Example: Find \( I, V \)

\[ \omega = 1000 \]

\( 4C_0, (1000t) \rightarrow 4 \)

0.1 H \( \rightarrow j \omega L = j (1000)(0.1) = j 100 \)

0.2 H \( \rightarrow j 200 \)

0.4 H \( \rightarrow j 400 \)

1.25 \( \mu F \) \( \rightarrow \frac{j}{wc} = \frac{-j}{1000 \times 1.25 \times 10^{-6}} = -j 800 \)

5 \( \mu F \) \( \rightarrow -j 200 \)

KCL:
\[ I = I_1 + I_2 \]

KVL:
\[ I_1 (j 100) + I_2 (j 400 - j 200) - 4 = 0 \]

\[ I_2 (j 200 - j 800) - I_1 (j 400 - j 200) = 0 \]

\[
\begin{cases}
I = I_1 + I_2 \\
(100)I_1 + j 200 I_2 = 4 \\
(j 600) I_2 - j 200 I_1 = 0
\end{cases}
\Rightarrow I_1 = -3 I_2
\]

\[
\begin{cases}
I = -3 I_2 + I_2 = -2 I_2 \\
(100)I_2 + j 600 I_2 = 4
\end{cases}
\Rightarrow -j 200 I_2 - j 600 I_2 = 4
\]

\[ I_2 = \frac{4}{j 800} = \frac{j}{200} = 0.005 j = 0.005 \angle 90 \]

\[ I = -2 I_2 = -0.01 j = 0.01 \angle -90 \]

\[ V = -j 800 I_2 = -j 800 (0.005 j) = 4 = 4 \angle 0 \]

\[ i_r = 0.005 C_0 (1000t + 90^\circ) \]

\[ i = 0.01 C_0 (1000t - 90^\circ) \]

\[ v = 4 C_0 (1000t) \]
Thevenin/Norton Transformation

\[ Z_T = Z_N \]
\[ V_T = Z_N I_N \]

Also

\[ V_T = V_{dc} \]
\[ I_N = I_{dc} \]
\[ Z_T = Z_N = \frac{V_{dc}}{I_{dc}} \]

Example:

\[ \omega = 2 \]
\[ 8 \text{Co. (2t)} \rightarrow 8 \]
\[ \frac{1}{\omega F} \rightarrow \frac{j}{2 \times (\frac{1}{\omega})} = -j2 \]
\[ 2 \times \rightarrow j4 \]
\[ 3 \times \rightarrow j6 \]

\[ Z_1 = 2 + j6 \]

\[ Z_T = 4 + j4 \quad V_T = 16 + j48 \]
\[ Z_N = 4 + j4 \]

\[ I_N = \frac{V_T}{Z_N} = \frac{16 + j48}{4 + j4} \]
\[ = \frac{4 + j12}{1 + j} \times \frac{1 + j}{1 + j} = \frac{4 + j12}{2} \]
\[ = 8 + j4 \]
Method 2:
1. \( V = V_{oc} \), \( I = 0 \)

\[ kC_1 + I_1 = 0 \]

\[ KVL: \quad -I(j^2) + I_1(j^6+j^2) - 2I - V_{oc} = 0 \]

\[ V_{oc} = I_1(j^6+j^2) = 8(j^6+j^2) = 16+j^8 \]

2. \( V = 0 \), \( I = I_{sc} \)

Using current divider formula:

\[ I_{sc} = \frac{V}{j^2} = \frac{I}{Z_{eq}} = \frac{1}{j^2 + 2} \frac{I}{2+1} \]

\[ I_{sc} = \frac{1}{2-j^2 + 2j6} \]

\[ I_{sc} = 8+j4 \]

3. \( Z_T \) (kill all independent source)

\[ Z_T = -j^2 + j6 + 2+2 = 4+j4 \]

Conversion to the domain:

\[ V_T = 16+j5e = 50.60 \angle 71.57^\circ \]

\[ I_{sc} = 8+j4 = 8.94 \angle 26.57^\circ \]

\[ Z_T = 4+j4 = \frac{jwL}{R} \]

A Norton / Thévenin Transformation is frequency dependent.
**Dependent Sources**

**Example:** Find $V_1$

\[ \omega = 2 \]

\[ 2 \cos(2t) \rightarrow 2 \]

\[ \frac{1}{8} \text{F} \rightarrow \frac{-j}{\text{F}} = \frac{-j}{2-\frac{1}{8}} = -j4 \]

\[ 2\mu \rightarrow j\omega L = j(\omega)(\mu) = j^4 \]

**KCL:**

\[ -I + 2V_1 + I_1 = 0 \Rightarrow I_1 = I + 2V_1 \]

**KVL:**

\[ 4I + I_1(\frac{-j4}{4}) + V_1 - 2 = 0 \Rightarrow 4I - \frac{j4}{4}(I + 2V_1) + V_1 = 2 \]

