{p}{\rho} \]  

(98)

Conservation of energy equates the rate of change of fluid energy with the electric power dissipated:

\[ \rho \frac{dH}{dt} = j \cdot \varepsilon \]  

(99)

With constant velocity, we can replace \( H \) by the static enthalpy \( h \):

\[ h = H - \frac{u^2}{2} \]  

(100)

For an ideal gas, \( h = C_p T \), so that the energy equation becomes:

\[ \rho C_p u \frac{dT}{dx} = j \varepsilon \]  

(101)

We can replace the current density and electric field in Eq. 101 using Eq. 18 and the definition of the loading parameter, \( K = \varepsilon / uB \):

\[ \rho C_p u \frac{dT}{dx} = \frac{1}{B} \frac{dp}{dx} \cdot uBK \]  

(102)

Solving for \( K \),
\[ K = \rho C_p \frac{dT}{dp} \]  \hspace{1cm} (103)

For an ideal gas, the temperature and pressure are related by:

\[ C_p T = \frac{\gamma - 1}{\gamma - 1} \rho \]  \hspace{1cm} (104)

If we use Eq. 103 to replace \( \rho \) in Eq. 104, we find:

\[ \frac{dT}{T} = K \left( \frac{\gamma - 1}{\gamma} \right) \frac{dp}{p} \]  \hspace{1cm} (105)

This can be integrated to give the relation between \( p \) and \( T \):

\[ \frac{T}{T_o} = \left( \frac{p}{p_o} \right)^{\frac{K(\gamma - 1)}{\gamma}} \]  \hspace{1cm} (106)

The pressure, temperature, flow area and Mach number are plotted in Fig. 16. The Mach number is:

\[ M = \frac{u}{c_s} = \frac{u}{\sqrt{\gamma P/\rho}} = \frac{u}{\sqrt{\gamma RT/W_m}} \]  \hspace{1cm} (107)

where \( c_s \) is the local speed of sound, \( R \) is the universal gas constant, and \( W_m \) is the molecular weight of the gas. For constant velocity:

\[ \frac{M}{M_o} = \left( \frac{T}{T_o} \right)^{1/2} = \left( \frac{p}{p_o} \right)^{-1/2 \frac{K(\gamma - 1)}{\gamma}} \]  \hspace{1cm} (108)
4.4 Working Fluid Conductivity

Conductivity of the working fluid is a critical parameter in a gaseous MHD topping unit. The conductivity is obtained by summing the contributions from each species. However, electrons contribute most of the conductivity due to their higher mobility.

\[ \sigma = n_e \mu_e = \frac{n_e e}{m_e N} \]  

(109)

At low ionization states, the electron mobility is inversely related to the collision frequency with neutral particles. As the degree of ionization increases, the mobility is reduced by electron-ion and electron-electron collisions, which have higher cross sections. At only 1% ionization, already this effect becomes significant. Therefore, a high degree of ionization is not necessary to achieve an appreciable fraction of the fully-ionized conductivity.

Temperatures for fossil-fuel combustion are in the range of 2500–3000 K. At these temperatures, thermal ionization of air, combustion product gases or inert gases is so low that the electron density is orders of magnitude below that necessary to obtain suitable conductivity (see Fig. 17). One might obtain marginally acceptable conductivity by reducing the gas pressure, however, this also would result in larger duct and heat exchanger sizes.

Fortunately, a large increase in conductivity can be obtained by seeding the gas with a small percentage of materials with much lower ionization potential. The ionization potential of the outermost electron in air is ~14 V, and those of inert gases are even higher. Alkali metals make exceptional seed materials, with ionization potential of 3.89 V for cesium, 4.34 V for potassium, and 5.4 V for lithium. Some calculated conductivities of seeded gases are shown in Fig. 18 as a function of temperature and pressure. In general, these curves are in close agreement with measured values (e.g., Ref. 41).
Unfortunately, alkali metals also carry a relatively high collision cross section, such that an increasing percentage of seed atoms not only increases the electron density but also decreases the mobility. An optimum seeding percentage is reached at about 0.1% for cesium and potassium in argon, and about 0.3% in neon [42].

5. Two-phase MHD

5.1 Flow characteristics

MHD of conducting two-phase mixtures, consisting of liquid and gas phases, raises new phenomena providing the potential for unique applications. Two-phase mixtures may arise from boiling or from mixing of distinct gas and liquid phases — for example, helium mixed with a liquid metal. Here we summarize the basic flow characteristics of two-phase mixtures and the application to liquid metal MHD (LMMHD) for power conversion.

