Comparators and Bistable Circuits

Analog-to-Digital (A/D or ADC) and Digital-to-Analog (D/A or DAC) circuits are in wide use today because of the need to translate analog signal to digital format for processing by microprocessors and translate the output of microprocessors (digital signal) to analog signal to drive loads. The heart of most ADC circuit is the “comparator” (see Rizosm Pt. Chap. 14 for some examples of ADC). A comparator compares the value of input signal to a reference voltage. If the input signal voltage is larger than the reference voltage, comparator output will be in a set voltage (for example, \( V^+ \), for a “HIGH” or “ON” state). If the input signal voltage is smaller than the reference voltage, comparator output will be in a different, yet set voltage (for example, \( V^- \), for a “LOW” or “OFF” state).

An example of a comparator circuit using an “ideal” OpAmp is show below. Since the open-loop gain of the ideal OpAmp is infinite, we have:

\[
\begin{align*}
  \text{if } v_d = v_i > 0 & \rightarrow v_o = Av_i \gg v_s^+ & \rightarrow & v_o = v_s^+ \\
  \text{if } v_d = v_i < 0 & \rightarrow v_o = Av_i \ll v_s^- & \rightarrow & v_o = v_s^-
\end{align*}
\]

The OpAmp compares the input signal with the reference value of “zero” (tests if \( v_i > 0 \)). If the result is positive, the OpAmp output will be “high” \( v_o = v_s^+ \), if the result is negative, the OpAmp output will be “low” \( v_o = v_s^- \). We can, for example, set \( v_s^+ = 5 \text{ V} \) and \( v_s^- = 0 \) to drive a 5-V digital logic circuit.

The above circuit can be generalized to any reference voltage (instead of zero):

\[
\begin{align*}
  v_i > v_{\text{ref}} & \rightarrow v_o = v_s^+ \\
  v_i < v_{\text{ref}} & \rightarrow v_o = v_s^-
\end{align*}
\]

Switching \( V_{\text{ref}} \) and \( v_i \) reverses the result of \( v_i > 0 \) test

\[
\begin{align*}
  v_i > v_{\text{ref}} & \rightarrow v_o = v_s^- \\
  v_i < v_{\text{ref}} & \rightarrow v_o = v_s^+
\end{align*}
\]
While ordinary OpAmps (with no negative feedback) can be used as comparators, special purpose chips (variants of OpAmp circuits) are typically used in order to increase the switching speed between the two states of the comparator. Modern comparator chips have typical “slew-rates” a thousand time faster than comparable OpAmp chips. (Note that the term “slew-rate” is not usually used for comparator chips, rather terms such as “propagation delay” is often used in the literature.) In addition to faster response time, most of comparator chips have a low maximum output current (again to speed up to chip) because they drive digital logic circuits.

Alternatively, while the comparator chips are similar to OpAmps, in practice, they are never used in OpAmp circuits with negative feedback because of problems in the circuit stability (because their gain is not reduced at high frequency in order to keep switching speed high).

From the point of analytical circuit analysis, comparators and OpAmp chip are similar and ideal OpAmp circuit models can be used for both.

**Bistable Circuits**

There are two problems with the comparator circuit above:

1. The gain of a practical comparator is not infinite. Output can be smaller than \( v_i^+ \) (or larger than \( -v_i^- \) when \( v_i \) is close to \( V_{ref} \). This can cause a major problem in some circuits which only expect the output voltage to be in one of two prescribed states. This is usually seen in slowly varying input signals, where the output may not switch rapidly from one voltage to another voltage.

2. When the input signal is near the reference voltage, noise in the circuit can trigger the comparator to switch its state repeatedly as seen below. This is also a major problem in circuits where the input signal changes slowly.

Figure below shows the response a comparator circuit to slowly varying input which includes noise. Note the repeated switching of the comparator output at time of about 0.6 s.
For a large number of application we need circuits which can have only two possible states (this solves concern 1 above). These circuits are called “Bistable” circuits.

A physical analogy for the operation of bistable circuits is shown. The ball cannot remain on the hill and it always drops to either side of the hill. The ball cannot even stay exactly at the top of hill for any length of time (a state of unstable equilibrium or a mesastate) as a small disturbance will forces to move down the hill to the either of two possible states.