**Ans:**

\[ V_1 = -j4I \]

\[ \begin{cases} I_1 = I + 2V_1 \\ I(4-j4) + V_1(1-j8) = 2 \\ V_1 = -j4I \end{cases} \]

\[ \Rightarrow I(4-j4 + j4(1-j8)) = 2 \Rightarrow I = \frac{2}{36} \text{ A} \]

\[ V_1 = j4I = \frac{j8}{36} \text{ A} \]

\[ \begin{cases} I = 0.0566 \text{ A} = 56.6^\circ \text{ mA} \\ V_1 = j0.224 = 0.224^\circ \text{ V} \end{cases} \]

\[ i(t) = 56.6 \cos(2t) \]

\[ v(t) = 0.224 \cos(2t + 90^\circ) = -0.224 \sin(2t) \]
SUPERPOSITION

If all sources have the same frequency, superposition is an alternative to other analysis methods.

If a circuit contains sources with different frequencies, superposition is the only applicable method.

Method:
1. Write the response in terms of circuit response to sources of the same frequency (kill other sources).
2. Solve each circuit in the frequency domain.
3. Transfer the solution to the time domain.
4. Sum over all sources.

Example: Find $v_2$

1. Superposition in the time domain

2. Solve circuits 1 and 2 in frequency domain

\[ w = 1 \]

\[ \frac{1}{\omega C} \rightarrow -\frac{j}{\omega C} = -\frac{j}{\frac{1}{2\pi}} = -j4 \]

\[ v_1 = -I_1 (j4) = -j4 \quad \frac{1}{2-j4} = \frac{4}{25} (4-j3) \]
\[ V_1 = \frac{4}{25} (4-j3) = 0.8 \angle -33^\circ \]

\[ v_1(t) = 0.8 \cos(t-37^\circ) \]

Circuit #2

\[ \omega = 3 \]

\[ 2C_0(3t+45^\circ) \rightarrow 2 e^{j\frac{\sqrt{2}}{2} + j\frac{\sqrt{2}}{2}} \]

\[ \frac{1}{4} I \rightarrow \frac{1}{j \Omega} = -\frac{j}{2nV_0} = -j\frac{\sqrt{3}}{3} \]

\[ V_2 = \frac{I_2}{j} \left( -j\frac{4}{3} \right) = \left( -j \frac{4}{3} \right) \frac{\sqrt{2} + j\sqrt{2}}{3 - j\sqrt{3}} \]

\[ V_2 = \frac{-j9((\sqrt{2} + j\sqrt{2})}{9-j4} = \frac{4\sqrt{2} \left( \frac{1 \cos(45^\circ) - j \sin(45^\circ)}{\sqrt{2}} \right)}{9-j4} = 0.8 \frac{\sqrt{2} \angle -45^\circ}{\sqrt{7} \angle -21^\circ} = \frac{8}{9.85} \angle -45+24^\circ \]

\[ V_2 = 0.81 \angle -21^\circ \]

\[ v_2 = 0.81 \ C_0(3t-21^\circ) \]

(1) Add in the domain.

\[ v(t) = v_1(t) + v_2(t) = 0.8 \ C_0(t-37^\circ) + 0.8 \ C_0(3t-21^\circ) \]

Notes:
- You do not need to divide the circuit into sub circuits each containing only one source. Divide the circuit into sub circuits containing sources of same frequency (or DC, i.e. 0 Hz). Then solve each circuit with an appropriate analysis method in the frequency domain.
- Always add responses in the time domain & not the frequency domain.
Maximum Power Transfer

\[ Z_s = R_s + jX_s \]

\[ Z = R + jX \]

\[ I = \frac{V}{R + X} \]

\[ P = \frac{1}{2} R I_m^2 = \frac{R}{2} \frac{V_m^2}{1 + (X/X_s)^2} = \frac{V_m^2}{2} \frac{R}{[\sigma + R_s + \omega X_s]^2} \]

To find \( R \) and \( X \) values which maximize \( P \), set \( \frac{\partial P}{\partial R} = 0 \), \( \frac{\partial P}{\partial X} = 0 \)

\[ \frac{\partial P}{\partial R} = \frac{V_m^2 R}{2} \frac{-2(R + X_s)}{[(R + R_s)^2 + (X + X_s)^2]^2} = 0 \Rightarrow X = -X_s \]

\[ \frac{\partial P}{\partial R} = \frac{V_m^2 R}{2} \frac{(R + R_s)^2 + (X + X_s)^2 - 2R(R + R_s)}{[(R + R_s)^2 + (X + X_s)^2]^2} = 0 \]

\[ \Rightarrow (R + R_s)^2 + (X + X_s)^2 - 2R(R + R_s) = 0 \Rightarrow R = R_s \]

Maximum Power Transfer \[ \begin{cases} R = R_s \\ X = -X_s \\ Z = \sqrt{R_s} \end{cases} \]

Maximum Available Power \[ P_{max} = \frac{V_m^2}{2} \frac{R_s}{(2R_s)^2} = \frac{V_m^2}{8R_s} = \frac{V_m^2}{4R_s} \]