Much of the early progress studying two-phase flows was based on empirical results [43–46], since the underlying flow structures can be very complex. Similar to single-phase flows, the effect of the magnetic field is to suppress turbulence and to alter the velocity profiles. In addition, modifications in the interface configuration and slip between phases occur, and the transition between flow regimes can be shifted.

Typical two-phase flow patterns are depicted in Fig. 19. As in ordinary two-phase flow, increasing superficial gas velocity causes the flow to transition from bubbly, to churn, to slug, and finally to annular mist flow regimes [47]. However, observable differences between MHD and non-MHD behavior occur, as summarized in Fig. 19.
5.2 MHD generators and power conversion

Liquid metal MHD power conversion using two-phase mixtures was contemplated as early as the 1960’s [48]. In the 1970’s, an extensive program was conducted at Argonne National Laboratory (ANL) [46,49–52], culminating in the development of a constant-velocity DC Faraday generator using N$_2$ with Na or NaK. Following this early work, a rather extensive program was initiated at Ben-Gurion University, where a variety of power conversion systems have been analyzed and/or tested [51].

The use of liquid metals for power conversion avoids the very high temperatures required to maintain an ionized gas in the conducting state. In that case, practically any heat source can be used, including solar, geothermal, nuclear, or even coal combustors. In addition, the higher conductivity of liquid metals makes possible higher power density with moderate magnetic fields, so that relatively small size generators are possible. For example, liquid metals offer conductivities of the order of $10^6$–$10^7$ $(\Omega\text{m})^{-1}$ at low temperature, as compared with $10$ $(\Omega\text{m})^{-1}$ for the case of He seeded with 0.45% Cs at 2000 K. Considerable support has been obtained for research on space-based power supplies, due in part to these advantages [52].

In general, a thermodynamic cycle requires a working medium (or “thermodynamic medium”) that can expand and contract with temperature, e.g., gas or steam. For MHD power conversion, the thermodynamic fluid is mixed with an electrodynamic fluid (the liquid metal) to allow MHD generation.

Because the heat capacity of the liquid phase significantly exceeds the gas phase, two-phase flow expansion (and compression) occurs nearly isothermally. This results in potentially higher thermal conversion efficiency, approaching that of the ideal Carnot cycle. For
example, Fig. 20 compares a standard gas Brayton cycle T-s diagram with a modified cycle using a LMMHD generator.

Several classes of thermodynamic cycles are possible, depending on the types of coolant (e.g., gas, liquid, and/or 2-phase fluid), the use of evaporation or gas mixing, and the use of bottom cycles. These are summarized in Table 4.

In the “homogeneous cycle”, the thermodynamic and electrodynamic fluids remain mixed throughout the cycle (see Fig. 21). The heat source causes the working fluid to evaporate, which drives expansion through the 2-phase generator.

In an Ericsson cycle, a mixer/separator combination provides the ability to operate over wider temperature ranges. The main steps are depicted in Fig. 22. In this example, the top cycle has four main steps: (1) the liquid metal is heated by a heat source, (2) a mixer combines the thermodynamic fluid with the liquid metal, (3) the mixture is expanded through a generator, and (4) the two phases are separated. The gas side of the cycle may itself take advantage of the useful heat by expansion through a Brayton cycle turbine, or it could utilize a 2-phase MHD compressor.

In the Rankine cycles, typically water is injected into a chemically compatible liquid metal (such as a lead-alloy). A steam turbine and/or 2-phase generator is used for electric generation.
Nomenclature

a  channel half-width parallel to B
b  channel half-width perpendicular to B
B  magnetic field intensity
Bo characteristic field strength, used to non-dimensionalize equations
c_e RMS electron thermal velocity in a Maxwellian distribution, \((3kT_e/m_e)^{1/2}\)
c_s speed of sound
e charge on an electron
Ha Hartmann number, \(aB\sqrt{\sigma/\mu_f}\)
j current density
j_f current density in fluid
j_w current density in wall
j_f current density in fluid
k Boltzmann constant
K loading parameter
K_H Hall loading parameter
l length
m mass
m_e electron mass
n_e number density of free electrons
p fluid pressure
P pressure gradient
q electric charge
Re_m Magnetic Reynolds number, \(\sigma\mu vl\)
R_L load resistance
T_e electron temperature
u fluid velocity
$u_e$  drift velocity of electrons
$v$  particle velocity
$v_b$  bulk average velocity
$v_o$  characteristic velocity, used to non-dimensionalize equations
$V$  nondimensional velocity
$V_{oc}$  open circuit voltage
$W_m$  molecular weight
$X$  non-dimensional x-coordinate
$Y$  non-dimensional y-coordinate
$\beta$  rectangular channel aspect ratio
$\delta$  wall thickness or skin depth
$\varepsilon$  electric field
$\lambda$  electron mean free path
$\mu_e$  electron mobility, $\omega \tau / B$
$\mu_f$  fluid dynamic viscosity
$\mu_m$  magnetic permeability
$\nu$  electron-atom collision frequency
$\nu_f$  kinematic viscosity, $\mu_f / \rho$
$\rho$  fluid density
$\sigma$  electrical conductivity
$\sigma_f$  electrical conductivity of fluid
$\sigma_w$  electrical conductivity of wall
$\tau$  electron mean collision time, $\lambda / \nu$
$\varphi$  electric scalar potential
$\Phi$  wall conductance ratio
$\omega_e$  electron cyclotron frequency, $eB / m_e = \mu_e B / \tau$
$\omega_c$  cyclotron frequency