Most amplifier circuits can be configured into a bistable circuit by using positive feedback. An example of such a circuit is shown. This circuit has positive feedback as the output of the OpAmp is connected to the non-inverting terminal of the OpAmp. As there is no negative feedback, the virtual short principle does not apply and \( v_p \neq v_n \).

To solve the circuit, we note that because no current flows intro OpAmp (or comparator), \( R_1 \) and \( R_2 \) form a voltage divider:

\[
v_p = \frac{R_2}{R_1 + R_2} v_o = \beta v_o \quad \beta = \frac{R_2}{R_1 + R_2}
\]

Let’s assume that the circuit is in one of its two states, e.g., \( v_o = v_s^+ \). Thus, \( v_p = \beta v_s^+ \). The circuit is in this state as long as \( v_d = v_p - v_n > 0 \), or since \( v_n = v_i \), as long as \( v_i < \beta v_s^+ \). This value \( v_i \) is called that the higher threshold voltage. For the above circuit, \( v_{th} = \beta v_s^+ \).

Suppose \( v_i \) starts increasing and at the same point it becomes slightly larger than \( v_{th} \) (no matter how small the difference is). In that case, \( v_d = v_p - v_i \) becomes negative and forces the output of OpAmp to become negative. Then, \( v_p = \beta v_o \) becomes negative making \( v_d \) much smaller (more negative), forcing the the output of OpAmp to become even more negative. This process continues until OpAmp becomes saturated and the output reaches its other state: \( v_o = v_s^- \). This is contrast with a regular comparator (with no feedback) in which if \( v_d \) is very small, OpAmp will not be not saturated.

One can perform a similar analysis for the second state of circuit. Assuming \( v_o = v_s^- \) and defining \( v_{tl} = \beta v_s^- \), we find that the circuit remain in its state as long as \( v_i > v_{tl} \) and switches to the other state when it falls below that value. Figure shows that transfer characteristics of the above bistable circuit.
As can be seen from the transfer characteristics, this is a bistable circuit, the output can have only one of the two possible values. (While case of $v_i = 0$ and $v_o = 0$ is a solution to the above circuit, it is an unstable point, similar to the ball sitting exactly on the top of the hill, and any small amount of disturbance forces the circuit to reaches one of its two possible states.)

The transfer characteristics of the above bistable circuit has certain features:

1. **Hysteresis:** As can be seen the circuit exhibits “hysteresis” – the circuit changes state at different values of $v_i$ depending on whether $v_i$ is decreasing or increasing. This “hysteresis” provides additional benefits in many applications. For example, because the state of the circuit depends on the value of previous trigger signal, the circuit exhibit memory! Indeed, bistable circuit is the basic memory element of digital systems. Hysteresis is also used to eliminate switching of comparators due to noise (see Schmidt Trigger circuit below).

2. **Trigger:** Suppose that $v_{tl} < v_i < v_{th}$. In this case, the circuit will be in one of its two possible states, suppose $v_s^-$. We can change the state of the circuit by applying an input signal $v_i > v_{th}$ to the circuit. This input pulse can be of a very short duration. As such, this input signal is referred to as a “trigger” signal. Alternatively, we can change the state of the circuit with $v_s^+$ by applying a negative pulse with $v_i < v_{tl}$.

3. **Non-inverting vs Inverting Characteristics:** The above circuit is a bistable circuit with an inverting transfer characteristics, *i.e.*, The output voltage is $v_s^+$ (positive) when the input is more negative than the negative trigger threshold and the output voltage is $v_s^-$ (negative) when the input is larger than the positive trigger threshold. Small modification in the circuit leads to a bistable circuit with non-inverting transfer characteristics as is shown.

4. **Location of Hysteresis Band:** The center of the “hysteresis” band can also be located at a different voltage than zero by adding a reference voltage to the circuit as is shown.
Application of the Bistable Circuit as a Comparator – Schmidt Trigger

The bistable circuit above can be used to resolve the two problems that we have identified for a comparator. 1) Because of the positive feedback, the output switches rapidly between the two states and is always one of the two values no matter how small the input signal is, and 2) The hysteresis characteristics can be used to eliminate switching of the comparator by the noise.

