Problem 1. Find the power supplied or absorbed by both sources (specify power supplied or absorbed).

Circuit is simple and can be solved by node-voltage method or KVL & KCL:

Method 1: Node-voltage Method:

Aux. Eq. for $i_s$: $v_1 = 20 \times 10^3 i_s$

KCL at node $v_1$:

\[
\frac{v_1 - 40}{10 \times 10^3} + \frac{v_1 - 15,000i_s}{5 \times 10^3} + \frac{v_1}{20 \times 10^3} = 0
\]
\[
2v_1 - 80 + 4v_1 - 60,000i_s + v_1 = 0
\]
\[
2v_1 - 80 + 4v_1 - 3v_1 + v_1 = 0
\]
\[
v_1 = 20 \text{ V} \quad \Rightarrow \quad i_s = 1 \text{ mA}
\]
\[
i_1 = \frac{40 - v_1}{10 \times 10^3} = 2 \text{ mA}
\]

KCL: $i_2 = i_1 - i_s = 1 \text{ mA}$

Method 2: KVL & KCL:

KVL on right loop: $5 \times 10^3 i_2 + 15,000i_s - 20 \times 10^3 i_s = 0 \quad \Rightarrow \quad i_2 = i_s$

KCL: $i_1 = i_2 + i_s = 2i_s$

KVL on left loop: $10 \times 10^3 i_1 + 20 \times 10^3 i_s - 40 = 0$

\[
40 \times 10^3 i_s = 40 \quad \Rightarrow \quad i_s = 1 \text{ mA}
\]
\[
i_1 = 2i_s = 2 \text{ mA} \quad i_2 = i_s = 1 \text{ mA}
\]

Powers:

\[
p_{40V} = -40 \times i_1 = -40 \times 2 \times 10^{-3} = -80 \text{ mW}
\]
\[
p_{15,000i_s} = 15,000i_s \times i_2 = 15,000(10^{-3})(10^{-3}) = 15 \text{ mW}
\]

60-V voltage source supplies (delivers) 80 mW of power and controlled voltage source absorbs (dissipates) 15 mW of power.
**Problem 2.** This circuit is the model of a power transistor amplifier. a) Find the maximum power available to $R_L$. b) Find value of $R_L$ which maximizes power transfer to $R_L$.

We need to find the Thevenin equivalent of the subcircuit to the left of $R_L$. As the subcircuit contains a controlled source and an independent source, the best parameters to calculate are $i_{sc}$ and $v_{oc}$. The circuits are simple enough to use KVL & KCL:

$i_{sc}$: By KCL, $i_{sc} = 50i_1$. KVL on the outer loop gives:

$$200i_1 - 10 = 0$$
$$i_1 = 0.05 \text{ A} \quad \Rightarrow \quad i_{sc} = 50i_1 = 2.5 \text{ A}$$

$v_{oc}$: By KCL, $i = 50i_1$. KVL on the outer loop gives:

$$200i_1 + 16(50i_1) - 10 = 0 \quad \Rightarrow \quad i_1 = 10 \text{ mA}$$
$$i = 50i_1 = 0.5 \text{ A} \quad \Rightarrow \quad v_{oc} = 16i = 8 \text{ V}$$

Thus: $v_T = v_{oc} = 8 \text{ V}$, $i_N = i_{sc} = 2.5 \text{ A}$, and $R_T = v_T/i_N = 3.2 \Omega$.

The maximum power available for $R_L$ is:

$$P_{\text{max}} = \frac{V_T^2}{4R_T} = \frac{64}{4 \times 3.2} = 5 \text{ W}$$

and $R_L = R_T = 3.2 \Omega$ maximizes power transfer to $R_L$. 
Problem 3. Find $v_o/v_i$.