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(assuming a=0.1 m, B=1 T, v=1 m/s)

<table>
<thead>
<tr>
<th></th>
<th>NaK (100°C)</th>
<th>Hg (20°C)</th>
<th>Electrolyte (20°C) 15% KOH</th>
<th>Air (3000°C) with 2% K</th>
</tr>
</thead>
<tbody>
<tr>
<td>Re</td>
<td>1.6×10^5</td>
<td>9.1×10^5</td>
<td>4.3×10^4</td>
<td>350</td>
</tr>
<tr>
<td>Ha</td>
<td>6800</td>
<td>2700</td>
<td>17.5</td>
<td>98</td>
</tr>
<tr>
<td>N</td>
<td>290</td>
<td>8.2</td>
<td>7×10^{-3}</td>
<td>27</td>
</tr>
<tr>
<td>Re_m</td>
<td>0.30</td>
<td>0.14</td>
<td>1.2×10^{-5}</td>
<td>1.3×10^{-5}</td>
</tr>
</tbody>
</table>
Table 2
Magnetic field end effect correction factors [26]

<table>
<thead>
<tr>
<th>L/D</th>
<th>$k_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.5</td>
<td>0.91</td>
</tr>
<tr>
<td>1.9</td>
<td>0.96</td>
</tr>
<tr>
<td>2.4</td>
<td>0.98</td>
</tr>
<tr>
<td>3.0</td>
<td>0.99</td>
</tr>
</tbody>
</table>


**Table 3.**
Gaseous MHD power generating system features

<table>
<thead>
<tr>
<th></th>
<th><strong>Open Cycle</strong></th>
<th><strong>Closed Cycle</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Heat Source</strong></td>
<td>coal</td>
<td>gas-cooled nuclear reactor</td>
</tr>
<tr>
<td></td>
<td>manufactured gas</td>
<td>coal</td>
</tr>
<tr>
<td></td>
<td>natural gas</td>
<td>natural gas</td>
</tr>
<tr>
<td></td>
<td>(\text{H}_2)</td>
<td>fuel oil</td>
</tr>
<tr>
<td></td>
<td>fuel oil</td>
<td></td>
</tr>
<tr>
<td><strong>Working Fluid</strong></td>
<td>potassium-seeded</td>
<td>cesium-seeded helium</td>
</tr>
<tr>
<td></td>
<td>combustion products</td>
<td></td>
</tr>
<tr>
<td><strong>Temperature</strong></td>
<td>(~2500^\circ \text{C})</td>
<td>(~1400^\circ \text{C})</td>
</tr>
<tr>
<td><strong>Magnetic Field Source</strong></td>
<td>DC superconducting magnets, 4–6 T</td>
<td>DC superconducting magnets, 4–6 T</td>
</tr>
<tr>
<td>Cycle Type</td>
<td>Working Fluids</td>
<td></td>
</tr>
<tr>
<td>------------------------------------------------</td>
<td>-----------------------</td>
<td></td>
</tr>
<tr>
<td>1. Homogeneous cycles</td>
<td>Na, K, Cs, <em>etc.</em></td>
<td></td>
</tr>
<tr>
<td>2. Ericsson cycle with LMMHD compressor or Brayton gas turbine/compressor</td>
<td>Na/He, Li/He</td>
<td></td>
</tr>
<tr>
<td>3. Rankine cycles with or without steam turbine</td>
<td>Pb-alloy/steam</td>
<td></td>
</tr>
<tr>
<td>4. Binary cycles <em>(e.g., homogeneous top plus Rankine bottom)</em></td>
<td></td>
<td></td>
</tr>
</tbody>
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