Note: 1) Closed book, notes, etc. (only one page of notes is allowed).
2) Explain what you are doing, formulas without explanation are not acceptable.

Problem 1. a) Find $i_\sigma$, b) Find the power produced by the 160 V voltage source (10pt).

20 and 30 Ω resistors are in series and can be combined into a 50 Ω resistor.

$N_{NV} = 4 - 1 - 2 = 1$ Equation

$N_{MC} = 2$ Equations

So proceed with node-voltage method:

KCL at $v_1$:

$$\frac{v_1 - 160}{10} + \frac{v_1 - 150i_\sigma}{50} + \frac{v_1}{100} = 0$$

Aux. Eq. for $i_\sigma$:

$$v_1 = -100i_\sigma$$

Substituting for $i_\sigma$ from 2nd into first equation, we get:

$$10v_1 - 1600 + 2v_1 - 300i_\sigma + v_1 = 0$$

$$10v_1 - 1600 + 2v_1 + 3v_1 + v_1 = 0$$

$$v_1 = 100 \text{ V}$$

$$i_\sigma = -0.01v_1 = -1 \text{ A}$$

To find power produced by 160-V source, we find the current in the source:

$$i = \frac{v_1 - 160}{100} = -6 \text{ A}$$

$$p = vi = 160 \times (-6) = -960 \text{ W}$$

Since we used passive sign convention, this source produces 960 W of power.
**Problem 2.** Find Thevenin Equivalent of this subcircuit (10pt).

As this subcircuit does not have an independent source, it should reduce to a resistor (If you try to calculate \( v_{oc} \) or \( i_{sc} \), you will find that both are equal to zero.). To find \( R_T \), we attach a voltage source to the subcircuit and proceed to compute \( i \) assuming \( v \) is known. We should get \( i = -v/R_T \) (active sign convention for Thevenin/Norton equivalent).

Checking the circuit we find that \( N_{NV} = 4 - 1 - 2 = 1 \) while \( N_{MC} = 3 \). So we proceed with node-voltage method. In addition, if we label the node in the middle as \( v_b \), there will be no need for any auxiliary equation for our controlled source:

\[
\text{KCL at } v_b: \quad \frac{v_b - 4v_b}{1000} + \frac{v_b - v}{1000} + \frac{v_b - 0}{1000} = 0 \quad \rightarrow \quad v_b = -v
\]

To find \( i \), we write KCL at node \( v \) and use \( v_b = -v \):

\[
+i + \frac{v - v_b}{1000} + \frac{v - 4v_b}{10,000} = 0
\]

\[
+i = -\frac{v + v}{1000} - \frac{v + 4v}{10,000} = -\frac{20v + 5v}{10,000} = -\frac{v}{400}
\]

\( R_T = 400 \, \Omega \)
Problem 3. Find $i_o$.

As the output of the OpAmp is attached to inverting input terminal, we have negative feedback: $v_n = v_p$

We now proceed with node-voltage method:

**Node $v_p$:**

\[
\frac{v_p - 1}{2.4 \times 10^3} + \frac{v_p}{5.6 \times 10^3} = 0 \quad \rightarrow \quad 5.6v_p - 5.6 + 2.4v_p = 0
\]

$v_p = 0.7 \text{ V}$

$v_n = v_p = 0.7 \text{ V}$

**Node $v_n$:**

\[
\frac{v_n - 0}{15 \times 10^3} + \frac{v_n - v_o}{75 \times 10^3} = 0 \quad \rightarrow \quad 5v_n + v_n - v_o = 0
\]

$v_o = 6v_n = 4.2 \text{ V}$

$i_o = \frac{v_o - 0}{6.8 \times 10^3} = 0.62 \text{ mA}$
**Problem 4.** The circuit below has been in DC steady state. The switch is moved to the position B at $t = 1$ s. Find $V_o$ for $t > 1$ s (10pt).

For $t < 1$, circuit is in DC steady-state:

$$v_C = 6 \text{ V}$$

No jump condition will give the initial condition for $t > 1$

$$v_c(t = 1^+) = v_c(t = 1^-) = 6 \text{ V}$$

For $t > 1$, $v_C$ can be found in two ways:

**Method 1:** using the formula for an RC circuit:

$$v_C(t) = (v_0 - v_s)e^{-\frac{t-t_0}{\tau}} + v_s$$

For our circuit, $t_0 = 1$ s, $v_0 = v_c(t = 1^+) = 6$ V $v_s = 0$, and $\tau = RC = 5 \times 10^3 \times 0.2 \times 10^{-6} = 0.001$ s:

$$v_C(t) = 6e^{-1000(t-1)} \text{ V}$$

**Method 2:** Solve the circuit:

\[
\begin{align*}
\text{KVL:} \quad & v_C + 5 \times 10^3 i_C = 0 \\
& v_C + 10^{-3} \frac{dv_C}{dt} = 0 \\
& v_C = Ke^{-1000t} \\
& v_C(t = 1^+) = 6 = Ke^{-1000} \\
& v_C(t) = 6e^{-1000(t-1)} \text{ V}
\end{align*}
\]