$v_{p1} = v_i$

Both OpAmps have negative feedback:

$v_{n1} = v_{p1} = v_i$  \hspace{1cm}  $v_{n2} = v_{p2}$

Proceeding with node-voltage method:

KCL at node $v_{n1}$:

$$\frac{v_{n1}}{R} + \frac{v_{n1} - v_{n2}}{kR} = 0$$

$$kv_{n1} + v_{n1} - v_{n2} = 0 \quad \rightarrow \quad v_{n2} = (k + 1)v_{n1} = (k + 1)v_i$$

KCL at node $v_{n2}$:

$$\frac{v_{n2} - v_{n1}}{kR} + \frac{v_{n2} - v_o}{kR} + \frac{v_{n2}}{R} = 0$$

$$v_{n2} - v_{n1} + v_{n2} - v_o + kv_{n2} = 0 \quad \rightarrow \quad v_o = (k + 2)v_{n2} - v_{n1}$$

$$v_o = (k + 2)(k + 1)v_i - v_i$$

$$\frac{v_o}{v_i} = (k + 2)(k + 1) - 1 = k^2 + 3k + 1$$
Problem 4. The switch has been in position A for a long time. At \( t = 0 \),
switch is moved to position B. Find \( v_C(t) \) for \( t > 0 \).

For \( t < 0 \), circuit is in DC steady state
and capacitor acts as an open circuit.

Then, by KVL:

\[
-12 + 10 \times 10^3 i + 10 \times 10^3 i + 10 \times 10^3 i = 0
\]

\[ i = 4 \text{ mA} \quad \rightarrow \quad v_c = 10 \times 10^3 i = 4 \text{ V} \]

No jump condition gives the initial condition for \( t > 0 \) circuit:
\( v_C(t = 0^+) = v_C(t = 0^-) = 4 \text{ V} \).

For \( t > 0 \), we can either reduce the circuit to a series RC circuit or
directly solves the circuit. Here, we solve
the circuit directly with node-voltage method:

\[
\frac{v_C - 12}{10 \times 10^3} + i_C + \frac{v_C}{10 \times 10^3} = 0
\]

\[
v_C - 12 + (10 \times 10^3) \times 10^{-6} \frac{dv_C}{dt} + v_C = 0
\]

\[
10^{-2} \frac{dv_C}{dt} + 2v_C = 12
\]

\[
5 \times 10^{-3} \frac{dv_C}{dt} + v_C = 6
\]

The natural solution for \( v_C \) is \( Ke^{-t/0.005} \) and the forced solution is 6:

\[
v_C(t) = Ke^{-t/0.005} + 6
\]

\[
v_C(t = 0^+) = 4 = K + 6 \quad \rightarrow \quad K = -2
\]

\[
v_C(t) = -2e^{-t/0.005} + 6 \quad (\text{Volt})
\]
Problem 5. Find $v_o(t)$ if the capacitor has zero charge for $t < 0$.

$v_p = 0$

Negative Feedback: $v_n = v_p = 0$

Circuit is simple and can be easily solved by KVL and KCL:

KVL: $R_1 i + v_C = v_i - v_n$

$R_1 C \frac{dv_C}{dt} + v_C = v_i$

The natural solution for $v_C$ is $Ke^{-t/\tau}$ ($\tau = R_1 C$) and the forced solution is $v_i$:

$v_C(t) = Ke^{-t/\tau} + v_i$

$v_C(t = 0^+) = 0 = K + v_i \quad \rightarrow \quad K = -v_i$

$v_C(t) = -v_i e^{-t/\tau} + v_i$

$i_C C \frac{dv_C}{dt} = + \frac{C v_i}{\tau} e^{-t/\tau} = \frac{v_i}{R_1} e^{-t/\tau}$

$i_C R_2 = v_n - v_o \quad \rightarrow \quad v_o = -i_C R_2 = -\frac{R_2}{R_1} v_i e^{-t/\tau}$
**Problem 6.** Show that \( V_o = 0 \) requires \( R_L R_C = L/C \).