As an example, consider a common application of comparators. It is required to design a circuit that detects and counts how many times an input signal crosses zero. Such a circuit can be implemented with a comparator with a threshold value of 0 V. Suppose now that the input signal has a noise superimposed on it (a common occurrence). As such, when the input signal is near zero, the noise can cause several crossing of zero voltage without input signal ever crossing zero (or crossing once). If we have an idea of the peak-to-peak amplitude of the noise, we can design a bistable circuit with a hysteresis band which is slightly larger than the expected noise level. In that case, noise will never trigger the comparator.

Such a circuit is shown and is commonly called a Schmidt Trigger. The method of solution of the circuit (as with all bistable circuits) is as follows. Assume that we are in one of the possible two states for \( v_o \). Compute \( v_d \) and find the range of \( v_i \) for which \( v_d > 0 \). As long as \( v_i \) remains in this range, the comparator state does not change. However, when \( v_i \) moves out of this range, \( v_d \) switches sign and comparator state changes. Before we start the analysis, we note:

\[
\begin{align*}
  v_d &= v_p - v_n = v_p - v_i \\
  i &= \frac{v_o - V_{ref}}{R_1 + R_2} \\
  v_p - V_{ref} &= iR_2 = \frac{R_2}{R_1 + R_2}(v_o - V_{ref}) \\
  v_p &= \frac{R_2}{R_1 + R_2} v_o + \frac{R_1}{R_1 + R_2} V_{ref}
\end{align*}
\]

**Case 1:** \( v_o = v_s^+ \) (requires \( v_d > 0 \)). Then:

\[
\begin{align*}
  v_p &= \frac{R_2}{R_1 + R_2} v_s^+ + \frac{R_1}{R_1 + R_2} V_{ref}
\end{align*}
\]

Since \( v_d = v_p - v_i > 0 \), the range of \( v_i \) for the Schmidt trigger to stay in its current state is \( v_i < v_p \) (or \( v_i < v_{th} \) is \( v_p \)).

\[
\begin{align*}
  v_i < v_{th} &= \frac{R_2}{R_1 + R_2} v_s^+ + \frac{R_1}{R_1 + R_2} V_{ref}
\end{align*}
\]
So as the comparator is in the high state and we increase \( v_i \), comparator stays in that state until the condition of \( v_i < v_{tl} \) is violated, as is shown in the transfer characteristic figure in the previous page (note direction of arrows).

**Case 2:** \( v_o = v_s^- \) (requires \( v_d < 0 \)). Then:

\[
v_p = \frac{R_2}{R_1 + R_2} v_s^- + \frac{R_1}{R_1 + R_2} V_{ref}
\]

Since \( v_d = v_p - v_i < 0 \), the range of \( v_i \) for the Schmidt trigger to stay in its current state is \( v_i > v_p \) (or \( v_d < v_p \)):

\[
v_i > v_{tl} = \frac{R_2}{R_1 + R_2} v_s^- + \frac{R_1}{R_1 + R_2} V_{ref}
\]

So as the comparator is in the low state and we decrease \( v_i \), comparator stays in that state until the condition of \( v_i > v_{p} \) is violated, as is shown in the transfer characteristic figure in the previous page (note direction of arrows).

The hysteresis region,

\[
\frac{R_2}{R_1 + R_2} v_s^- + \frac{R_1}{R_1 + R_2} V_{ref} < v_i < \frac{R_2}{R_1 + R_2} v_s^+ + \frac{R_1}{R_1 + R_2} V_{ref}
\]

is called the dead band. The input signal should pass completely through this band before the output of trigger switches from one state to another. The value of \( v_s^- \) and \( v_s^+ \) are chosen based on the desired value of the comparator output voltages. Value of \( V_{ref} \) sets the “average” switching input voltage as shown. Values of \( R_2/(R_1 + R_2) \) are usually set based on the level of the noise in the circuit. Typically the width of the dead band is chosen to be slightly larger than the maximum expected peak-to-peak amplitude of the noise.

Figure below shows the response of the Schmidt trigger to the same noisy input as previous stage. Note that multiple triggering of the comparator by the noise has been eliminated.