Terminated RL and RC filters

Terminated two-port networks are referred to those with a finite load resistance. For example, consider this terminated low-pass RC filter:

![Terminated RC Filter Diagram]

**Voltage Transfer Function:** From the circuit,

\[
H(j\omega) = \frac{V_o}{V_i} = \frac{1/(j\omega C) || R_L}{R + [1/(j\omega C) || R_L]} = \frac{R'/R}{1 + j(\omega R'C)}
\]

This is similar to the transfer function for unterminated RC filter but with resistance \( R \) being replaced by \( R' \). Therefore,

\[
\omega_c = \frac{1}{R'\omega C} = \frac{1}{(R || R_L)\omega C}
\]

and

\[
H(j\omega) = \frac{R'/R}{1 + j\omega/\omega_c}
\]

We see that the impact of the load is to reduce the filter gain \((K = R'/R < 1)\) and to shift the cut-off frequency to a higher frequency as \( R' = R || R_L < R \).

**Input Impedance:** \( Z_i = R + \frac{1}{j\omega C} || R_L \) \( Z_{i|\text{min}} = R \)

**Output Impedance:** \( Z_o = R || \frac{1}{j\omega C}, \quad Z_{o|\text{max}} = R \)

As long as \( R_L \gg Z_o \) or \( R_L \gg Z_{o|\text{max}} = R \) (our condition for good voltage coupling), \( R' \approx R \) and the terminated RC filter will look exactly like an unterminated filter. The filter gain is one, the shift in cut-off frequency disappears, and input and output resistance become the same as before.

**Terminated RL low-pass filters**

The parameters of the terminated RL filters can be found similarly:

**Voltage Transfer Function:** \( H(j\omega) = \frac{V_o}{V_i} = \frac{1}{1 + j\omega/\omega_c}, \quad \omega_c = (R || R_L)/L. \)

**Input Impedance:** \( Z_i = j\omega L + R || R_L, \quad Z_{i|\text{min}} = R || R_L \)

**Output Impedance:** \( Z_o = (j\omega L) || R, \quad Z_{o|\text{max}} = R \)

Here, the impact of load is to shift the cut-off frequency to a lower value. Filter gain is not affected. Again for \( R_L \gg Z_o \) or \( R_L \gg Z_{o|\text{max}} = R \) (our condition for good voltage coupling), the shift in cut-off frequency disappears and the filter will look exactly like an unterminated filter.