We first transform the circuit to frequency domain. Let the voltage at the node on top of the circuit to be \( V_a \). Then:

\[
I_1 = \frac{V_a}{R_C + \left( \frac{1}{j\omega C} \right)}
\]

\[
V_1 = R_C I_1 = \frac{R_C}{R_C + \left( \frac{1}{j\omega C} \right)} V_a
\]

Alternatively, we could have written the expression for \( V_1 \) using voltage divider formula. The same way, we have:

\[
V_2 = \frac{j\omega L}{j\omega L + R_L} V_a
\]

\[
V_o = V_1 - V_2 = 0 \quad \rightarrow \quad V_1 = V_2
\]

\[
\frac{R_C}{R_C + \left( \frac{1}{j\omega C} \right)} V_a = \frac{j\omega L}{j\omega L + R_L} V_a
\]

\( V_a \) cancels out from both side. Inversing both sides, we get:

\[
\frac{R_C + \left( \frac{1}{j\omega C} \right)}{R_C} = \frac{j\omega L + R_L}{j\omega L}
\]

\[
1 + \left( \frac{1}{j\omega C} \right) \frac{R_C}{R_L} = 1 + \frac{R_L}{j\omega L}
\]

\[
\frac{1}{j\omega C R_L} = \frac{R_L}{j\omega L}
\]

\[
R_L R_C = \frac{L}{C}
\]
**Problem 7.** The circuit is driven by \( v_i = \cos(\omega t) \).

For what \( \omega \), we will have \( v_o(t) = -2\cos(\omega t) \).

As the source is a sinusoidal source, we can use phasors. Transforming the circuit to frequency domain, we have \( V_I = 1, V_o = -2, Z_1 = R \) and \( Z_2 = (2R) \parallel (j\omega L) \parallel (1/j\omega C) \).

The circuit is an inverting amplifier. Therefore,

\[
\frac{V_o}{V_i} = -\frac{Z_2}{Z_1}
\]

\[
-2 = -\frac{Z_2}{R} \quad \rightarrow \quad Z_2 = 2R
\]

\[
\frac{1}{Z_2} = \frac{1}{2R} + \frac{1}{j\omega L} + \frac{1}{1/j\omega C}
\]

\[
\frac{1}{2R} = \frac{1}{2R} + \frac{1}{j\omega L} + \frac{1}{1/j\omega C}
\]

\[
\frac{1}{j\omega L} = -\frac{1}{1/j\omega C} = -j\omega C
\]

\[
-j\omega C(j\omega L) = 1 \quad \rightarrow \quad \omega^2 = \frac{1}{LC}
\]

\[
\omega = \frac{1}{\sqrt{LC}}
\]
**Problem 8.** Find $v_o$.

As the sources are both sinusoidal sources and have the same frequency, we can use phasors. Transforming the circuit to frequency domain, we have

$$\omega = 2500$$

$$5 \cos(2500t) \rightarrow 5$$

$$100 \cos(2500t) \rightarrow 100$$

$$8 \text{ mH} \rightarrow j \omega L = j20$$

$$20 \mu \text{F} \rightarrow \frac{1}{j \omega C} = -j20$$

The resulting resistive circuit can be solved by many methods. For example, using nodal voltage method, we have:

$$\frac{V_o}{20 + j20} + \frac{V_o}{20 - j20} + \frac{V_o - 100}{50} - 5 = 0$$

$$V_o \frac{20 - j20}{(20 + j20)(20 - j20)} + V_o \frac{20 + j20}{(20 + j20)(20 - j20)} + 0.02V_o - 2 - 5 = 0$$

$$V_o \frac{20 - j20 + 20 + j20}{800} + 0.02V_o - 2 - 5 = 0$$

$$0.05V_o + 0.02V_o - 7 = 0$$

$$V_o = 100 \text{ V}$$

$$v_o = 100 \cos(2500t) \text{ (Volts)}$$