To find $v_o$, we first find $i_C$:

$$i_C = -\frac{v_C}{5 \times 10^3} = 1.2^{-1000(t-1)} \text{ mA}$$

$$v_o = -3 \times 10^3 i_C = 3.6e^{-1000(t-1)} \text{ V}$$
Problem 5. The switch in the circuit below has been in position A for a long time. The switch is moved to the position B at \( t = 0 \) s. Find \( i_o \) for \( t > 0 \) s (10pt).

For \( t < 0 \), circuit is in DC steady-state. Replacing the capacitor with an open circuit, we get \( i_o = 0 \), and by KVL, \( v_C = 25 \) V. No jump condition will give the initial condition for \( t > 0 \)

\[ v_C(t = 0^+) = v_C(t = 0^-) = 25 \text{ V} \]

For \( t > 0 \):

KVL: \[ v_C + 60 \times 10^3 i_o + 20 \times 10^3 i_o = 0 \]
\[ v_C + 80 \times 10^3 \times 25 \times 10^{-9} \frac{dv_C}{dt} = 0 \]
\[ 2 \times 10^3 \frac{dv_C}{dt} + v_C = 0 \]
\[ v_C = Ke^{-500t} \]
\[ v_C(t = 0^+) = 25 = K \]
\[ v_C(t) = 25e^{-500t} \text{ V} \]
\[ i_o = C \frac{dv_C}{dt} = 25 \times 10^{-9} \frac{d}{dt} (25e^{-500t}) \]
\[ i_o = -0.31e^{-500t} \text{ mA} \]
Problem 6. Find $v_o$ for $t > 0$ s assuming $v_c(t = 0) = 0$ (10pt).

The non-inverting input terminal of OpAmp is attached to the ground, so $v_p = 0$.

As the output of the OpAmp is attached to inverting input terminal, we have negative feedback: $v_n = v_p = 0$

We now proceed with node-voltage method:

Node $v_n$: \[ \frac{v_n - v_i}{R} + \frac{v_n - v_o}{R} + i_C = 0 \quad \rightarrow \quad -v_i - v_o + R i_c = 0 \]

\[ v_C = v_n - v_o = -v_o \]

\[ -v_i + v_C + RC \frac{dv_C}{dt} = 0 \]

\[ RC \frac{dv_C}{dt} + v_C = v_i \]

The natural solution to the differential equation is $v_C(t) = Ke^{-t/\tau}$, where $\tau = RC$, and the forced solution is $v_i$:

\[ v_C(t) = Ke^{-t/\tau} + v_i \]

\[ v_C(t = 0) = 0 = K + v_i \quad \rightarrow \quad K = -v_i \]

\[ v_C(t) = v_i(1 - e^{-t/\tau}) \]

\[ v_o(t) = -v_C(t) = -v_i(1 - e^{-t/\tau}) \]
**Problem 7.** The switch is closed at \( t = 0 \). a) Find \( v_C(t) \). b) Is this circuit over damped or underdamped?

For \( t < 0 \), the circuit is in DC steady state. \( i_L = 0 \) because capacitor is open circuit. By KVL, \( v_C = 10 \text{ V} \). No jump conditions give initial conditions for \( t > 0 \) case:

\[
v_C(t = 0^+) = 10 \quad \quad i_L(t = 0^+) = 0
\]

For \( t > 0 \) the circuit is a series RLC circuit. The differential equation for this circuit can be found by KVL (or using formulas). Noting \( i_L = i_C = i = C(dv_C/dt) \), we get:

\[
\begin{align*}
\text{KVL:} \quad & Ri + v_L + v_C = 0 \\
& L \frac{di}{dt} + R \frac{dv_C}{dt} + v_C = 0 \\
& LC \frac{d^2v_C}{dt^2} + RC \frac{dv_C}{dt} + v_C = 0
\end{align*}
\]

Use notation of \( \omega_0 \) and \( \zeta \) from the textbook:

\[
\begin{align*}
\omega_0^2 &= \frac{1}{LC} = \frac{1}{32 \times 10^{-3} \times 50 \times 10^{-9}} = 6.25 \times 10^8 \\
2\zeta\omega_0 &= \frac{R}{L} = \frac{10^3}{32 \times 10^{-3}} = 3.125 \times 10^4 \quad \rightarrow \quad \zeta = 0.625
\end{align*}
\]

Since \( \zeta < 1 \) the circuit is underdamped and the solution is:

\[
v_C(t) = K_1 e^{-\alpha t} \cos(\beta t) + K_2 e^{-\alpha t} \sin(\beta t) \\
\alpha = \zeta\omega_0 = 1.5625 \times 10^4 \quad \beta = \omega_0 \sqrt{1 - \zeta^2} = 1.95 \times 10^4
\]

I.C.: \( v_C(t = 0^+) = 10 \quad \frac{dv_C}{dt} \bigg|_{(t=0^+)} = \frac{1}{C}i_C = \frac{1}{C} i_L(t = 0^+) = 0 \)

Using initial conditions, we find \( K_1 = 10 \) and \( K_2 = 0 \). Thus:

\[
v_C(t) = 10e^{-1.56 \times 10^4 t} \cos(1.95 \times 10^4 t)
\]
**Problem 8.** a) Find \(i_0\). b) Find the real and reactive powers in the capacitor (10pt).

Since the source is sinusoidal, we can use phasors:

\[
\begin{align*}
\omega &= 5000 \\
14 \text{ mH} &\rightarrow j\omega L = j70 \\
0.5 \mu\text{F} &\rightarrow \frac{1}{j\omega C} = -j400
\end{align*}
\]

For the phasor circuit, we note \(V_{NV} = 3 - 2 = 1\) while \(N_{MC} = 2\). Proceeding with node-voltage method:

KVL at \(V_1\):

\[
\frac{V_1 - 72}{j70} + \frac{V_1 + 590I_0}{50} + \frac{V_1}{160 - j400} = 0
\]

Aux. Eq. for \(I_0\):

\[
I_0 = \frac{V_1}{160 - j400}
\]

Typically we substitute for control parameter \((I_0)\) in node equation. But since in this problem we want to find \(I_0\), we substitute for \(V_1\) in the node equation:

\[
\frac{(160 - j400)I_0 - 72}{j70} + \frac{(160 - j400)I_0 + 590I_0}{50} + \frac{(160 - j400)I_0}{160 - j400} = 0
\]

\[
(3600 - j3600)I_0 = -j360
\]

\[
I_0 = 0.05 - j0.05 = 0.071\angle-45^\circ
\]

\[
i_0 = 0.071\cos(5000t - 45^\circ)
\]

Power in the capacitor:

\[
S = 0.5Z_C|I_0|^2 = 0.5(-j400)(0.071)^2 = -j
\]

\[
P = 0 \text{ W} \quad Q = -1 \text{ VA}
\]
Problem 9.  a) What is the maximum power that can be transferred to $Z_L$.  b) Find value of $Z_L$ which maximizes power delivered to $Z_L$ (15pt).

We first need to find the Thevenin equivalent of subcircuit to the left of $Z_L$. Because this subcircuit includes a controlled source and an independent source, it is best to find $I_{sc}$ and $V_{oc}$:

To find $I_{sc}$, we note that $V_o = 0$. Thus, no current flows through the 40 Ω resistor and the strength of the controlled source is zero. By KCL, current $I_{sc}$ flows in the outer loop of the circuit. Then:

KVL: 

$$j150I_{sc} + 600I_{sc} - j150I_{sc} - 75 = 0$$

$$I_{sc} = 0.125 \text{ A}$$

To find $V_{oc}$, we note $N_{NV} = 4 - 1 - 1 = 2$ and $N_{MC} = 2 - 1 = 1$. Proceeding with mesh current method,

Aux. Eq. for $V_o$: 

$$V_o = 40I_2$$

$$I_2 - I_1 = 0.02V_o = 0.8I_2 \rightarrow I_1 = 0.2I_2$$

Supermesh $I_1$ and $I_2$: 

$$j150I_1 + 600I_1 - j150I_2 + 40I_2 - 75 = 0$$

$$(j150 + 600)(0.2I_2) + (40 - j150)I_2 = 75$$

$$I_2 = \frac{75}{160 - j120}$$

$$V_{oc} = V_o = 40I_2 = 12 + j9$$

Thus: 

$$I_N = I_{sc} = 0.125 \text{ A} \quad V_T = V_{oc} = 12 + j9 \text{ V} \quad Z_T = \frac{V_{oc}}{I_{sc}} = 96 + j72 \text{ Ω}$$

Maximum power available is:

$$P_{max} = \frac{|V_T|^2}{8R_T} = \frac{(12 + j9)(12 - j9)}{8 \times 96} = \frac{225}{8 \times 96} = 0.29 \text{ W}$$

The value of $Z_L$ which maximizes power into $Z_L$ is

$$Z_L = Z_T^* = 96 - j72 \text{ Ω}$$